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NATIONAL GEOSPATIAL-INTELLIGENCE AGENCY
STANDARDIZATION DOCUMENT
Implementation Practice

The Universal Grids and the Transverse Mercator and Polar Stereographic Map Projections

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OFFICE OF GEOMATICS

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(Revision of DMA Technical Manual 8358.2 dated 18 September 1989)

The Universal Grids and the Transverse Mercator and Polar Stereographic Map Projections

March 25, 2014

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a_2, a_4...	Coefficients in the series for forward transverse Mercator	3.2
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b	Semi-minor axis of a reference ellipsoid	2.1
b_2, b_4...	Coefficients in the series for inverse transverse Mercator	3.5
B2, B4...	Coefficients in the series for inverse transverse Mercator	4 intro
deg	Size of one degree, in radians	1.6
f	Flattening of the reference ellipsoid	2.1
f^{-1}	Inverse flattening; reciprocal flattening	2.1
f_1, f_2 ...	(various functions for forward mapping equations)	3.2, 6.3, 8.1
g_1, g_2 ...	(various functions for inverse mapping equations)	3.5, 8.2
k_0	Scale factor mandated for the central meridian or for the Pole	5.1, 9.1
P	(an intermediate variable for some latitude conversions)	2.8
P_n	(an intermediate variable for some latitude conversions)	2.9
R_4	Meridional isoperimetric radius	3.2
u	(an intermediate variable for basic transverse Mercator)	3.2
v	(an intermediate variable for basic transverse Mercator)	3.2
w	(an intermediate variable depending on ϕ)	2.2
x	Map projection plane abscissa; distance on the horizontal axis; Easting	2.1, 5.1, 8.1...
X	First of three Cartesian coordinates for 3D Euclidean space	2.1
y	Map projection plane ordinate; distance on the vertical axis; Northing	2.1, 5.1, 8.1...
Y	Second of three Cartesian coordinates for 3D Euclidean space	2.1
Z	Third of three Cartesian coordinates for 3D Euclidean space	2.1
γ	Convergence-of-meridians angle; grid declination	6.3, 8.1
e	(First) eccentricity of the reference ellipsoid	2.1
λ	Longitude	2.2
λ_0	Longitude of the central meridian	5.1, 9.1
π	Pi, the ratio of a circle's circumference to its diameter	1.6
σ	Local scale (factor)	6.3, 8.1
ϕ	Geodetic latitude; latitude	2.2
χ	Conformal latitude	2.4
ψ	Geocentric latitude	2.4

1. General

1.1 Introduction

Earth features are commonly referenced by geographic coordinates — longitude and latitude. However, these coordinates are not suitable in all situations to report positions or to calculate distances or directions. To perform these functions conveniently, grids and grid coordinate systems have been invented. A national grid is devised by a national authority and covers a single country (or part of it). The universal grids, Universal Transverse Mercator (UTM) and Universal Polar Stereographic (UPS), were devised by the U.S. Department of Defense (DoD) and taken together cover the whole Earth. The Military Grid Reference System (MGRS) is the pair, UTM and UPS, after some reformatting (*e.g.* lettering) is applied to each.

1.2 Purpose and scope

This document defines the UTM, UPS and MGRS systems of coordinates and provides *some* information toward their understanding and use in surveying, cartography, and geographic-information analysis.

Mainly, though, this document provides guidance to DoD and DoD contractors for the software implementation of algorithms to convert between longitude/latitude, UTM or UPS, and MGRS coordinates. As a necessary step toward that end, this document provides guidance for the software implementation of the transverse Mercator and polar stereographic map projections. These map projections are endowed with parameters for general utility, of which UTM and UPS are particular instances.

It should be noted that the previous edition, [3], had these same purposes: to define UTM and to provide the formulas for its implementation in software. Moreover, this should be accomplished without partiality to a particular programming language or software environment. Existing software, even if it were open source and government provided (*e.g.* GeoTrans) and most modern and up-to-date, would not be a substitute for this document. Management of specific DoD procurements is outside the scope of this document. Likewise also are the policies and procedures for quality assurance of these procurements. Yet, a general recommendation can be stated: if the above conversions are to be implemented anew, or if existing software is to be modified (for the benefits below or for other reasons), then this document should be used to direct the development or redevelopment. This will yield the benefits explained below under "What's new".

A companion to this document is *NGA Standardization Document NGA.STND.0037_2.0_GRIDS*, "Universal Grids and Grid Reference Systems" [11]. DoD mapping and charting production elements should refer to it for guidance on the proper depiction of UTM and UPS grids and MGRS labels on standard products.

Although some explanations are offered in defense of what is new, this document is not designed as a tutorial. It is recommended to consult the map projection literature for the meaning and usefulness of grid coordinates in general and UTM, UPS and MGRS coordinates in particular.

1.3 Previous edition

This document replaces technical manual DMA TM8358.2 Edition 1, "The Universal Grids: Universal Transverse Mercator (UTM) and Universal Polar Stereographic (UPS)", dated 18 September, 1989. Chapters 1-4 of the 1989 technical manual are superseded by this document. Chapter 5 dealt with datum transformations, which is a separate topic and is not included in this document. Datum transformations are included in Edition 3 and Edition 4 (in preparation) of [12].

1.4 What's new

The transverse Mercator map projection formulas in Section 3 are new, as explained in Subsections 5.6 and 5.7. The new formulas provide improved efficiency and expanded coverage of the ellipsoid. Using them, the software is shorter and simpler to write, and, by implication, less likely to have bugs.

New to this document are several sections on MGRS (Sections 11, 12, and 13). The one-dimensional tables in Subsections 11.2 and 11.3 offer simpler logic for grid-square lettering than the traditional two-dimensional tables in [2], but produce the same result. Some secondary matters concerning MGRS, namely non-WGS-84 lettering (Subsection 11.4) and latitude-band-letter leniency (Subsection 12.9), have remained ambiguous (not standardized) for years. This is corrected here for the first time in a DMA, NIMA, or NGA document. "MGRS Quick-start" (Section 13) may be read after reading Sections 1, 2, and 3. Because it is so close to MGRS, there is a brief section (Section 14) on the U.S. National Grid (USNG).

This document advocates layering of software modules, so that, for example, MGRS is a layer over UTM; UTM is a layer over transverse Mercator with parameters; and the latter is a layer over basic transverse Mercator. Then, within each of UTM, UPS and MGRS, some rules are described as "administrative rules" (*e.g.* Subsection 7.4). These are usage oriented and not required by the theory. The recommendation is to not bundle these with the theory-required formulas and logic, but make them a separate layer.

As a help to developers of geographic metadata formats and as a furtherance of general functionality, map projection parameters for the transverse Mercator and polar stereographic projections are discussed in detail in Sections 5 and 9. This yields software that is capable of both grid calculations and general cartography (map-sheet design) — a boon to the desired consistency between these capabilities.

It is hoped that the plots and diagrams in Section 15 (all newly produced) will be useful to many. They illustrate the principles in this document.

1.5 What's old

The new formulas for transverse Mercator and UTM are consistent with the previous edition formulas where they overlap. MGRS-needed UTM calculations, for example, are unchanged.

1.6 Meters, radians, pi

All lengths and distances in this document are given in meters. Readers interested in English units should be aware that the international foot and the U.S. survey foot are slightly different. For both, a foot is 12 inches. For the U.S. survey foot, one meter equals 39.37 (U.S. survey) inches exactly; for the international foot, one (international) inch equals 2.54 centimeters exactly.

All angles occurring in the formulas are assumed to be in radians. One radian equals $\frac{180}{\pi}$ degrees and one degree equals $\frac{\pi}{180}$ radians. When it is convenient to refer to an angle by its degree-equivalent, the notation "deg" is used as a multiplier. Its value is $\text{deg} = \frac{\pi}{180}$. For example, $\lambda = 23\ \text{deg} = \frac{23\,\pi}{180}$. An angle occurring in a numerical table will be in degrees, if its column heading includes the notation "(deg)".

If the programming language does not have a built-in function for π, the developer may establish a value for it with a statement like `pi = 4 * atan (1)` taking the benefit of the arc-tangent function, which might be spelled "atan". This statement provides all the digits for π within the chosen arithmetic precision type — single, double, or other type.

1.7 Inverse trigonometric functions

The (circular) trigonometric functions cosine (cos), sine (sin) and tangent (tan) take a single argument in radians. Their inverses are defined:

$$\begin{aligned}
\arccos(\cos\theta) &= \theta, \quad \text{if } 0 \le \theta \le \pi \\
\arcsin(\sin\theta) &= \theta, \quad \text{if } \frac{-\pi}{2} \le \theta \le \frac{\pi}{2} \\
\arctan(\tan\theta) &= \theta, \quad \text{if } \frac{-\pi}{2} < \theta < \frac{\pi}{2}
\end{aligned}$$

The following function is needed because some angles have values in all four quadrants and because the determination of a first-quadrant angle is numerically more robust if its cosine and sine are given than if its tangent is given. It is called the two-argument version of arc-tangent and satisfies these identities:

$$\begin{aligned}
&\arctan(\cos\theta,\ \sin\theta) = \theta, && \text{if } -\pi < \theta \le \pi \\
&\arctan(a\,x,\ a\,y) = \arctan(x,\ y), && \text{if } x \text{ and } y \text{ are any real numbers and } a > 0 \\
&\arctan(x,\ y) = \arctan\left(\frac{y}{x}\right), && \text{if } x > 0 \text{ and } y \text{ is any real number}
\end{aligned} \tag{1}$$

The order of the arguments for arctan as they appear in Eq. (1.1) and in [18] might be called "x before y". The other convention might be called "numerator before denominator" and is the convention used in Fortran and C, where the two-argument version of arc-tangent function is spelled "atan2". Its relationship to arctan in this document is:

$$\arctan(x,\ y) = \operatorname{atan2}(y,\ x)$$

Some computer languages might not have the inverse hyperbolic tangent. It is:

$$\operatorname{arctanh}(x) = \frac{1}{2}\operatorname{Ln}\left(\frac{1+x}{1-x}\right)$$

where Ln is the natural logarithm function, that is, logarithms to the base 2.71828 ...

1.8 Sign, Floor, Round

Signum (sign) is the function that returns $+1$ if the argument is positive, -1 if the argument is negative, and 0 if the argument is zero.

Floor is the function that returns the greatest integer less than or equal to the given number. Some examples are: $\operatorname{Floor}(1.1) = 1$, $\operatorname{Floor}(1) = 1$, $\operatorname{Floor}(-1) = -1$, and $\operatorname{Floor}(-1.1) = -2$

Round is the function that returns the integer nearest to the given number, with half-integers rounded up. It can also be defined:

$$\operatorname{Round}(x) = \operatorname{Floor}\left(x + \frac{1}{2}\right)$$

2. Reference Ellipsoid

Essential for the construction of the universal grids are a reference ellipsoid and the concepts of longitude and latitude, which are based upon it. These and related matters are discussed in this section.

2.1 The reference ellipsoid

In this document, the Earth is represented by a reference ellipsoid, defined as a surface whose points' three-dimensional Cartesian coordinates $\{X, Y, Z\}$ satisfy the equation:

$$\frac{X^2}{a^2} + \frac{Y^2}{a^2} + \frac{Z^2}{b^2} = 1 \tag{2}$$

where a and b are constants called the semi-major and semi-minor axes, respectively. It is required that $a > b$. The quantities a and b determine the flattening, f, and the eccentricity-squared, e^2, as follows:

$$f = \frac{a - b}{a} = 1 - \frac{b}{a}$$

$$e^2 = \frac{a^2 - b^2}{a^2} = 1 - \left(\frac{b}{a}\right)^2$$

The flattening and the eccentricity-squared are inter-convertible as follows:

$$e^2 = f\,(2 - f)$$

$$f = \frac{e^2}{1 + \sqrt{1 - e^2}}$$

Instead of the pair $\{a, b\}$ as the defining parameters, the reference ellipsoid can be defined by $\{a, f\}$, $\{a, f^{-1}\}$, $\{a, e\}$, or $\{a, e^2\}$ in which case b is given by either of these equations:

$$b = a\,(1 - f)$$

$$b = a\sqrt{1 - e^2}$$

The reference ellipsoid is a mathematical idealization. How it is attached to the physical Earth is outside the scope of this document. For a treatment of this topic in general, see the geodetic literature. For its part in the establishment of some modern terrestrial reference systems see [12] and [13].

2.2 Longitude λ and geodetic latitude ϕ

As stated above, a point in space lies on the reference ellipsoid if its coordinates $\{X, Y, Z\}$ satisfy Eq. (2.2). Equivalently, a point in space lies on the reference ellipsoid if its coordinates $\{X, Y, Z\}$ can be generated by the following formulas:

$$\begin{aligned} X &= \frac{a}{w}(\cos\phi)(\cos\lambda) \\ Y &= \frac{a}{w}(\cos\phi)(\sin\lambda) \\ Z &= \frac{a\left(1 - e^2\right)}{w}(\sin\phi) \end{aligned} \tag{3}$$

where

$$w = \sqrt{1 - e^2 \sin^2\phi} \tag{4}$$

and λ and ϕ are any two real numbers. The quantity λ, which is longitude in radians, can be restricted to any interval of length 2π such as $-\pi < \lambda \le \pi$. The quantity ϕ, which is geodetic latitude in radians, should be restricted to the interval $-\pi/2 \le \phi \le \pi/2$.

In this section, the term for ϕ is "geodetic latitude", to distinguish it from other quantities that are 0° at the equator and ±90° at the Poles (see Subsection 2.4). After this section and in keeping with standard usage in geography and cartography, geodetic latitude is shortened to “latitude”.

2.3 Ellipsoid numerical example

The International ellipsoid (1924) is defined by $a = 6\,378\,388$ meters and $f^{-1} = 297.000000$. Using the formulas in Subsection 2.1, the other parts of this ellipsoid are:

Name	name	International 1924
NGA two-letter code	twolet	IN
inverse flattening	1/f	297.000000000000000
flattening	f	0.00336700336700336700
eccentricity-squared	e^2	0.00672267002233332200
eccentricity	e	0.0819918899790297674
semi-major axis	a	6378388.00000000000
semi-minor axis	b	6356911.94612794613

A particular point on the International ellipsoid has longitude $\lambda = 23\text{ deg} = \frac{23\pi}{180}$ and geodetic latitude $\phi = 47\text{ deg} = \frac{47\pi}{180}$. Using Eqs. (2.3 and 2.4), the Cartesian coordinates $\{X,\ Y,\ Z\}$ of the particular point are:

$X = 4\,011\,461.001914537$

$Y = 1\,702\,764.171519670$

$Z = 4\,641\,850.497100156$

2.4 Geocentric latitude ψ and conformal latitude χ

As stated above, each point on a reference ellipsoid has a longitude λ and geodetic latitude ϕ. These quantities are sufficient to locate the point without ambiguity. Other quantities needed in this document are the geocentric latitude ψ and the conformal latitude χ, whose dependencies on ϕ are given by:

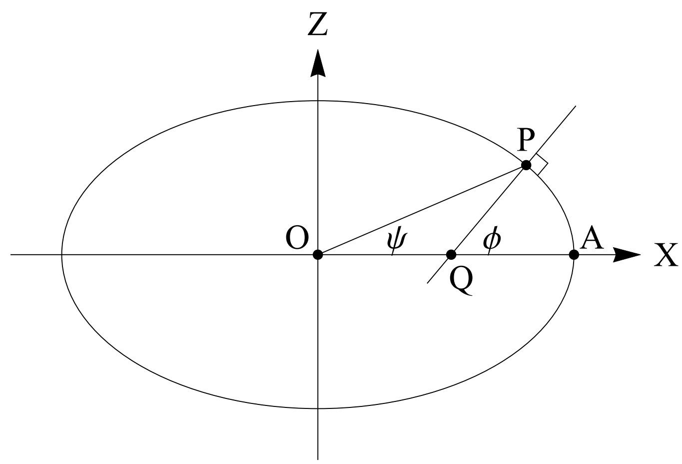
$$\tan\psi = \left(1 - e^2\right)\tan\phi \tag{5}$$

$$\operatorname{arctanh}(\sin\chi) = \operatorname{arctanh}(\sin\phi) - e\,\operatorname{arctanh}(e\sin\phi) \tag{6}$$

At the Equator, $\phi = \psi = \chi = 0$, and at the north Pole, $\phi = \psi = \chi = 90\text{ deg}$. For the southern hemisphere, changing $\phi \to (-\phi)$ implies $\psi \to (-\psi)$ and $\chi \to (-\chi)$. The recommended steps for converting between ϕ and χ are given in Subsections 2.8 and 2.9.

2.5 Illustration of ϕ and ψ

The following illustrates the concepts of reference ellipsoid, geodetic latitude ϕ and geocentric latitude ψ. The reference ellipsoid (with greatly exaggerated flattening) is shown by its intersection with the *XZ* plane, *i.e.* the plane of the prime meridian ($\lambda = 0$). Point *P* is on the prime meridian. The line *PQ* is perpendicular to the ellipsoid at *P*. Then $\phi = \angle PQA$ is the geodetic latitude of *P* and $\psi = \angle POA$ is the geocentric latitude of *P*.



2.6 Given ϕ, compute ψ

This subsection gives the formulas to convert geodetic latitude ϕ to geocentric latitude ψ.

Eq. (2.5) succinctly states the relationship between ϕ and ψ, but a computational algorithm is given by:

$$\psi = \frac{\pi}{2} - \arctan\left(\frac{\cot\phi}{1-e^2}\right), \qquad \text{if} \quad \phi > \frac{\pi}{4}$$

$$\psi = \arctan\left(\left(1-e^2\right)\tan\phi\right), \qquad \text{if} \quad \frac{-\pi}{4} \le \phi \le \frac{\pi}{4}$$

$$\psi = \frac{-\pi}{2} - \arctan\left(\frac{\cot\phi}{1-e^2}\right), \qquad \text{if} \quad \phi < \frac{-\pi}{4}$$

where arctan is the inverse tangent function and cot is the cotangent function. The latter is defined $\cot\phi = \tan\left(\frac{\pi}{2} - \phi\right)$ for the occasion that it is not available in the programming language. The choice of endpoint $\frac{\pi}{4} = 45\,\text{deg}$ and its negative for the above intervals of ϕ is mostly arbitrary; other choices such as 50 deg and 1 radian would work just as well.

Let the function defined by the above formulas be given the name "PhiToPsi" so that the above is equivalent to:

$$\psi = \text{PhiToPsi}\,(\phi)$$

2.7 Given ψ, compute ϕ

This subsection gives the formulas to convert geocentric latitude ψ to geodetic latitude ϕ.

$$\phi = \frac{\pi}{2} - \arctan\left(\left(1-e^2\right)\cot\psi\right), \qquad \text{if} \quad \psi > \frac{\pi}{4}$$

$$\phi = \arctan\left(\frac{\tan\psi}{1-e^2}\right), \qquad \text{if} \quad \frac{-\pi}{4} \le \psi \le \frac{\pi}{4}$$

$$\phi = \frac{-\pi}{2} - \arctan\left(\left(1-e^2\right)\cot\psi\right), \qquad \text{if} \quad \psi < \frac{-\pi}{4}$$

Let the function defined by the above formulas be given the name "PsiToPhi" so that the above is equivalent to:

$$\phi = \text{PsiToPhi}\,(\psi)$$

See comments in the previous subsection.

2.8 Given ϕ, compute {cos χ, sin χ}

Eq. (2.6) succinctly states the relationship between ϕ and χ, but the need for χ later in this document is only through its cosine and sine. Therefore, the conversion from ϕ to χ as needed in this document is the following:

$$\cos\chi = \frac{2\cos\phi}{(1+\sin\phi)/P + (1-\sin\phi)\,P}$$

$$\sin\chi = \frac{(1+\sin\phi)/P - (1-\sin\phi)\,P}{(1+\sin\phi)/P + (1-\sin\phi)\,P}$$

where

$$P = \exp\left(e \operatorname{arctanh}\left(e\sin\phi\right)\right) = \left(\frac{1+e\sin\phi}{1-e\sin\phi}\right)^{e/2}$$

See Subsection 1.7 for the definition of arctanh. Of the two formulas given for P, the one using arctanh is preferred. Let the function defined by the above formulas be given the name "PhiToChi" so that the above may be summarily written:

$$(\cos\chi,\, \sin\chi) = \text{PhiToChi}\,(\phi)$$

2.9 Given {cos χ, sin χ}, compute ϕ

The procedure to compute geodetic latitude ϕ given the cosine and sine of the conformal latitude χ is the following:

$$\phi = \arctan(\cos\phi,\ \sin\phi)$$

where cos ϕ is computed from sin ϕ and P by:

$$\cos\phi = \left(\frac{(1+\sin\phi)/P + (1-\sin\phi)\,P}{2}\right)\cos\chi$$

where sin ϕ is the limit (within desired resolution) of s_1, s_2, s_3, ... and P is the corresponding limit of P_1, P_2, P_3, ...

and where:

$$s_1 = \sin\chi$$

$$s_{n+1} = \frac{(1+\sin\chi)\,{P_n}^2 - (1-\sin\chi)}{(1+\sin\chi)\,{P_n}^2 + (1-\sin\chi)}$$

$$P_n = \exp\left(e \operatorname{arctanh}(e\,s_n)\right) = \left(\frac{1+e\,s_n}{1-e\,s_n}\right)^{e/2}$$

Of the two formulas given for P_n, the one using arctanh is preferred. Let the function defined by the above formulas be given the name "ChiToPhi" so that the above conversion is written:

$$\phi = \text{ChiToPhi}(\cos\chi,\ \sin\chi)$$

2.10 Using ψ as a substitute for χ

The difference between ψ and χ is small and $\phi \leftrightarrow \psi$ conversions are faster than $\phi \leftrightarrow \chi$ conversions. Software developers could substitute ψ for χ in situations that require extreme performance and loose accuracy. Numerical investigation of the loss of accuracy would be an obligation of the developer, but here is some initial guidance:

For an ellipsoid no flatter than the ellipsoids in Section 4, the worst case occurs for the Clark 1880 ellipsoid at $\phi = \pm 60.1184$ deg where $|\chi - \psi|$ reaches a maximum of 0.5207 arc-seconds. An error in χ of some amount under one second (*e.g.* because the formula for ψ is used instead) propagates to an error in ϕ of roughly the same amount.

3. Basic Transverse Mercator

One of the universal grids, namely Universal Transverse Mercator (UTM), is based on the transverse Mercator map projection. This section gives the formulas for transverse Mercator in its basic form. Later in Section 5, various parameters such as central meridian and central scale factor will be introduced. They will enable transverse Mercator to be offered in its commonly-used general form.

The theory of map projections and the theory of conformal mapping between surfaces are outside the scope of this document. However, one idea from these theories is presented. The formulas for transverse Mercator will be new, and the theoretical definition of transverse Mercator in Subsection 3.1 is appropriate as a bridge between old, *e.g.* [16], and new.

In this section, any constant dependent on a reference ellipsoid will have the value pertaining to the WGS 84 ellipsoid. Transverse Mercator for other reference ellipsoids is given in Section 4.

3.1 Definition of transverse Mercator

Transverse Mercator in its basic form is defined by the following requirements:

- Requirement 1: The prime meridian, *i.e.* the meridian at longitude $\lambda = 0$, is portrayed on the $\{x, y\}$ plane of the map projection as a segment of the vertical line $x = 0$.
- Requirement 2: The point of intersection of the prime meridian with the Equator corresponds to the point $\{x, y\} = \{0, 0\}$ on the map projection plane.
- Requirement 3: If two points lie on the prime meridian, the distance between them on the map projection plane will be the same as the length of meridional arc joining them on the reference ellipsoid. In other words, “distance is preserved” (on the prime meridian).
- Requirement 4: The map projection is conformal

It is notable that the only requirement dealing with points *not* on the prime meridian is Requirement 4. After the prime meridian’s points are properly placed, Requirement 4 is enough to determine the map projection’s placement of all other points.

For readers who are familiar with transverse Mercator or who have looked ahead to Section 5, it can be stated that the parameter choices implied by the above definition are (i) a central meridian of longitude 0 deg, (ii) a central scale factor of 1.0000, (iii) an “Origin” point given as longitude 0 deg and latitude 0 deg, and (iv) a False Easting and a False Northing of 0 mE and 0 mN, respectively, assigned to that origin. This is the basic form of transverse Mercator.

The formulas for transverse Mercator to follow are new (in a sense to be explained), but they adhere to the above definition, which is not new (in effect). The above definition is implicit in the map projection literature, and both old and new formulas are based upon it. A discussion of the relationship of this document to other authorities on transverse Mercator must await the conclusion of Section 5.

3.2 Given $\{\lambda, \phi\}$, compute $\{x, y\}$

This subsection gives the forward mapping equations for the basic form of the transverse Mercator projection. Given the longitude λ and latitude ϕ of a point on the reference ellipsoid, the functions f_1 and f_2, specified below, produce the easting $x = f_1(\lambda, \phi)$ and northing $y = f_2(\lambda, \phi)$ of the corresponding point on the map projection plane. They satisfy the requirements of Subsection 3.1.

$$\begin{aligned} x &= f_1(\lambda, \phi) \\ &= R_4\,(u + a_2 \sinh(2u)\cos(2v) + a_4 \sinh(4u)\cos(4v) + \ldots + a_{12} \sinh(12u)\cos(12v)) \\ y &= f_2(\lambda, \phi) \\ &= R_4\,(v + a_2 \cosh(2u)\sin(2v) + a_4 \cosh(4u)\sin(4v) + \ldots + a_{12} \cosh(12u)\sin(12v)) \end{aligned} \tag{7}$$

where cosh and sinh are the hyperbolic cosine and hyperbolic sine, respectively, and R_4 and a_2, a_4, a_6, a_8, a_{10} and a_{12} are constants, and where u and v are determined by:

$$\begin{aligned} u &= \operatorname{arctanh}((\cos\chi)(\sin\lambda)) \\ v &= \arctan((\cos\chi)(\cos\lambda),\ \sin\chi) \end{aligned} \tag{8}$$

and cos χ and sin χ are computed according to Subsection 2.8, *i.e.* the function PhiToChi is applied:

$$(\cos\chi,\ \sin\chi) = \text{PhiToChi}(\phi)$$

For the WGS 84 ellipsoid ($a = 6\,378\,137$, $f^{-1} = 298.257223563$), the numerical values of the constants are:

$$\begin{aligned}
R_4 &= 6\,367\,449.1458234153093 && \text{meters}\\
a_2 &= 8.3773182062446983032\text{ E }{-04} && \text{(unitless)}\\
a_4 &= 7.608527773572489156\text{ E }{-07} && \text{(unitless)}\\
a_6 &= 1.19764550324249210\text{ E }{-09} && \text{(unitless)}\\
a_8 &= 2.4291706803973131\text{ E }{-12} && \text{(unitless)}\\
a_{10} &= 5.711818369154105\text{ E }{-15} && \text{(unitless)}\\
a_{12} &= 1.47999802705262\text{ E }{-17} && \text{(unitless)}
\end{aligned} \tag{9}$$

The quantity R_4 has a name — the meridional isoperimetric radius. It is the radius of a semicircle having the same arclength as a meridian. Its notation, R_4, was chosen after seeing that notations R_1, R_2, R_3 were adopted by [10] and [12] for the tri-axial arithmetic-mean radius, the authalic radius, and the isovolumetric radius, respectively.

3.3 Notes to the developer

The previous subsection is complete for the mathematics of the forward mapping equations of the basic form of transverse Mercator. This subsection offers additional information that might be helpful.

A series of numbers should be added from small (absolute values) to large, so as not to risk losing the full contribution of the small numbers to the sum. Therefore, the series for x in Eq. (3.7) should begin with the last term and add each preceding term in turn. Likewise for the series for y.

Simplicity of computer code and high performance of computer code are competing requirements for algorithm design; it is usually not possible to achieve both. This document leans toward the former, but not exclusively, and the following improvement for performance (speed) might be of interest to some developers. Toward the numerical outcome required by Eq. (3.7), after $\cos(2\,v)$ and $\sin(2\,v)$ have been computed, the remaining multiple-angle sines and cosines can be computed by:

$$\begin{aligned}
\cos(4\,v) &= 2\cos^2(2\,v) - 1\\
\sin(4\,v) &= 2\cos(2\,v)\sin(2\,v)\\
\cos(6\,v) &= \cos(4\,v)\cos(2\,v) - \sin(4\,v)\sin(2\,v)\\
\sin(6\,v) &= \cos(4\,v)\sin(2\,v) + \cos(2\,v)\sin(4\,v)
\end{aligned} \tag{10}$$

and the pattern continues with:

$$\begin{aligned}
\cos(8\,v) &= 2\cos^2(4\,v) - 1\\
\sin(8\,v) &= 2\cos(4\,v)\sin(4\,v)\\
\cos(10\,v) &= \cos(8\,v)\cos(2\,v) - \sin(8\,v)\sin(2\,v)\\
\sin(10\,v) &= \cos(8\,v)\sin(2\,v) + \cos(2\,v)\sin(8\,v)\\
\cos(12\,v) &= 2\cos^2(6\,v) - 1\\
\sin(12\,v) &= 2\cos(6\,v)\sin(6\,v)
\end{aligned} \tag{11}$$

For the hyperbolic functions, the formulas are:

$$\begin{aligned}
\cosh(4\,u) &= 2\cosh^2(2\,u) - 1\\
\sinh(4\,u) &= 2\cosh(2\,u)\sinh(2\,u)\\
\cosh(6\,u) &= \cosh(2\,u)\cosh(4\,u) + \sinh(2\,u)\sinh(4\,u)\\
\sinh(6\,u) &= \cosh(4\,u)\sinh(2\,u) + \cosh(2\,u)\sinh(4\,u)
\end{aligned} \tag{12}$$

and the pattern continues with:

$$\begin{aligned}
&\cosh(8\,u) = 2\cosh^2(4\,u) - 1\\
&\sinh(8\,u) = 2\cosh(4\,u)\sinh(4\,u)\\
&\cosh(10\,u) = \cosh(2\,u)\cosh(8\,u) + \sinh(2\,u)\sinh(8\,u)\\
&\sinh(10\,u) = \cosh(8\,u)\sinh(2\,u) + \cosh(2\,u)\sinh(8\,u)\\
&\cosh(12\,u) = 2\cosh^2(6\,u) - 1\\
&\sinh(12\,u) = 2\cosh(6\,u)\sinh(6\,u)
\end{aligned} \tag{13}$$

It is in the nature of these mathematical functions that Eqs. (3.10 and 3.12) look so much alike as do Eqs. (3.11 and 3.13). A careful look at the formulas for $\cos(6\,v)$, and $\cosh(6\,u)$ will reveal that they are not totally alike. The above is correct, despite looking like there is a mistake in sign.

Eq. (3.7) as written above implies 24 calls to trigonometric functions (circular or hyperbolic). With the use of Eqs. (3.10 through 3.13), this is reduced to merely four calls — $\cos(2\,v)$, $\sin(2\,v)$, $\cosh(2\,u)$ and $\sinh(2\,u)$. The time for the extra additions and multiplications is minuscule compared to the performance savings of fewer calls to trigonometric functions. The extra effort to use Eqs. (3.10 through 3.13) will not suit the needs of all software developers.

It may be argued that for practical applications of transverse Mercator and UTM, Eq. (3.9) contains an excessive number of digits. However, developers are encouraged to cut and paste the numbers as given into their code. The computer memory locations must be filled somehow; the extra digits cause no performance degradation; and they are not entirely inconsequential in software-testing.

The transverse Mercator projection is symmetric about the Equator and about the prime meridian. These symmetries are contained in Eqs. (3.7, 3.8, and 3.9), which therefore apply to all four quadrants, not merely to $\lambda > 0$ with $\phi > 0$. There is no need for additional code to convert points in other quadrants. Additionally and likewise, Eqs. (3.7, 3.8, and 3.9) get correct the (lesser known) symmetry about the meridians $\lambda = \pm 90$ deg in the polar regions.

Lastly, some developers might be interested in a trade-off between accuracy and speed. Eqs. (3.10 to 3.13) were an attempt to meet the developer's need for speed. They do so without loss of accuracy. If that effort is insufficient, it is admitted that fewer terms of Eq. (3.7) would be possible under a more lax accuracy requirement (Subsection 3.9) or a more restricted reference-ellipsoid coverage requirement (Subsection 3.7), or both.

3.4 Forward mapping: a numerical example

Let $\{\lambda, \phi\} = \{-10 \text{ deg}, 3 \text{ deg}\}$ define a point on the WGS 84 ellipsoid. Then $\cos\chi = 0.998647785036631316$ and $\sin\chi = 0.0519865505821477812$ by Subsection 2.8. Then $u = -0.175183729646051084$ and $v = 0.0528108539283539197$ by Eq. (3.8). Finally, $x = -1\,117\,373.87527102019$ and $y = 336\,868.939627688401$ by Eq. (3.7).

3.5 Given $\{x, y\}$, compute $\{\lambda, \phi\}$

This subsection gives the inverse mapping equations for the basic form of the transverse Mercator projection. Given the easting x and northing y of a point on the map projection plane, the functions g_1 and g_2, specified below, produce the longitude λ and latitude ϕ of the corresponding point on the ellipsoid.

$$\lambda = g_1(x,\ y) = \arctan(\cos v,\ \sinh u) \tag{14}$$

where u and v are computed below;

$$\phi = g_2(x,\ y) = \text{ChiToPhi}(\cos\chi,\ \sin\chi)$$

where the function ChiToPhi is defined in Subsection 2.9 and $\cos\chi$ is computed from u, and v as follows:

$$\cos\chi = \frac{\sinh u}{(\cosh u)(\sin\lambda)}$$

unless $(\sin\lambda)$ is close to zero, that is, unless:

$$|\lambda| < 0.01 \text{ or } |\lambda \pm \pi| < 0.01 \text{ or } |\lambda \pm 2\pi| < 0.01$$

in which case the calculation should be:

$$\cos\chi = \frac{\sqrt{\sinh^2 u + \cos^2 v}}{\cosh u}$$

and $\sin\chi$ is computed

$$\sin\chi = \frac{\sin v}{\cosh u}$$

where u and v are computed from x and y as follows:

$$u = \frac{x}{R_4} + b_2 \sinh\left(\frac{2x}{R_4}\right)\cos\left(\frac{2y}{R_4}\right) + b_4 \sinh\left(\frac{4x}{R_4}\right)\cos\left(\frac{4y}{R_4}\right) + \ldots + b_{12} \sinh\left(\frac{12x}{R_4}\right)\cos\left(\frac{12y}{R_4}\right)$$
$$v = \frac{y}{R_4} + b_2 \cosh\left(\frac{2x}{R_4}\right)\sin\left(\frac{2y}{R_4}\right) + b_4 \cosh\left(\frac{4x}{R_4}\right)\sin\left(\frac{4y}{R_4}\right) + \ldots + b_{12} \cosh\left(\frac{12x}{R_4}\right)\sin\left(\frac{12y}{R_4}\right)$$

where R_4 is defined in Subsection 3.2 and b_2, b_4, ... b_{12} are unitless constants. In the case of the WGS 84 ellipsoid, the values are:

b_2 = -8.3773216405794867707E-04
b_4 = -5.905870152220365181E-08
b_6 = -1.67348266534382493E-10
b_8 = -2.1647981104903862E-13
b_{10} = -3.787930968839601E-16
b_{12} = -7.23676928796690E-19

Longitude at the Poles is ambiguous, *i.e.* not well defined. For the forward mapping equations (Section 3.2) this was not a problem. The formulas there will correctly convert $\phi = \pm 90$ deg no matter what numerical value is used for λ. In this subsection, the ambiguity is a problem. The attempted computation of λ in Eq. (3.14) will fail when the math-library routine for arctangent encounters arctan(0, 0). This will happen at a Pole, where $u = 0$ and $v = \pm\pi/2$, derived from $x = 0$ and $y = \pm R_4(\pi/2)$. To get around this, let the software define a constant, $\lambda_{\text{pole}} = 0$ (suggested), and execute $\lambda = \lambda_{\text{pole}}$ if $u = 0$ and $v = \pm\pi/2$, and execute Eq. (3.14) otherwise.

See the notes to the developer in Subsection 3.3.

■ 3.6 Inverse mapping: a numerical example

Let the reference ellipsoid be WGS 84 and let $x = 400\,000$ and $y = 7\,000\,000$ be given. Then, in order of calculation, $u = 0.0628815005045996857$, $v = 1.09865807573984195$, $\lambda = 0.137482740770994122$ which in degrees is 7.87718080206913254, $\cos\chi = 0.458217667193810883$, and $\sin\chi = 0.888840013428435821$. Then, by the methods of Subsection 2.9, $\phi = 1.09753532362197469$ which in degrees is 62.8841419100641123.

■ 3.7 Coverage of the ellipsoid

For reasons beyond the scope of this document, the forward mapping equations in Subsection 3.2 are not valid for the entire ellipsoid (*i.e.* the WGS 84 ellipsoid, in this section). An area surrounding each of the two points $\{\lambda, \phi\} = \{\pm 90\text{ deg},\ 0\text{ deg}\}$ must be omitted. Without trying to make the omitted area as small as possible, it is possible and permitted to specify the region of validity as those points $\{\lambda, \phi\}$ which satisfy one or more of the inequalities in the following list:

$$|\lambda| \le 70\text{ deg}$$
$$|\lambda - \pi| \le 70\text{ deg}$$
$$|\lambda + \pi| \le 70\text{ deg}$$
$$\frac{\pi}{2} - \phi \le 70\text{ deg}$$
$$\phi + \frac{\pi}{2} \le 70\text{ deg}$$

(Recall from Subsection 1.6 that deg $= \pi/180$ is a multiplier so that 70 deg $= 7\pi/18$). In words, by the above rule, any point to be placed on a transverse Mercator map must be within 70° of longitude to the prime or anti-prime meridian or within 70° of latitude to the North or South Pole.

There is a corresponding region of validity for the inverse mapping equations. A simple, non-maximal, but adequate choice for it is the set of all points $\{x,\ y\}$ such that:

$$|\,x\,| \le 10\,000\,000 \text{ meters} \quad \text{and} \quad |\,y\,| \le 20\,000\,000 \text{ meters}$$

The above regions of validity permit all calculations of the form $\{x,\ y\} \to \{\lambda,\ \phi\} \to \{x',\ y'\}$, *i.e.* the forward mapping equations can always be used to check an inverse-mapping-equation calculation.

3.8 Index δ

As a measure of how well a point given by $\{\lambda,\ \phi\}$ falls within the ellipsoid coverage (Subsection 3.7) and as an index to computational-error bounds in Subsection 3.9, the following function of $\{\lambda,\ \phi\}$ is defined:

$$\delta = \text{Minimum}\left(\ |\,\lambda\,|,\ \ |\,\lambda - \pi\,|,\ \ |\,\lambda + \pi\,|,\ \ \frac{\pi}{2} - \phi,\ \ \phi + \frac{\pi}{2}\right)$$

The quantity δ is the minimum of the 5 quantities listed above. The ellipsoid coverage can be restated simply as $\delta \le 70$ deg. In words, δ is the smaller of the latitude-difference to the nearest Pole and the longitude-difference to the nearest "special" meridian (*i.e.* central or anti-central meridian).

3.9 Computational accuracy

The theoretical definition of transverse Mercator in Subsection 3.1 is the standard by which approximate formulations such as in Subsections 3.2 and 3.5 are judged for computational accuracy. The forward mapping equations (Subsection 3.2 using all terms) have the following computational-error bounds, depending on the index δ:

index δ (deg)	bound (meters)
30	10^{-9}
40	10^{-8}
50	0.5×10^{-6}
60	10^{-5}
70	10^{-2}

For example, if a point P has index $\delta \le 60$ deg, then $\sqrt{(x - x')^2 + (y - y')^2} < 10^{-5}$meters where $\{x,\ y\}$ are the computed coordinates and $\{x',\ y'\}$ are the true coordinates of the conversion of P.

The inverse mapping equations have corresponding accuracies. In other words, the inverse mapping followed by the *true* forward mapping would produce round-trip discrepancies in meters within the bounds given above.

Software developers competent in iterative numerical methods will know how to build an *accurate* inverse of this document's *approximate* forward mapping equations. This is discouraged, as it will not produce a more accurate inverse mapping than the one given here.

4. Transverse Mercator for other Ellipsoids

Section 3 was limited to one choice for the reference ellipsoid, namely the WGS 84 ellipsoid. In particular, the constants R_4, e, a_2, ..., a_{12}, b_2, ... b_{12} all depend on the choice of the reference ellipsoid. This section provides the values of these constants for each ellipsoid in Appendix A of [12]. A method of calculating these is found in [9]. This provision extends the formulations of transverse Mercator in Subsections 3.2 and 3.5 to these other ellipsoids.

In this section, subscripted notations are replaced by non-subscripted notations. For example, a_2 is replaced by A2 and b_2 is replaced by B2.

The ellipsoids are listed in order of increasing flattening (decreasing inverse flattening).

4.1 Everest 1956 (India) ellipsoid

```
Name                             name    Everest 1956 (India)
NGA two-letter code              twolet  EC
Semi-major axis                  a       6377301.2430000000000
Semi-minor axis                  b       6356100.2283681013106
Inverse flattening               1/f     300.80170000000000000
(First) eccentricity             e       0.081472980982652689208
Eccentricity squared             e²      0.0066378466301996867553
Meridional isoperimetric radius  R4      6366705.1481254190443

A2  =   8.3064943111192510534E-04
A4  =   7.480375027595025021E-07
A6  =   1.16750772278215999E-09
A8  =   2.3479972304395461E-12
A10 =   5.474212231879573E-15
A12 =   1.40642257446745E-17

B2  =  -8.3064976590443772201E-04
B4  =  -5.805953517555717859E-08
B6  =  -1.63133251663416522E-10
B8  =  -2.0923797199593389E-13
B10 =  -3.630200927775259E-16
B12 =  -6.87666654919219E-19
```

4.2 Other "Everest" ellipsoids

There are other ellipsoids listed in Appendix A of [12] having "Everest" in their names. They differ from the Everest 1956 (India) ellipsoid in size but not in shape. Therefore they have the same values for f, f^{-1}, e, e^2, a_2, a_4, ..., b_{12}. The value of R_4 is obtained from the value of the semi-major axis, a, by multiplying by the constant `0.99833846724957337010` or by referring to the following table. (This multiplier pertains only to ellipsoids having this shape, *i.e.* an inverse flattening of `300.8017`).

Name	code	a	b	R4
Everest (India 1830)	EA	6377276.345000	6356075.413140	6366680.291494
Everest (E. Malaysia, Brunei)	EB	6377298.556000	6356097.550301	6366702.465590
Everest 1956 (India)	EC	6377301.243000	6356100.228368	6366705.148125
Everest 1969 (West Malaysia)	ED	6377295.664000	6356094.667915	6366699.578395
Everest 1948(W.Malaysia,Singapore)	EE	6377304.063000	6356103.038993	6366707.963440
Everest (Pakistan)	EF	6377309.613000	6356108.570542	6366713.504218

4.3 Airy 1830 ellipsoid

```
Name                             name    Airy 1830
NGA two-letter code              twolet  AA
Semi-major axis                  a       6377563.3960000000000
Semi-minor axis                  b       6356256.9092372851202
Inverse flattening               1/f     299.32496460000000000
(First) eccentricity             e       0.081673373874141892673
Eccentricity squared             e²      0.0066705399999853634746
Meridional isoperimetric radius  R4      6366914.6089252214441

A2  =   8.3474517669594013740E-04
A4  =   7.554352936725572895E-07
A6  =   1.18487391005135489E-09
```

```
A8  =   2.3946872955703565E-12
A10 =   5.610633978440270E-15
A12 =   1.44858956458553E-17

B2  =  -8.3474551646761162264E-04
B4  =  -5.863630361809676570E-08
B6  =  -1.65562038746920803E-10
B8  =  -2.1340335537652749E-13
B10 =  -3.720760760132477E-16
B12 =  -7.08304328877781E-19
```

■ 4.4 Modified Airy ellipsoid

This ellipsoid has the same flattening (and inverse flattening) as the Airy 1830 ellipsoid above.

Name	name	Modified Airy
NGA two-letter code	twolet	AM
Semi-major axis	a	6377340.1890000000000
Semi-minor axis	b	6356034.4479385342568
Inverse flattening	1/f	299.32496460000000000
(First) eccentricity	e	0.081673373874141892673
Eccentricity squared	e^2	0.0066705399999853634746
Meridional isoperimetric radius	R4	6366691.7746198806757

The coefficients, a_2, a_4, ..., b_{12} are the same as for the Airy 1830 ellipsoid.

■ 4.5 Bessel 1841 (Ethiopia, Asia) ellipsoid

Name	name	Bessel 1841 (Ethiopia, Asia)
NGA two-letter code	twolet	BR
Semi-major axis	a	6377397.1550000000000
Semi-minor axis	b	6356078.9628181880963
Inverse flattening	1/f	299.15281280000000000
(First) eccentricity	e	0.081696831222527503120
Eccentricity squared	e^2	0.0066743722318021446801
Meridional isoperimetric radius	R4	6366742.5202340428423

```
A2  =   8.3522527226849818552E-04
A4  =   7.563048340614894422E-07
A6  =   1.18692075307408346E-09
A8  =   2.4002054791393298E-12
A10 =   5.626801597980756E-15
A12 =   1.45360057224474E-17

B2  =  -8.3522561262703079182E-04
B4  =  -5.870409978661008580E-08
B6  =  -1.65848307463131468E-10
B8  =  -2.1389565927064571E-13
B10 =  -3.731493368666479E-16
B12 =  -7.10756898071999E-19
```

■ 4.6 Bessel 1841 (Namibia) ellipsoid

This ellipsoid has the same flattening (and inverse flattening) as Bessel 1841 (Ethiopia, Asia), above.

Name	name	Bessel 1841 (Namibia)
NGA two-letter code	twolet	BN
Semi-major axis	a	6377483.8650000000000
Semi-minor axis	b	6356165.3829663254699
Inverse flattening	1/f	299.15281280000000000
(First) eccentricity	e	0.081696831222527503120
Eccentricity squared	e^2	0.0066743722318021446801
Meridional isoperimetric radius	R4	6366829.0853687697376

The coefficients, a_2, a_4, ..., b_{12} are the same as for Bessel 1841 (Ethiopia, Asia).

■ 4.7 Krassovsky 1940 ellipsoid

```
Name                             name    Krassovsky 1940
NGA two-letter code              twolet  KA
Semi-major axis                  a       6378245.0000000000000
Semi-minor axis                  b       6356863.0187730472679
Inverse flattening               1/f     298.30000000000000000
(First) eccentricity             e       0.081813334016931147358
Eccentricity squared             e2       0.0066934216229659432280
Meridional isoperimetric radius  R4      6367558.4968749794253

A2  =   8.3761175713442343106E-04
A4  =   7.6063462008147201 97E-07
A6  =   1.19713032035541037E-09
A8  =   2.4277772986483520E-12
A10 =   5.707722772225013E-15
A12 =   1.47872454335773E-17

B2  =  -8.3761210042019176501E-04
B4  =  -5.9041691540785462 37E-08
B6  =  -1.67276212891429215E-10
B8  =  -2.1635549847939549E-13
B10 =  -3.785212121016612E-16
B12 =  -7.23053625983667E-19
```

■ 4.8 Helmert 1906 ellipsoid

This ellipsoid has the same flattening (and inverse flattening) as the Krassovsky 1940 ellipsoid above.

```
Name                             name    Helmert 1906
NGA two-letter code              twolet  HE
Semi-major axis                  a       6378200.0000000000000
Semi-minor axis                  b       6356818.1696278913845
Inverse flattening               1/f     298.30000000000000000
(First) eccentricity             e       0.081813334016931147358
Eccentricity squared             e2       0.0066934216229659432280
Meridional isoperimetric radius  R4      6367513.5722707412102
```

The coefficients, a_2, a_4, ..., b_{12} are the same as for Krassovsky 1940.

■ 4.9 Modified Fischer 1960 ellipsoid

This ellipsoid has the same flattening (and inverse flattening) as the Krassovsky 1940 ellipsoid above.

```
Name                             name    Modified Fischer 1960
NGA two-letter code              twolet  FA
Semi-major axis                  a       6378155.0000000000000
Semi-minor axis                  b       6356773.3204827355012
Inverse flattening               1/f     298.30000000000000000
(First) eccentricity             e       0.081813334016931147358
Eccentricity squared             e2       0.0066934216229659432280
Meridional isoperimetric radius  R4      6367468.6476665029951
```

The coefficients, a_2, a_4, ..., b_{12} are the same as for Krassovsky 1940.

■ 4.10 WGS 72 ellipsoid

```
Name                             name    WGS 72
NGA two-letter code              twolet  WD
Semi-major axis                  a       6378135.0000000000000
Semi-minor axis                  b       6356750.5000000000000
Inverse flattening               1/f     298.25972082583179406
(First) eccentricity             e       0.081818848890064648207
Eccentricity squared             e2       0.0066943240336952331159
Meridional isoperimetric radius  R4      6367447.2386241894462

A2  =   8.3772481044362217923E-04
A4  =   7.6084003888635609 36E-07
A6  =   1.19761541904924067E-09
A8  =   2.4290893081322466E-12
A10 =   5.711579173743133E-15
```

```
A12 =   1.47992364667635E-17

B2  =  -8.3772515386847544554E-04
B4  =  -5.905770828762463028E-08
B6  =  -1.67344058948464124E-10
B8  =  -2.1647255130188214E-13
B10 =  -3.787772179729998E-16
B12 =  -7.23640523525528E-19
```

4.11 WGS 84 ellipsoid

The subsection repeats some information for the WGS 84 ellipsoid in the format of this section.

```
Name                            name    WGS 84
NGA two-letter code             twolet  WE
Semi-major axis                 a       6378137.0000000000000
Semi-minor axis                 b       6356752.3142451794976
Inverse flattening              1/f     298.25722356300000000
(First) eccentricity            e       0.081819190842621494335
Eccentricity squared            e²      0.0066943799901413169961
Meridional isoperimetric radius R4      6367449.1458234153093

A2  =   8.3773182062446983032E-04
A4  =   7.608527773572489156E-07
A6  =   1.19764550324249210E-09
A8  =   2.4291706803973131E-12
A10 =   5.711818369154105E-15
A12 =   1.47999802705262E-17

B2  =  -8.3773216405794867707E-04
B4  =  -5.905870152220365181E-08
B6  =  -1.67348266534382493E-10
B8  =  -2.1647981104903862E-13
B10 =  -3.787930968839601E-16
B12 =  -7.23676928796690E-19
```

4.12 GRS 80 ellipsoid

```
Name                            name    GRS 80
NGA two-letter code             twolet  RF
Semi-major axis                 a       6378137.0000000000000
Semi-minor axis                 b       6356752.3141403558479
Inverse flattening              1/f     298.25722210100000000
(First) eccentricity            e       0.081819191042815790146
Eccentricity squared            e²      0.0066943800229007876254
Meridional isoperimetric radius R4      6367449.1457710475269

A2  =   8.3773182472855134012E-04
A4  =   7.608527848149655006E-07
A6  =   1.19764552085530681E-09
A8  =   2.4291707280369697E-12
A10 =   5.711818509192422E-15
A12 =   1.47999807059922E-17

B2  =  -8.3773216816203523672E-04
B4  =  -5.905870210369121594E-08
B6  =  -1.67348268997717031E-10
B8  =  -2.1647981529928124E-13
B10 =  -3.787931061803592E-16
B12 =  -7.23676950110361E-19
```

4.13 South American 1969 ellipsoid

```
Name                            name    South American 1969
NGA two-letter code             twolet  SA
Semi-major axis                 a       6378160.0000000000000
Semi-minor axis                 b       6356774.7191953059514
Inverse flattening              1/f     298.25000000000000000
(First) eccentricity            e       0.081820179996059878869
Eccentricity squared            e²      0.0066945418545876371598
Meridional isoperimetric radius R4      6367471.8485322822248
```

```
A2  =   8.3775209887947194075E-04
A4  =   7.608896263599627157E-07
A6  =   1.19773253021831769E-09
A8  =   2.4294060763606098E-12
A10 =   5.712510331613028E-15
A12 =   1.48021320370432E-17

B2  =  -8.3775244233790270051E-04
B4  =  -5.906157468586898015E-08
B6  =  -1.67360438158764851E-10
B8  =  -2.1650081225048788E-13
B10 =  -3.788390325953455E-16
B12 =  -7.23782246429908E-19
```

4.14 Australian National 1966 ellipsoid

The Australian National 1966 ellipsoid is identical to the South American 1969 ellipsoid. Its NGA two-letter code is "AN". The numerical values of all the parameters are the same as those for South American 1969.

4.15 Indonesian 1974 ellipsoid

Name	name	Indonesian 1974
NGA two-letter code	twolet	ID
Semi-major axis	a	6378160.0000000000000
Semi-minor axis	b	6356774.5040855398378
Inverse flattening	1/f	298.24700000000000000
(First) eccentricity	e	0.081820590809460040025
Eccentricity squared	e^2	0.0066946090804090967678
Meridional isoperimetric radius	R4	6367471.7410677818465

```
A2  =   8.3776052087969078729E-04
A4  =   7.609049308144604484E-07
A6  =   1.19776867565343872E-09
A8  =   2.4295038464530901E-12
A10 =   5.712797738386076E-15
A12 =   1.48030257891140E-17

B2  =  -8.3776086434848497443E-04
B4  =  -5.906276799395007586E-08
B6  =  -1.67365493472742884E-10
B8  =  -2.1650953495573773E-13
B10 =  -3.788581120060625E-16
B12 =  -7.23825990889693E-19
```

4.16 International 1924 ellipsoid

Name	name	International 1924
NGA two-letter code	twolet	IN
Semi-major axis	a	6378388.0000000000000
Semi-minor axis	b	6356911.9461279461279
Inverse flattening	1/f	297.00000000000000000
(First) eccentricity	e	0.081991889979029767433
Eccentricity squared	e^2	0.0067226700223333219966
Meridional isoperimetric radius	R4	6367654.5000575837475

```
A2  =   8.4127599100356448089E-04
A4  =   7.673066923431950296E-07
A6  =   1.21291995794281190E-09
A8  =   2.4705731165688123E-12
A10 =   5.833780550286833E-15
A12 =   1.51800420867708E-17

B2  =  -8.4127633881644851945E-04
B4  =  -5.956193574768780571E-08
B6  =  -1.69484573979154433E-10
B8  =  -2.2017363465021880E-13
B10 =  -3.868896221495780E-16
B12 =  -7.42279219864412E-19
```

4.17 Hough 1960 ellipsoid

This ellipsoid has the same flattening (and inverse flattening) as the International 1924 ellipsoid.

```
Name                            name    Hough 1960
NGA two-letter code             twolet  HO
Semi-major axis                 a       6378270.0000000000000
Semi-minor axis                 b       6356794.3434343434343
Inverse flattening              1/f     297.00000000000000000
(First) eccentricity            e       0.08199188997902976743
Eccentricity squared            e²      0.0067226700223333219966
Meridional isoperimetric radius R4      6367536.6986270331452
```

The coefficients, a_2, a_4, ..., b_{12} are the same as for International 1924.

4.18 War Office 1924 ellipsoid

```
Name                            name    War Office 1924
NGA two-letter code             twolet  WO
Semi-major axis                 a       6378300.5800000000000
Semi-minor axis                 b       6356752.2672297297297
Inverse flattening              1/f     296.00000000000000000
(First) eccentricity            e       0.08213003906177850016
Eccentricity squared            e²      0.0067453433162892622352
Meridional isoperimetric radius R4      6367530.9812114439907

A2  =   8.4411652150600103279E-04
A4  =   7.7249897501725834 27E-07
A6  =   1.22525529789972041E-09
A8  =   2.5041361775549209E-12
A10 =   5.933026083631383E-15
A12 =   1.54904908794521E-17

B2  =  -8.4411687285559594196E-04
B4  =  -5.9966816870643225 48E-08
B6  =  -1.71209836918814857E-10
B8  =  -2.2316811233502163E-13
B10 =  -3.934782433323038E-16
B12 =  -7.57474665717687E-19
```

4.19 Clarke 1866 ellipsoid

```
Name                            name    Clarke 1866
NGA two-letter code             twolet  CC
Semi-major axis                 a       6378206.4000000000000
Semi-minor axis                 b       6356583.8000000000000
Inverse flattening              1/f     294.97869821390582076
(First) eccentricity            e       0.08227185422300325877 0
Eccentricity squared            e²      0.0067686579972910991438
Meridional isoperimetric radius R4      6367399.6891697827298

A2  =   8.4703742793654652315E-04
A4  =   7.7785645176581152 12E-07
A6  =   1.23802665917879731E-09
A8  =   2.5390045684252928E-12
A10 =   6.036484469753319E-15
A12 =   1.58152259295850E-17

B2  =  -8.4703778294785813001E-04
B4  =  -6.0384598746001835 55E-08
B6  =  -1.72996106059227725E-10
B8  =  -2.2627911073545072E-13
B10 =  -4.003466873888566E-16
B12 =  -7.73369749524777E-19
```

4.20 Clarke 1880 (IGN) ellipsoid

```
Name                            name    Clarke 1880 (IGN)
NGA two-letter code             twolet  CG
Semi-major axis                 a       6378249.2000000000000
Semi-minor axis                 b       6356514.9999634416278
Inverse flattening              1/f     293.46602080000000000
(First) eccentricity            e       0.082483256832670385055
Eccentricity squared            e^2     0.0068034876577242657616
Meridional isoperimetric radius R4      6367386.7366550997514

A2  =   8.5140099460764136776E-04
A4  =   7.8589454560381877774E-07
A6  =   1.25727085106103462E-09
A8  =   2.5917718627340128E-12
A10 =   6.193726879043722E-15
A12 =   1.63109098395549E-17

B2  =  -8.5140135513650084564E-04
B4  =  -6.1011454750630334994E-08
B6  =  -1.75687742410879760E-10
B8  =  -2.3098718484594067E-13
B10 =  -4.107860472919190E-16
B12 =  -7.97633133452512E-19
```

4.21 Clarke 1880 ellipsoid

```
Name                            name    Clarke 1880
NGA two-letter code             twolet  CD
Semi-major axis                 a       6378249.1450000000000
Semi-minor axis                 b       6356514.8695497759528
Inverse flattening              1/f     293.46500000000000000
(First) eccentricity            e       0.082483400044185038061
Eccentricity squared            e^2     0.0068035112828490643388
Meridional isoperimetric radius R4      6367386.6439805112873

A2  =   8.5140395445291970541E-04
A4  =   7.8590001194641409780E-07
A6  =   1.25728397182445579E-09
A8  =   2.5918079321459932E-12
A10 =   6.193834639108787E-15
A12 =   1.63112504092335E-17

B2  =  -8.5140431498554106268E-04
B4  =  -6.1011881061870921840E-08
B6  =  -1.75689577596504470E-10
B8  =  -2.3099040312610703E-13
B10 =  -4.107932016207395E-16
B12 =  -7.97649804397335E-19
```

4.22 Coverage of the ellipsoid

The statements about regions of validity in Subsection 3.7 are true also for the above ellipsoids. This is because the ellipsoids above, listed after "WGS 84 ellipsoid" are not *severely* flatter than the WGS 84 ellipsoid, and because the validity regions defined in Subsection 3.7 are more restrictive than what is theoretically possible.

4.23 Sphere

For a sphere of radius a, the formulas of Section 3 are applicable by setting $f = e^2 = e = 0$ and $b = R_4 = a$ and setting all the coefficients $a_2, a_4, ..., b_{12}$ to zero.

5. Transverse Mercator with Parameters

Sections 3 and 4 presented the basic form of transverse Mercator. In this section, the basic form is extended two ways: Firstly, where those sections measured longitude from the prime meridian, this section will allow longitude to be measured from any specified meridian ("central meridian"). Secondly, the easting-northing-pairs $\{x, y\}$ obtained from those sections will be subjected to a homothetic transformation in this section. (A transformation is homothetic if it consists of a translation and/or a proportional re-sizing. Rotations and other modes of stretching/shrinking are not allowed).

This section concludes with a review of the sources consulted in the development of this document.

5.1 Preliminary general form

Let f_1 and f_2 be the functions from Subsection 3.2 that define the forward mapping of the transverse Mercator projection in its basic form. Let λ_0 be a constant in radians, let $k_0 > 0$ be a unitless constant, and let $x_{\mathrm{cm}}$and y_{eq} be constants in meters. Then a preliminary general form of the transverse Mercator forward mapping equations is:

$$\begin{aligned} x &= k_0\, f_1(\lambda - \lambda_0, \phi) + x_{\mathrm{cm}} \\ y &= k_0\, f_2(\lambda - \lambda_0, \phi) + y_{\mathrm{eq}} \end{aligned} \tag{15}$$

The constants, also called parameters, have these notation, names, and units:

λ_0	central meridian, CM	radians
k_0	central scale factor, central scale	(unitless)
x_{cm}	central meridian easting, CM easting	meters
y_{eq}	Equator northing	meters

The parameter k_0 controls the proportional re-sizing and the parameters x_{cm} and y_{eq} control the translation mentioned above. The corresponding inverse mapping equations are:

$$\begin{aligned} \lambda &= \lambda_0 + g_1\left(\frac{x - x_{\mathrm{cm}}}{k_0}, \frac{y - y_{\mathrm{eq}}}{k_0}\right) \\ \phi &= g_2\left(\frac{x - x_{\mathrm{cm}}}{k_0}, \frac{y - y_{\mathrm{eq}}}{k_0}\right) \end{aligned} \tag{16}$$

where functions g_1 and g_2 are the inverse mapping equations of the basic form of transverse Mercator specified in Subsection 3.5.

The quantity λ computed according to Eq. (5.16) lies in the interval $\lambda_0 - \pi < \lambda \le \lambda_0 + \pi$. To convert it to a longitude lying in a different interval (of length 2π), the quantity 2π should be added or subtracted to it as necessary.

The list, $\{\lambda_0, k_0, x_{\mathrm{cm}}, y_{\mathrm{eq}}\}$, is a set of unique independently-specifiable parameters.

5.2 Origin

The equations and parameters of Subsection 5.1 accomplish the goals stated in the Section 5 introduction, which were to (i) specify a meridian of reference (the meridian λ_0), (ii) apply a proportional re-sizing (the factor k_0) and (iii) apply a translation (the vector $\{x_{\mathrm{cm}}, y_{\mathrm{eq}}\}$). We should be done. However, for convenience, an alternate method to accomplish the translation has been adopted. This is now explained:

A point on the reference ellipsoid is selected for special treatment. It must lie in the transverse Mercator coverage area (*i.e.* lie within 70° of longitude from the central or anti-central meridian or lie within 70° of latitude from the North or South Pole), and is called the Origin. Let its longitude and latitude be notated $\lambda_{\mathrm{origin}}$ and ϕ_{origin}, respectively. On the map projection plane, the Origin is to have rectangular coordinates $\{x, y\} = \{x_{\mathrm{origin}}, y_{\mathrm{origin}}\}$. This will determine the translation under consideration.

The above parameters have these notations, names, abbreviations, and units:

$\lambda_{\mathrm{origin}}$	Origin longitude	radians
ϕ_{origin}	Origin latitude	radians
x_{origin}	(Origin easting), False Easting, FE	meters

y_{origin} (Origin northing), False Northing, FN meters

(If there was an opportunity to revise the terminology, "Origin easting" and "Origin northing" would make sense. Accepted terminology is "False Easting" and "False Northing").

5.3 Given $\{\lambda_{\text{origin}}, \phi_{\text{origin}}, x_{\text{origin}}, y_{\text{origin}}\}$, compute $\{x_{\text{cm}}, y_{\text{eq}}\}$

Let the reference ellipsoid and transverse Mercator parameters λ_0 and k_0 be fixed. Let the parameters $\{\lambda_{\text{origin}}, \phi_{\text{origin}}, x_{\text{origin}}, y_{\text{origin}}\}$ be given. To obtain values for the parameters $\{x_{\text{cm}}, y_{\text{eq}}\}$ that yield the same translation, the following applies:

$$\begin{aligned} x_{\text{cm}} &= x_{\text{origin}} - k_0\, f_1(\lambda_{\text{origin}} - \lambda_0, \phi_{\text{origin}}) \\ y_{\text{eq}} &= y_{\text{origin}} - k_0\, f_2(\lambda_{\text{origin}} - \lambda_0, \phi_{\text{origin}}) \end{aligned} \tag{17}$$

5.4 General form of transverse Mercator

The general form of transverse Mercator is Eqs. (5.15 and 5.16) with the further stipulations that x_{cm} and y_{eq} are taken as intermediate variables computed according to Eq. (5.17) and that the list $\{\lambda_0, k_0, \lambda_{\text{origin}}, \phi_{\text{origin}}, x_{\text{origin}}, y_{\text{origin}}\}$ is adopted as the general form's set of (non-unique) independently-specifiable parameters.

Not all authorities provide the option to allow an Origin longitude distinct from the central meridian. When the set of parameters has only one special longitude, $\lambda_{\text{origin}} = \lambda_0$ should be assumed.

5.5 Coverage of the ellipsoid

The statements in Subsection 3.7 about the regions of validity for the forward and inverse mapping equations carry over to the general form of transverse Mercator if λ is replaced by $\lambda - \lambda_0$ and x is replaced by $(x - x_{\text{cm}})/k_0$ and y is replaced by $(y - y_{\text{eq}})/k_0$.

5.6 History and sources

A history of the development of transverse Mercator is outside the scope of this document, but some aspects should be mentioned. Transverse Mercator as defined in Subsection 3.1 and extended in Subsections 5.1 or 5.4 for parameters is sometimes given the name Gauss-Krüger or the phrase "of Gauss-Krüger type" after its inventors C. F. Gauss and L. Krüger. This is done to distinguish it from some historical versions (Gauss-Lambert, Gauss-Schreiber) that do not adhere to Requirement 3 of Subsection 3.1.

The formulas in Subsections 3.2 and 3.5 are extensions of the work of Krüger (1912). Krüger carried out an expansion to 4th order, *i.e.* obtaining coefficients a_2, a_4, a_6, a_8 to some precision, and this resulted in equations which were accurate to within 10^{-6} meters for points located within 1000 km of the central meridian. The algorithms given in Section 3 extend Krüger's method to 6th order and are based on the work of [4], [9], and [15]. Variations of Krüger's algorithms are in use by the national geodetic institutes of several European countries. Recently the Oil and Gas Producers (formerly EPSG) added some version of this method to their compendium of coordinate conversion formulas [6]. Another reference for the basic idea of Krüger's method is Section 5.1.6, "Gauss-Kruger projection for a wide zone" of [1].

An international standard for spatial reference frames and their coordinates, including some map projections, is presented in [8]. The mathematical formulas it adopted for transverse Mercator do imply and are implied by the theoretical definition in this document (Subsection 3.1). Its choice of parameters is the same as Subsection 5.4 with $\lambda_{\text{origin}} = \lambda_0$.

5.7 Old *v.* new

Reference [3], *i.e.*, Edition 1 of this document, and references [16] and [17] used algorithms based on an expansion in $(\lambda - \lambda_0)$. The major drawback of this approach is that it has a much more restricted domain of applicability, particularly at high latitudes. In contrast, the algorithms given in Subsections 3.2 and 3.5 are vast improvements. They offer better accuracy, greater ellipsoid coverage, faster execution, simpler logic, and easier software coding.

The choice of parameters in Subsection 5.4 follows current practice except that providing λ_{origin} as a parameter distinct from λ_0 is new. This is recommended for its naturalness (see Subsection 5.2) and its flexibility in specifying electronic-drafting-table coordinates, especially when the map sheet has multiple plans.

Assessments of software packages in current use at DoD are outside the scope of this document. If the transverse Mercator routines are satisfactory with respect to accuracy, ellipsoid coverage, execution speed, and code maintainability, they need not be replaced with the algorithms specified here.

6. Transverse Mercator Auxiliary Functions

Every conformal map projection comes with two auxiliary functions: point-scale and convergence-of-meridians (CoM). The formulas for these for transverse Mercator are the subject of this section. Detailed explanations of the importance and usefulness of these functions are outside the scope of this document, but some introductory definitions will be offered.

6.1 Point-scale

Loosely, point-scale is the function which tells how the map projection enlarges or reduces *small* distances when transferring them from the reference ellipsoid to the map projection plane. It is location specific (it varies from point to point); it is independent of direction (conformality is required) and it is a unitless ratio (proportionality is assumed). Let σ (sigma, for "s" in "scaling") be the notation for this function so that $\sigma(P)$ is the value of this function at position P. If points A and B on the reference ellipsoid are one meter apart, then on the map projection plane they will be $\sigma(A) \approx \sigma(B)$ meters apart.

A precise definition using the differential calculus is available in the map projection literature [1], [8], [16], or [17], where it might be called scale, local scale, local scale function, scale distortion, or point distortion.

6.2 Convergence-of-meridians

Convergence-of-meridians (CoM) is the function which gives the angles of intersection between the meridians and the map projection's vertical lines, *i.e.* the lines x = constant. More precisely, it is the angle from true north to map north at such an intersection point, where the positive sense of the rotation is clockwise. True north is tangent to the meridian and points in the direction of increasing latitude. Map north is tangent to (and coincident with) the line x = constant and points in the direction of increasing y coordinate. All this takes place on the map projection plane.

The symbol for CoM will be γ (gamma, for "g" in "grid declination" and "grid convergence", synonyms for CoM when the map projection is one of the universal grids UTM or UPS).

6.3 Given $\{\lambda, \phi\}$, compute $\{\sigma, \gamma\}$ — basic case

The basic form of transverse Mercator (Section 3) is handled first. Let a be the semi-major axis and e be the eccentricity of the reference ellipsoid. When it is desired to emphasize the functional dependence of point-scale σ and CoM γ on longitude λ and latitude ϕ, the notations f_3 and f_4 will be used.

The formulas for σ and γ are:

$$\sigma = f_3(\lambda, \phi) = \frac{2\,(R_4/a)\,w\,(\cosh u)\sqrt{{\sigma_1}^2 + {\sigma_2}^2}}{(1 + \sin\phi)/P + (1 - \sin\phi)\,P}$$

$$\gamma = f_4(\lambda, \phi) = \arctan(\cos\lambda,\ (\sin\chi)\sin\lambda) + \arctan(\sigma_1, \sigma_2)$$

where:

$$\sigma_1 = 1 + 2\,a_2\cosh(2\,u)\cos(2\,v) + 4\,a_4\cosh(4\,u)\cos(4\,v) + \ldots + 12\,a_{12}\cosh(12\,u)\cos(12\,v)$$

$$\sigma_2 = 2\,a_2\sinh(2\,u)\sin(2\,v) + 4\,a_4\sinh(4\,u)\sin(4\,v) + \ldots + 12\,a_{12}\sinh(12\,u)\sin(12\,v)$$

$$w = \sqrt{1 - e^2\sin^2\phi}$$

$$P = \exp(e\,\mathrm{arctanh}(e\sin\phi)) = \left(\frac{1 + e\sin\phi}{1 - e\sin\phi}\right)^{e/2}$$

and where u and v are computed by Eq. (3.8) of Subsection 3.2 and R_4 and the coefficients a_2, a_4, ..., a_{12} have the same values as in Sections 3 and 4.

Depending on their requirements, software developers should consider bundling the equations of this subsection with those of Subsection 3.2 to obtain a single module which could be described, "Given $\{\lambda, \phi\}$, compute $\{x, y, \sigma, \gamma\}$".

6.4 Given $\{\lambda, \phi\}$, compute $\{\sigma, \gamma\}$ — general case

The general form of transverse Mercator is now considered. Let the parameters $\{\lambda_0, k_0, \lambda_{\text{origin}}, \phi_{\text{origin}}, x_{\text{origin}}, y_{\text{origin}}\}$ be given. The subset $\{\lambda_{\text{origin}}, \phi_{\text{origin}}, x_{\text{origin}}, y_{\text{origin}}\}$ is irrelevant to the computation of σ and γ. Subsection 6.3 gave the formulas for σ and γ for the case that $\lambda_0 = 0$ and $k_0 = 1$. The formulas for the general case are:

$$\sigma = k_0\, f_3\,(\lambda - \lambda_0, \phi)$$
$$\gamma = f_4\,(\lambda - \lambda_0, \phi)$$

where f_3 and f_4 are the functions defined in Subsection 6.3. Software developers could bundle the above with Eq. (5.15) as part of a module, "transverse Mercator preliminary general form".

7. Universal Transverse Mercator (UTM)

This section gives the definition of UTM, some numerical examples of it, and the administrative rules added to it.

7.1 Definition of UTM

UTM is a family of 120 instances of the general form of the transverse Mercator projection. Each instance is called a zone and is given a zone number Z between −60 and +60 excluding zero. (As a connection to other explanations, the zone numbers can be arranged suggestively this way):

+1	+2	...	+30	+31	...	+59	+60
−1	−2	...	−30	−31	...	−59	−60

UTM zone Z is the transverse Mercator projection whose parameters $\{\lambda_0, k_0, \lambda_{\text{origin}}, \phi_{\text{origin}}, x_{\text{origin}}, y_{\text{origin}}\}$ are specified:

$\lambda_0 = -183 \text{ deg } + (6 \text{ deg}) \,|\, Z \,|$ (using the absolute value function applied to Z)

$k_0 = 0.9996$ exactly

$\lambda_{\text{origin}} = \lambda_0$

$\phi_{\text{origin}} = 0$

$x_{\text{origin}} = 500\,000$ meters

$y_{\text{origin}} = 0$ if $Z > 0$ but $y_{\text{origin}} = 10\,000\,000$ meters if $Z < 0$

Some comments apply: East longitude is positive; west longitude is negative. For $Z = \pm 1$, the central meridian in degrees is $-183\,° + 6\,° \times 1 = -177\,°$, which by the above rule may be notated 177°W. The notation "−177 °W" is incorrect. Never use both a prefix (plus or minus sign) and a suffix ("E" or "W"). A longitude in degrees can be a UTM central meridian if and only if it is a whole number divisible by three but not by two.

7.2 Examples of computing $\{x, y, \sigma, \gamma\}$, given $\{\lambda, \phi, Z\}$

This subsection gives numerical examples of the computation of the easting x, northing y, point-scale σ, and grid-declination γ, given the longitude λ, latitude ϕ, and UTM zone number Z.

The following points in the Indian Ocean are symmetrically arrayed about the Equator and 75°E, which is the central meridian for $Z = \pm 43$. Lon., Lat., and CoM are in degrees; easting and northing are in meters, and point-scale ("`pt-scale`") is a unitless ratio. The computations pertain to the WGS 84 ellipsoid.

E.g.	Lon	Lat	Z	easting	northing	pt-scale	CoM
---	(deg)	(deg)	---	(meters)	(meters)	---	(deg)
1	65	3	43	-616926.925721	336734.192052	1.015083	-0.528835
2	74	3	43	388870.867643	331643.938073	0.999753	-0.052341
3	75	3	43	500000.000000	331593.179548	0.999600	0.000000
4	76	3	43	611129.132357	331643.938073	0.999753	0.052341
5	85	3	43	1616926.925721	336734.192052	1.015083	0.528835
6	65	-3	43	-616926.925721	-336734.192052	1.015083	0.528835
7	74	-3	43	388870.867643	-331643.938073	0.999753	0.052341
8	75	-3	43	500000.000000	-331593.179548	0.999600	0.000000
9	76	-3	43	611129.132357	-331643.938073	0.999753	-0.052341
10	85	-3	43	1616926.925721	-336734.192052	1.015083	-0.528835

Example 1, above, devolves to the example in Subsection 3.4.

The same points are re-computed for zone $Z = -43$, and the only change is the northing. An offset of 10,000,000 meters has been added:

E.g.	Lon	Lat	Z	easting	northing	pt-scale	CoM
---	(deg)	(deg)	---	(meters)	(meters)	---	(deg)
11	65	3	-43	-616926.925721	10336734.192052	1.015083	-0.528835
12	74	3	-43	388870.867643	10331643.938073	0.999753	-0.052341
13	75	3	-43	500000.000000	10331593.179548	0.999600	0.000000
14	76	3	-43	611129.132357	10331643.938073	0.999753	0.052341
15	85	3	-43	1616926.925721	10336734.192052	1.015083	0.528835
16	65	-3	-43	-616926.925721	9663265.807948	1.015083	0.528835
17	74	-3	-43	388870.867643	9668356.061927	0.999753	0.052341

18	75	-3	-43	500000.000000	9668406.820452	0.999600	0.000000
19	76	-3	-43	611129.132357	9668356.061927	0.999753	-0.052341
20	85	-3	-43	1616926.925721	9663265.807948	1.015083	-0.528835

The following points in the Arctic region are symmetrically arrayed about the North Pole, and about the central meridian 75°E and its anti-meridian 105 °W = 255 °E. (The first and last points are the same):

E.g.	Lon	Lat	Z	easting	northing	pt-scale	CoM
---	(deg)	(deg)	---	(meters)	(meters)	---	(deg)
21	-105	80	43	500000.000000	11114344.070054	0.999600	-180.000000
22	-45	80	43	-469262.805167	10560437.037836	1.011097	-120.381138
23	15	80	43	-469262.805167	9435492.848206	1.011097	-59.618862
24	75	80	43	500000.000000	8881585.815988	0.999600	0.000000
25	135	80	43	1469262.805167	9435492.848206	1.011097	59.618862
26	195	80	43	1469262.805167	10560437.037836	1.011097	120.381138
27	255	80	43	500000.000000	11114344.070054	0.999600	180.000000

7.3 Examples of computing $\{\lambda, \phi\}$, given $\{Z, x, y\}$

This subsection gives numerical examples of the computation of the longitude and latitude, given the zone number, easting and northing. Easting and northing are in meters; longitude and latitude are in degrees. The reference ellipsoid is WGS 84.

E.g.	Z	easting	northing	Lon	Lat
---	---	(meters)	(meters)	(deg)	(deg)
1	43	500000	0	75.0000000000	0.0000000000
2	43	600000	0	75.8986376602	0.0000000000
3	43	1000000	0	79.4887438844	0.0000000000
4	43	500000	2000000	75.0000000000	18.0887089431
5	43	600000	2000000	75.9450469497	18.0863946381
6	43	1000000	2000000	79.7195800291	18.0310022588
7	43	500000	4000000	75.0000000000	36.1447180988
8	43	600000	4000000	76.1114780322	36.1395604499
9	43	1000000	4000000	80.5461340659	36.0161920195
10	43	500000	6000000	75.0000000000	54.1481041039
11	43	600000	6000000	76.5307012564	54.1383733178
12	43	1000000	6000000	82.6176089075	53.9061008395
13	43	500000	8000000	75.0000000000	72.0992225251
14	43	600000	8000000	77.9124923218	72.0775365270
15	43	1000000	8000000	89.2856856739	71.5657403285
16	43	500000	10000000	-105.0000000000	89.9817727747
17	43	600000	10000000	166.1657933474	89.1041886301
18	43	1000000	10000000	165.2329617955	85.5261156460
19	43	500000	15000000	-105.0000000000	45.1168391850
20	43	600000	15000000	-106.2712189672	45.1097638704
21	43	1000000	15000000	-111.3373820793	44.9406465210
22	43	500000	20000000	-105.0000000000	-0.0368235977
23	43	600000	20000000	-105.8986378445	-0.0368190381
24	43	1000000	20000000	-109.4887448015	-0.0367098873

7.4 Administrative rules

For standard uses at DoD, there are amendments to UTM as defined above, called administrative rules. The mathematics does not require them. The most important of these rules are:

For $Z > 0$, UTM zone Z is intended for the portion of the reference ellipsoid given by:

$$\lambda_0 - 3 \text{ deg} \le \lambda < \lambda_0 + 3 \text{ deg} \quad \text{and} \quad 0 \le \phi < 84 \text{ deg}$$

For $Z < 0$, UTM zone Z is intended for:

$$\lambda_0 - 3 \text{ deg} \le \lambda < \lambda_0 + 3 \text{ deg} \quad \text{and} \quad -80 \text{ deg} \le \phi < 0$$

The inequalities above are strict or non-strict according to the administrative rule that a zone owns its southern and western boundaries. In other words, points on a zone's southern and western boundaries belong to the zone but points on its northern and eastern boundaries do not.

The above has exceptions (more administrative rules) for parts of Norway and the Arctic. These are given in [11] and are included in Subsection 7.5.

Examples 2,3,4 in Subsection 7.2 comply with the administrative rules; Examples 1,5-10 do not.

Software developers should be aware that the administrative rules cannot be applied in every situation, such as when overlapping or partly overlapping UTM grids are mandated for a map sheet. Again, see [11]. Geographic information analysts should be aware that *for analytical purposes* the administrative rules can be put aside in favor of obtaining continuous coordinates for a region of interest. For example, UTM zone 16 coordinates could be used for hurricane Katrina damage studies even where some of the damage is west of 90°W.

7.5 Given {λ, ϕ}, compute *Z*

This subsection gives the procedure to determine the value of Z for which UTM zone Z contains the point $\{\lambda, \phi\}$ in compliance with the administrative rules. At the outset, if $\lambda = 180\,\text{deg}$ is given, it should be converted to $\lambda = -180\,\text{deg}$. The inequalities $(-180\,\text{deg}) \le \lambda < 180\,\text{deg}$ and $(-80\,\text{deg}) \le \phi < 84\,\text{deg}$ should be confirmed. Then, in pseudo-code, the procedure is:

$$Z = \text{Floor}\left(\frac{\lambda + 180\,\text{deg}}{6\,\text{deg}}\right) + 1$$

if $\phi < 0$
  $Z = -Z$
if $Z = 31$ and $56\,\text{deg} \le \phi < 64\,\text{deg}$ and $\lambda \ge 3\,\text{deg}$
  $Z = 32$
else if $Z = 32$ and $\phi \ge 72\,\text{deg}$
  if $\lambda < 9\,\text{deg}$
    $Z = 31$
  else
    $Z = 33$
else if $Z = 34$ and $\phi \ge 72\,\text{deg}$
  if $\lambda < 21\,\text{deg}$
    $Z = 33$
  else
    $Z = 35$
else if $Z = 36$ and and $\phi \ge 72\,\text{deg}$
  if $\lambda < 33\,\text{deg}$
    $Z = 35$
  else
    $Z = 37$

7.6 Hierarchy of subroutines

Software design considerations are mostly beyond the scope of this document, but the following is recommended. Let the foregoing formulas and logic be gathered into subroutines under the following hierarchy, where each subroutine is a client of the one below it:

- UTM with administrative rules
- UTM
- transverse Mercator general form
- transverse Mercator preliminary general form
- transverse Mercator basic form

8. Basic Polar Stereographic

The other universal grid system is Universal Polar Stereographic (UPS) and is based on the polar stereographic projection. This section gives the formulas for the polar stereographic in its basic form. Later in Section 9, various parameters such as zone (north or south), central meridian and central scale factor will be introduced. They will enable polar stereographic to be offered in its commonly-used general form.

Stereographic projections in general are outside the scope of this document. The only stereographic projection treated herein is the *polar* stereographic projection.

8.1 Given $\{\lambda, \phi\}$, compute $\{x, y, \sigma, \gamma\}$

The basic form of the polar stereographic projection chosen for this document is centered at the north Pole and has the following for its forward mapping equations. Let $\{\lambda, \phi\}$ be the longitude and latitude, respectively, of a point on the reference ellipsoid excepting the south Pole. The rectangular coordinates $\{x, y\}$, the point-scale σ (Subsection 6.1) and the convergence-of-meridians γ (Subsection 6.2) corresponding to the given point are:

$$
\begin{aligned}
x &= f_1(\lambda, \phi) = \frac{2\,a\,(\sin\lambda)\cos\chi}{k_{90}\,(1+\sin\chi)} \\
y &= f_2(\lambda, \phi) = \frac{-2\,a\,(\cos\lambda)\cos\chi}{k_{90}\,(1+\sin\chi)} \\
\sigma &= f_3(\lambda, \phi) = \frac{2\sqrt{1-e^2\sin^2\phi}\;\exp\left(e\,\mathrm{arctanh}\,(e\sin\phi)\right)}{k_{90}(1+\sin\phi)} = \frac{2\sqrt{(1+e\sin\phi)^{1+e}\,(1-e\sin\phi)^{1-e}}}{k_{90}(1+\sin\phi)} \\
\gamma &= f_4\,(\lambda, \phi) = \lambda
\end{aligned}
\tag{18}
$$

where, from Subsection 2.8,

$$(\cos\chi,\ \sin\chi) = \mathrm{PhiToChi}\,(\phi)$$

and where, from Section 2, $\{a, e\}$ are the semi-major axis and eccentricity of the reference ellipsoid. The constant k_{90} depends only on the reference ellipsoid and is computed by:

$$k_{90} = \sqrt{1-e^2}\;\exp\,(e\,\mathrm{arctanh}\,e) = \sqrt{(1+e)^{1+e}\,(1-e)^{1-e}} \tag{19}$$

Where two formulas are given, namely for $\sigma = f_3(\lambda, \phi)$ and for k_{90}, the one using arctanh is preferred. (The notation "k_{90}" was chosen because its value is determined by the desire to have $\sigma = 1$ at the north Pole for the basic form).

For readers who are familiar with polar stereographic, or who have looked ahead to Section 9, it can be stated that the parameter choices implied by Eq. (8.18) are (i) a central meridian of longitude 0 deg, (ii) a central scale factor of 1.0000, (iii) the north Pole adopted as the "Origin" point, and (iv) a False Easting and a False Northing of 0 mE and 0 mN, respectively, assigned to that origin. This is the basic form of polar stereographic.

8.2 Given $\{x, y\}$, compute $\{\lambda, \phi\}$

The inverse mapping equations for the basic form are:

$$
\begin{aligned}
\lambda &= g_1(x,\ y) = \arctan\,(-y,\ x) \\
\phi &= g_2(x,\ y) = \mathrm{ChiToPhi}\,(\cos\chi,\ \sin\chi)
\end{aligned}
\tag{20}
$$

where the function ChiToPhi is defined in Subsection 2.9 and $\cos\chi$ and $\sin\chi$ are computed by:

$$
\begin{aligned}
\cos\chi &= \frac{2\,r}{1+r^2} \\
\sin\chi &= \frac{1-r^2}{1+r^2}
\end{aligned}
$$

where:

$$r^2 = \left(\frac{k_{90}\, x}{2\, a}\right)^2 + \left(\frac{k_{90}\, y}{2\, a}\right)^2 \quad \text{and} \quad r = \sqrt{r^2}$$

Longitude at the Poles is ambiguous, *i.e.* not well defined. For the forward mapping equations (Section 8.1) this was not a problem. The formulas there will correctly convert $\phi = \pm 90$ deg no matter what numerical value is used for λ. In this subsection, the ambiguity is a problem. The attempted computation of λ in Eq. (8.20) will fail when the math-library routine for arctangent encounters arctan(0, 0). This will happen at a Pole, where $x = 0$ and $y = 0$. To get around this, let the software define a constant, $\lambda_{\text{pole}} = 0$ (suggested), and execute $\lambda = \lambda_{\text{pole}}$ if $x = 0$ and $y = 0$, and execute Eq. (8.20) otherwise.

9. Polar Stereographic with Parameters

Section 8 presented the basic form of the polar stereographic projection. In this section, the basic form is extended two ways: (i) Where Section 8 measured the longitude from the prime meridian, this section will allow longitude to be measured from any specified meridian ("central meridian"). Then (ii), the easting-northing pairs $\{x, y\}$ obtained from that section will be subjected to a homothetic transformation in this section, *i.e.* subjected to a translation and/or a proportional re-sizing. This follows the pattern of Section 5. (For polar stereographic but not for transverse Mercator, the combination of (i) and (ii) is a similarity transformation).

9.1 General form (k_0)

Let $Z = \pm 1$ be a flag such that $Z = 1$ (respectively, $Z = -1$) indicates the north (respectively, south) polar stereographic projection. Let λ_0 be a constant in radians, $k_0 > 0$ be a unitless constant, and x_{pole} and y_{pole} be constants in meters. Then a general form of the polar stereographic forward mapping equations is:

$$
\begin{aligned}
&\text{For } Z = +1,\\
&\qquad x = k_0\, f_1(\lambda - \lambda_0, \phi) + x_{\text{pole}}\\
&\qquad y = k_0\, f_2(\lambda - \lambda_0, \phi) + y_{\text{pole}}\\
&\qquad \sigma = k_0\, f_3(\lambda - \lambda_0, \phi)\\
&\qquad \gamma = f_4(\lambda - \lambda_0, \phi) = \lambda - \lambda_0\\
&\text{For } Z = -1,\\
&\qquad x = k_0\, f_1(\lambda - \lambda_0, -\phi) + x_{\text{pole}}\\
&\qquad y = -k_0\, f_2(\lambda - \lambda_0, -\phi) + y_{\text{pole}}\\
&\qquad \sigma = k_0\, f_3(\lambda - \lambda_0, -\phi)\\
&\qquad \gamma = -f_4(\lambda - \lambda_0, -\phi) = -\lambda + \lambda_0
\end{aligned}
\tag{21}
$$

where $\{x, y, \sigma, \gamma\}$ are the easting, northing, point-scale and CoM corresponding to the reference ellipsoid point at longitude λ and latitude ϕ, and where functions f_1, f_2, f_3, and f_4 are defined in Subsection 8.1.

The constants, also called parameters, have these notations, names and units:

λ_0	central meridian, longitude down from the Pole ($Z = 1$), longitude up from the Pole ($Z = -1$).	radians
k_0	central scale, point-scale at the Pole, scale at the Pole	(unitless)
x_{pole}	easting of the Pole	meters
y_{pole}	northing of the Pole	meters

The parameter k_0 controls the proportional re-sizing and the parameters x_{pole} and y_{pole} control the translation mentioned in the introduction to this section. The corresponding inverse mapping equations are:

$$
\begin{aligned}
&\text{For } Z = 1,\\
&\qquad \lambda = \lambda_0 + g_1\!\left(\frac{x - x_{\text{pole}}}{k_0}, \frac{y - y_{\text{pole}}}{k_0}\right)\\
&\qquad \phi = g_2\!\left(\frac{x - x_{\text{pole}}}{k_0}, \frac{y - y_{\text{pole}}}{k_0}\right)\\
&\text{For } Z = -1,\\
&\qquad \lambda = \lambda_0 + g_1\!\left(\frac{x - x_{\text{pole}}}{k_0}, \frac{y - y_{\text{pole}}}{-k_0}\right)\\
&\qquad \phi = -g_2\!\left(\frac{x - x_{\text{pole}}}{k_0}, \frac{y - y_{\text{pole}}}{-k_0}\right)
\end{aligned}
\tag{22}
$$

The quantity λ computed according to Eq. (9.22) lies in the interval $\lambda_0 - \pi < \lambda \le \lambda_0 + \pi$. To convert it to a longitude lying in a different interval (of length 2π), the quantity 2π should be added or subtracted to it as necessary.

The list $\{Z, \lambda_0, k_0, x_{\text{pole}}, y_{\text{pole}}\}$ is a set of unique independently-specifiable parameters.

9.2 Origin

(This subsection is deliberately almost the same wording as Subsection 5.2 “Origin”).

The equations and parameters of Subsection 9.1 accomplish the goals stated in the Section 9 introduction, which were to (i) specify a meridian of reference (the meridian λ_0), (ii) apply a proportional re-sizing (the factor k_0) and (iii) apply a translation (the vector $\{x_{\text{pole}}, y_{\text{pole}}\}$). More options are not a necessity. However, for convenience, an alternate method to accomplish the translation is possible, and is now explained.

A point on the reference ellipsoid is selected for special treatment. It must lie in the ellipsoid coverage area (*i.e.* outside a small region around the opposite Pole) and is called the Origin. Let its longitude and latitude be notated λ_{origin} and ϕ_{origin}, respectively. On the map projection plane, the Origin is to have rectangular coordinates $\{x, y\} = \{x_{\text{origin}}, y_{\text{origin}}\}$. This will determine the translation under consideration.

The above parameters have these notations, names, and units:

λ_{origin}	Origin longitude	radians
ϕ_{origin}	Origin latitude	radians
x_{origin}	(Origin easting), False Easting, FE	meters
y_{origin}	(Origin northing), False Northing, FN	meters

(If there was an opportunity to revise the terminology, “Origin easting” and “Origin northing” would make sense. Accepted terminology is “False Easting” and “False Northing”).

9.3 Given $\{\lambda_{\text{origin}}, \phi_{\text{origin}}, x_{\text{origin}}, y_{\text{origin}}\}$, compute $\{x_{\text{pole}}, y_{\text{pole}}\}$

Let the reference ellipsoid and polar stereographic parameters Z, λ_0, and k_0 be fixed. Let the parameters $\{\lambda_{\text{origin}}, \phi_{\text{origin}}, x_{\text{origin}}, y_{\text{origin}}\}$ be given. To obtain values for the parameters $\{x_{\text{pole}}, y_{\text{pole}}\}$ that yield the same translation, the following applies:

$$
\begin{aligned}
&\text{For } Z = 1,\\
&\qquad x_{\text{pole}} = x_{\text{origin}} - k_0\, f_1(\lambda_{\text{origin}} - \lambda_0,\ \phi_{\text{origin}})\\
&\qquad y_{\text{pole}} = y_{\text{origin}} - k_0\, f_2(\lambda_{\text{origin}} - \lambda_0,\ \phi_{\text{origin}})\\
&\text{For } Z = -1,\\
&\qquad x_{\text{pole}} = x_{\text{origin}} - k_0\, f_1(\lambda_{\text{origin}} - \lambda_0,\ -\phi_{\text{origin}})\\
&\qquad y_{\text{pole}} = y_{\text{origin}} + k_0\, f_2(\lambda_{\text{origin}} - \lambda_0,\ -\phi_{\text{origin}})
\end{aligned}
\tag{23}
$$

9.4 Other meanings of “Origin”

The polar stereographic projection as defined in this document should be treated as a map projection in its own right and not as a special case of more general kinds of map projections that are called “stereographic”. Consequently, the set of parameters has been tailored to this purpose. Moving the origin as accomplished by choosing values for $\{\lambda_{\text{origin}}, \phi_{\text{origin}}, x_{\text{origin}}, y_{\text{origin}}\}$ and applying Eq. (9.23) has no affect on the shape, size, or orientation of any feature portrayed on the map. It affects only the up/down placement of the x-axis and left/right placement of the y-axis on the map projection plane.

By contrast, literature and software that treats the more general “stereographic” projection (not defined in this document) might use the term “Origin” differently. Its use might include a parameter called “latitude of Origin” and require it to be 90° to obtain the *polar* stereographic projection.

In this document, the concept of origin and the meaning of $\{\lambda_{\text{origin}}, \phi_{\text{origin}}, x_{\text{origin}}, y_{\text{origin}}\}$ are consistent between polar stereographic and transverse Mercator. This is to the advantage of cartographers and geographic information analysts having to try both map projections.

9.5 General form (k_0, arbitrary origin)

An alternate general form of the polar stereographic projection is Eqs. (9.21 and 9.22) with the further stipulations that x_{pole} and y_{pole} are taken as intermediate variables computed according to Eq. (9.23) and that the list

$\{Z, \lambda_0, k_0, \lambda_{\text{origin}}, \phi_{\text{origin}}, x_{\text{origin}}, y_{\text{origin}}\}$ is adopted as the general form's set of (non-unique) independently-specifiable parameters. (The list is non-unique because more than one quadruple $\{\lambda_{\text{origin}}, \phi_{\text{origin}}, x_{\text{origin}}, y_{\text{origin}}\}$ will define the same translation, i.e. the same $\{x_{\text{pole}}, y_{\text{pole}}\}$ values).

9.6 Standard parallel

The standard parallel, also called the latitude of unity scale, is the value of ϕ in Eq. (9.21) that gives $\sigma = 1$. It will exist if $k_0 \le 1$. Its notation in this document is ϕ_1.

9.7 Given ϕ_1, compute k_0

Given the standard parallel ϕ_1, the formula to find the scale factor k_0 at the relevant Pole is:

For $Z = 1$,

$$k_0 = \frac{1}{f_3(0, \phi_1)} = \frac{k_{90}(1 + \sin\phi_1)}{2\sqrt{(1 + e\sin\phi_1)^{1+e}(1 - e\sin\phi_1)^{1-e}}}$$

For $Z = -1$, (24)

$$k_0 = \frac{1}{f_3(0, -\phi_1)} = \frac{k_{90}(1 - \sin\phi_1)}{2\sqrt{(1 - e\sin\phi_1)^{1+e}(1 + e\sin\phi_1)^{1-e}}}$$

9.8 Given k_0, compute ϕ_1

For $Z = \pm 1$, let a value $k_0 < 1$ for the scale factor at the Pole be given. Then the method to compute the standard parallel ϕ_1 is:

$$\phi_1 = Z \arcsin s$$

where s is the limit (within the desired resolution) of the sequence $s_1, s_2, s_3, \ldots$ whose members are computed by:

$$s_1 = -1 + 2\,k_0$$

$$s_{n+1} = \frac{2\,k_0\sqrt{(1 + e\,s_n)^{1+e}(1 - e\,s_n)^{1-e}}}{k_{90}} - 1$$

9.9 General form (ϕ_1, arbitrary origin)

Another general form of the polar stereographic projection is Eqs. (9.21 and 9.22) with the further stipulations that x_{pole}, y_{pole}, and k_0 are taken as intermediate variables computed according to Eqs. (9.23 and 9.24) and that the list $\{Z, \lambda_0, \phi_1, \lambda_{\text{origin}}, \phi_{\text{origin}}, x_{\text{origin}}, y_{\text{origin}}\}$ is adopted as the general form's set of (non-unique) independently-specifiable parameters. Note that ϕ_1 replaces k_0 in the list.

The adjectives "unique" and "independently-specifiable" and their negatives have been used carefully in this section when describing lists of polar stereographic parameters. As another example, consider the list $\{Z, \lambda_0, k_0, \phi_1, x_{\text{pole}}, y_{\text{pole}}\}$. Its parameters are all unique, but they are not all independently-specifiable because both k_0 and ϕ_1 are listed.

9.10 Examples of conversions between ϕ_1 and k_0

The following two tables pertain to the north polar stereographic projection ($Z = 1$) of the WGS 84 ellipsoid ($a = 6\,378\,137$ and $f^{-1} = 298.257223563$).

In the table at left, the values of ϕ_1 are exact and the values of k_0 are computed to as many digits shown. In the table at right, the values of k_0 are exact, and the values of ϕ_1 are computed to as many digits as are shown.

φ1 degree	k0 unitless
−75	0.017 259 384 673
−60	0.067 773 950 243
−45	0.147 883 853 421
−30	0.251 891 492 664
−15	0.372 562 837 459
0	0.501 678 277 625
15	0.630 570 160 065
30	0.750 629 794 742
45	0.853 799 593 615
60	0.933 069 071 736
75	0.982 966 757 777
80	0.992 404 648 246
81	0.993 844 677 874
82	0.995 134 351 941
83	0.996 273 262 333
84	0.997 261 048 527
85	0.998 097 397 746
86	0.998 782 045 101
87	0.999 314 773 702
88	0.999 695 414 760
89	0.999 923 847 656
90	1.000 000 000 000

k0 unitless	φ1 degree
0.1000	−53.337 403 999 811
0.2000	−37.116 011 177 617
0.3000	−23.825 251 373 649
0.4000	−11.763 627 302 241
0.5000	−0.192 963 050 538
0.6000	11.385 608 705 462
0.7000	23.471 956 301 947
0.8000	36.808 078 424 089
0.9000	53.106 923 780 672
0.9100	55.064 894 505 323
0.9200	57.123 352 185 495
0.9300	59.302 785 064 482
0.9400	61.631 355 734 180
0.9500	64.149 649 327 832
0.9600	66.920 027 216 673
0.9700	70.047 603 511 896
0.9800	73.737 632 650 010
0.9900	78.520 890 585 055
0.9910	79.111 860 671 964
0.9920	79.736 353 686 644
0.9930	80.400 910 884 378
0.9940	81.114 517 868 594
0.9950	81.890 113 174 369
0.9960	82.747 558 146 254
0.9970	83.720 292 647 173
0.9980	84.873 530 910 724
0.9990	86.375 668 096 133
0.9991	86.561 716 515 710
0.9992	86.758 411 486 352
0.9993	86.967 824 204 285
0.9994	87.192 799 401 974
0.9995	87.437 432 634 553
0.9996	87.708 009 826 129
0.9997	88.015 112 711 542
0.9998	88.379 374 422 928
0.9999	88.854 064 538 034
1.0000	90.000 000 000 000

10. Universal Polar Stereographic (UPS)

This section gives the definition of UPS, some numerical examples of it, and the administrative rules added to it.

10.1 Definition of UPS

The Universal Polar Stereographic (UPS) system is these two instances of the polar stereographic projection with parameters. The parameters fit Eqs. (9.21 and 9.22).

North UPS is defined by:

$Z = 1$
$\lambda_0 = 0$ (longitude down from the Pole)
$k_0 = 0.994$ (exactly)
$x_{\text{pole}} = 2\,000\,000$
$y_{\text{pole}} = 2\,000\,000$

South UPS is defined by:

$Z = -1$
$\lambda_0 = 0$ (longitude up from the Pole)
$k_0 = 0.994$ (exactly)
$x_{\text{pole}} = 2\,000\,000$
$y_{\text{pole}} = 2\,000\,000$

10.2 Examples of computing $\{x, y, \sigma, \gamma\}$, given $\{\lambda, \phi, Z\}$

The following computations pertain to the WGS 84 ellipsoid.

E.g.	Lon	Lat	Z	easting	northing	pt-scale	CoM
---	(deg)	(deg)	---	(meters)	(meters)	---	(deg)
1	---	90	1	2000000.000000	2000000.000000	0.994000	---
2	-179	89	1	1998062.320046	2111009.610243	0.994076	-179
3	-90	88	1	1777930.731071	2000000.000000	0.994303	-90
4	-1	87	1	1994185.827038	1666906.254073	0.994682	-1
5	0	86	1	2000000.000000	1555731.570643	0.995212	0
6	1	85	1	2009694.068153	1444627.207468	0.995895	1
7	89	84	1	2666626.157825	1988363.997132	0.996730	89
8	90	83	1	2778095.750322	2000000.000000	0.997718	90
9	91	82	1	2889442.490749	2015525.276426	0.998860	91
10	179	81	1	2017473.190606	3001038.419357	1.000156	179
11	180	80	1	2000000.000000	3112951.136955	1.001608	180
12	0	40	1	2000000.000000	-3918313.984953	1.209619	0
13	-179	3	1	1790630.987261	13994742.706481	1.883453	-179
14	-90	2	1	-10206568.118587	2000000.000000	1.914973	-90
15	-1	1	1	1783239.204558	-10418217.653909	1.947589	-1
16	0	0	1	2000000.000000	-10637318.498257	1.981349	0
17	1	-1	1	2224408.737826	-10856367.979638	2.016305	1
18	90	-2	1	15083269.373905	2000000.000000	2.052510	90
19	179	-3	1	2232331.498720	15310262.647286	2.090020	179
20	180	-4	1	2000000.000000	15545537.944524	2.128897	180

10.3 Examples of computing $\{\lambda, \phi\}$, given $\{Z, x, y\}$

The following computations pertain to the WGS 84 ellipsoid.

E.g.	Z	easting	northing	Lon	Lat
---	---	(meters)	(meters)	(deg)	(deg)
1	-1	0	0	-135.0000000000	-64.9164123332
2	-1	1000000	0	-153.4349488229	-70.0552944014
3	-1	2000000	0	-180.0000000000	-72.1263610163
4	-1	3000000	0	153.4349488229	-70.0552944014
5	-1	4000000	0	135.0000000000	-64.9164123332
6	-1	0	1000000	-116.5650511771	-70.0552944014
7	-1	1000000	1000000	-135.0000000000	-77.3120791908
8	-1	2000000	1000000	180.0000000000	-81.0106632645

9	-1	3000000	1000000	135.0000000000	-77.3120791908
10	-1	4000000	1000000	116.5650511771	-70.0552944014
11	-1	0	2000000	-90.0000000000	-72.1263610163
12	-1	1000000	2000000	-90.0000000000	-81.0106632645
13	-1	2000000	2000000	---	-90.0000000000
14	-1	3000000	2000000	90.0000000000	-81.0106632645
15	-1	4000000	2000000	90.0000000000	-72.1263610163
16	-1	0	3000000	-63.4349488229	-70.0552944014
17	-1	1000000	3000000	-45.0000000000	-77.3120791908
18	-1	2000000	3000000	0.0000000000	-81.0106632645
19	-1	3000000	3000000	45.0000000000	-77.3120791908
20	-1	4000000	3000000	63.4349488229	-70.0552944014
21	-1	0	4000000	-45.0000000000	-64.9164123332
22	-1	1000000	4000000	-26.5650511771	-70.0552944014
23	-1	2000000	4000000	0.0000000000	-72.1263610163
24	-1	3000000	4000000	26.5650511771	-70.0552944014
25	-1	4000000	4000000	45.0000000000	-64.9164123332

10.4 Administrative rules

For standard uses at DoD, there are amendments to UPS as defined above, called administrative rules. The mathematics does not require them. They are: (i) north UPS coordinates ($Z = 1$) may be used for the region defined by $\phi \geq 84$ deg, and (ii) south UPS coordinates ($Z = -1$) may be used for the region defined by $\phi < -80$ deg.

10.5 Hierarchy of subroutines

The suggested hierarchy of subroutines (where calls are made to subroutines further down the list) is the following:

- UPS with administrative rules
- UPS
- polar stereographic general forms
- routines to convert between k_0 and ϕ_1
- polar stereographic basic form

11. Military Grid Reference System (MGRS)

The Military Grid Reference System (MGRS) is the pair, UTM and UPS taken together, with some digits dropped or replaced by letters and with other notations and rules added. Subsections 11.1 to 11.8 specify the UTM to MGRS conversion, and Subsections 11.9 to 11.12 specify the UPS to MGRS conversion. The inverse conversion, MGRS to UTM or UPS , is explained in Subsections 11.13 and 11.14. The section ends with a re-print of some old but still valid tables about MGRS lettering.

The agenda of this document is the programming logic needed by the software developer. Basic explanations of MGRS for land navigation and policies for tactical forces to report positions or define operational areas are outside the scope of this document.

11.1 Character string for the UTM portion of MGRS

The UTM portion of MGRS is the following sequence of letters and digits. From left to right they are:

(i) One or two decimal digits, representing the UTM zone number in absolute value
(ii) A letter in the range "C" to "X", representing an interval of latitude
(iii) Two letters — an easting letter and a northing letter — representing a square that is 100 000 meters on a side
(iv) Zero to five decimal digits, representing the UTM easting to desired precision
(v) The same number of decimal digits, representing the UTM northing to the same precision

To facilitate machine-to-machine communication, an MGRS string is to have no intermediate spaces or punctuation marks and all the letters are to be capitals. Letters "I" and "O" are never used. For (i), if the UTM zone number is less than 10 in absolute value, a leading zero is preferred but not mandated. Consequently, software for information processing should accept both 5NAB123123 and 05NAB123123, for example, but should produce only 05N-AB123123. End-user devices and map margin notes may show 5NAB123123.

11.2 Lettering scheme "AA"

This subsection specifies one of the schemes for picking two letters to represent the 100 000 meter square, *i.e.* item (iii) of Subsection 11.1.

Let Z be the UTM zone and $\{x,\ y\}$ be the UTM easting and northing (in meters) of a point within these limits:

$$100\,000 \le x < 900\,000$$
$$0 \le y < 9\,700\,000 \quad \text{if} \quad Z > 0$$
$$300\,000 \le y < 10\,000\,000 \quad \text{if} \quad Z < 0$$

The 100 000 meter square identifier consists of an easting letter followed by a northing letter. The easting letter is the conversion of $\text{Floor}\,(x/100\,000)$ according to the following set of tables:

1	2	3	4	5	6	7	8
A	B	C	D	E	F	G	H

, if $\text{Mod}\,(|Z|,\ 3) = 1$

1	2	3	4	5	6	7	8
J	K	L	M	N	P	Q	R

, if $\text{Mod}\,(|Z|,\ 3) = 2$

1	2	3	4	5	6	7	8
S	T	U	V	W	X	Y	Z

, if $\text{Mod}\,(|Z|,\ 3) = 0$

where, for $n > 0$, $\text{Mod}\,(n,\ 3)$ is the remainder when n is divided by 3.

The northing letter is the conversion of $\text{Floor}\,(\text{Mod}\,(y,\ 2\,000\,000)/100\,000)$ according to the following set of tables:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q	R	S	T	U	V

, if $|Z|$ is odd

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
F	G	H	J	K	L	M	N	P	Q	R	S	T	U	V	A	B	C	D	E

, if $|Z|$ is even

Notice that the letters "I" and "O" are deliberately omitted from the above tables. The notation "AA" for this lettering scheme comes from the fact that for $Z = 1$ (and other Z), the southwest corner of allowed values of $\{x, y\}$ is square AA.

11.3 Lettering scheme "AL"

This subsection specifies another scheme for picking two letters to represent the 100 000 meter square, *i.e.* item (iii) of Subsection 11.1.

Let Z be the UTM zone and $\{x, y\}$ be the UTM easting and northing of a point within the same limits as for scheme "AA".

The 100 000 meter square identifier consists of an easting letter followed by a northing letter. The easting letter is the same as for scheme "AA".

The northing letter is the conversion of Floor (Mod (y, 2 000 000) / 100 000) according to the following set of tables:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
L	M	N	P	Q	R	S	T	U	V	A	B	C	D	E	F	G	H	J	K

, if $|Z|$ is odd

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
R	S	T	U	V	A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q

, if $|Z|$ is even

Notice that the letters "I" and "O" are deliberately omitted from the above tables. The notation "AL" for this lettering scheme comes from the fact that for $Z = 1$ (and other Z), the southwest corner of allowed values of $\{x, y\}$ is square AL.

11.4 Which lettering scheme to use

This subsection pertains to item (iii) of Subsection 11.1

For all usages of MGRS within the WGS 84 datum and ellipsoid, the lettering scheme to use should be "AA".

If not operating within the WGS 84 datum, the lettering scheme to use depends on the reference ellipsoid to which the UTM coordinates refer. If the reference ellipsoid is Bessel 1841 (Ethiopia, Asia) (BR), or Bessel 1841 (Namibia) (BN), or Clarke 1866 (CC), or Clarke 1880 (CD), or Clarke 1880 (IGN) (CG), then scheme "AL" is to be used. For all other ellipsoids, scheme "AA" is to be used.

11.5 Lettering schemes on old maps

MGRS predates the establishment of WGS 84 and was invented when no global 3D geodetic datum had yet gained preeminence. Consequently, MGRS historically employed at least the two lettering schemes explained — "AA" and "AL". This was done to highlight a change of datum when crossing into an adjacent area on a competing datum. (A change in datum is usually accompanied by a change in reference ellipsoid). A review of the inventory of U.S. and NATO maps and charts to investigate this further is outside the scope of this document, but the following should be mentioned as an example of the Subsection 11.4 rule:

The horizontal datum for the United States for many decades of the 20th century was the North American Datum of 1927 which uses the Clarke 1866 ellipsoid. When an MGRS position is specified using this datum, as may happen with old maps of U.S. military installations, lettering scheme "AL" is used.

Also to be found are usages of "AA" and "AL" outside of the Subsection 11.4 rule and letterings compliant with neither "AA" nor "AL". Edition 1 of [11] contains an advisory worth repeating here: "Users are cautioned that deviations from the combined AA-or-AL lettering schemes were made in the past. These deviations were an attempt to provide unique grid references within a complicated and disparate world-wide mapping system."

The foregoing has implications for the software developer. The Subsection 11.4 rule should be segregated and made into a separate table with room for amendment and not combined with the logic of Subsections 11.2 and 11.3. Further, if new lettering schemes are discovered and software support for them is wanted, the logic for them should be patterned after Subsections 11.2 and 11.3. For example, lettering scheme "AF" is built on the pattern of "AA" and "AL".

11.6 Precision and digits

Let $\{x, y\}$ be the UTM coordinates to be converted to an MGRS string. The rules for MGRS provide a choice of six levels of precision. With each level of precision, there is a fixed number of digits for the easting and the same number of digits for the northing. See items (iv) and (v) of Subsection 11.1.

Precision (meters)	no. of digits (n)
1	5
10	4
100	3
1000	2
10,000	1
100,000	0

Let n be the number of easting digits to be displayed in the MGRS string. For $n = 0$, there are no digits to be displayed. For $n > 0$, the easting digits are those of the number $\text{Floor}\left(\text{Mod}\left(x,\ 10^5\right)/10^{5-n}\right)$ and the northing digits are those of the number $\text{Floor}\left(\text{Mod}\left(y,\ 10^5\right)/10^{5-n}\right)$. The number 10^{5-n} is the precision in meters corresponding to n. This completes the specification of items (iv) and (v) of Subsection 11.1.

11.7 Latitude band letter

The MGRS latitude band letter, *i.e.* item (ii) of Subsection 11.1, is the conversion of $\text{Floor}(\phi/(8\,\text{deg}))$ to a letter according to the following table:

−11	−10	−9	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8	9	10
C	C	D	E	F	G	H	J	K	L	M	N	P	Q	R	S	T	U	V	W	X	X

where latitude ϕ lies in the interval $-88\ \text{deg} \le \phi < 88\ \text{deg}$. The letters "C" and "X" occur twice as shown.

Consequently, the UTM to MGRS conversion requires also the UTM to Lon./Lat. conversion. This means executing the inverse mapping equations for transverse Mercator (Eq. 5.16) with, of course, the parameters for UTM (Subsection 7.1). This is necessary to obtain the latitude ϕ required above.

11.8 Latitude band letter example

Here is an example of the UTM portion of MGRS and the nuisance caused by the latitude band letter. The example uses the WGS 84 ellipsoid.

`1 3 V F C 4 9 6 6 1 0 8 6 7 9`	A point in western Canada near 102.6°W, 56°N
`. . . 1 0 . . . . .`	Move 10 m in the easting direction
`1 3 V F C 4 9 6 7 1 0 8 6 7 9`	New position after the move, so it would seem, but the latitude band letter "V" is not correct
`1 3 U F C 4 9 6 7 1 0 8 6 7 9`	Correct new position

UTM is independent of longitude/latitude when doing displacement calculations of the above kind. This is not true for MGRS as the above example shows. Application-software developers should be aware of this and do all plane-geometry calculations in UTM and only use MGRS to convert the inputs or outputs, as needed.

11.9 Character string for the UPS portion of MGRS

For the UPS portion of MGRS, a sequence of letters and digits is specified from left to right as:

(i) Three letters — two easting letters and one northing letter — representing a square that is 100 000 meters on a side

(ii) Zero to five decimal digits, representing the UTM easting to desired precision

(iii) The same number of decimal digits, representing the UTM northing to the same precision

To be strictly correct and to facilitate machine-to-machine communication, an MGRS string is to have no intermediate spaces or punctuation marks and all the letters are to be capitals. Letters "I" and "O" are never used.

11.10 Lettering scheme "UPS north"

This subsection specifies the scheme for picking two letters to represent the 100 000 meter square for the UPS portion of MGRS, *i.e.* item (i) of Subsection 11.9, when the UPS zone is north, i.e. $Z = 1$.

Let $Z = 1$ be the UPS zone, and let $\{x,\ y\}$ be the UPS easting and northing (in meters) of a point within these limits:

$$1\,300\,000 \le x < 2\,700\,000$$
$$1\,300\,000 \le y < 2\,700\,000$$

The 100 000 meter square identifier consists of two easting letters followed by a northing letter. The two easting letters are the conversion of Floor($x/100\,000$) according to the following table:

13	14	15	16	17	18	19	20	21	22	23	24	25	26
YR	YS	YT	YU	YX	YY	YZ	ZA	ZB	ZC	ZF	ZG	ZH	ZJ

Note that YU is followed on the right by YX (skipping YV and YW) and ZC is followed on the right by ZF (skipping ZD and ZE).

The northing letter is the conversion of Floor($y/100\,000$) according to the following table:

13	14	15	16	17	18	19	20	21	22	23	24	25	26
A	B	C	D	E	F	G	H	J	K	L	M	N	P

Notice that the letters “I” and “O” are deliberately omitted from the above tables.

■ 11.11 Lettering scheme “UPS south”

This subsection specifies the scheme for picking two letters to represent the 100 000 meter square for the UPS portion of MGRS, *i.e.* item (i) of Subsection 11.9, when the UPS zone is south, i.e. $Z = -1$.

Let $Z = -1$ be the UPS zone, and let $\{x,\ y\}$ be the UPS easting and northing (in meters) of a point within these limits:

$$800\,000 \le x < 3\,200\,000$$
$$800\,000 \le y < 3\,200\,000$$

The 100 000 meter square identifier consists of two easting letters followed by a northing letter. The two easting letters are the conversion of Floor ($x/100\,000$) according to the following table (shown in two pieces):

8	9	10	11	12	13	14	15	16	17	18	19
AJ	AK	AL	AP	AQ	AR	AS	AT	AU	AX	AY	AZ

20	21	22	23	24	25	26	27	28	29	30	31
BA	BB	BC	BF	BG	BH	BJ	BK	BL	BP	BQ	BR

Note that AL is followed on the right by AP (skipping AM and AN) and that other skips occur. The northing letter is the conversion of Floor ($y/100\,000$) according to the following table (shown in two pieces):

8	9	10	11	12	13	14	15	16	17	18	19
A	B	C	D	E	F	G	H	J	K	L	M

20	21	22	23	24	25	26	27	28	29	30	31
N	P	Q	R	S	T	U	V	W	X	Y	Z

Notice that the letters “I” and “O” are deliberately omitted from the above tables.

■ 11.12 Precision and digits

Let $\{x,\ y\}$ be the UPS coordinates to be converted to an MGRS string. There is a choice of six levels of precision. The rules about this are the same as for the UTM portion of MGRS in Subsection 11.6 and are repeated here. With each level of precision, there is a fixed number of digits for the easting and the same number of digits for the northing as follows:

Precision (meters)	no. of digits (n)
1	5
10	4
100	3
1000	2

10,000	1
100,000	0

Let n be the number of easting digits to be displayed in the MGRS string. For $n = 0$, there are no digits to be displayed. For $n > 0$, the easting digits are those of the number $\text{Floor}\left(\text{Mod}\left(x,\ 10^5\right) / 10^{5-n}\right)$ and the northing digits are those of the number $\text{Floor}\left(\text{Mod}\left(y,\ 10^5\right) / 10^{5-n}\right)$. The number 10^{5-n} is the precision in meters corresponding to n. This completes the specification of items (ii) and (iii) of Subsection 11.9.

This subsection completes the specification of the UPS portion of MGRS.

11.13 Conversion of MGRS to UTM or UPS

If the first character of an MGRS string is a digit, the string belongs to the UTM portion of MGRS and can be converted to UTM coordinates. Otherwise the string belongs to the UPS portion of MGRS and can be converted to UPS coordinates. In all cases, the easting x is obtained by:

$$x = 100\,000\, x_{\text{letter}} + 10^{5-n}\, x_{\text{digits}}$$

where x_{letter} is the number listed in the appropriate lettering-scheme table for the given easting letter(s) and x_{digits} is the number defined by the n given easting digits, assuming some easting digits were given. If no easting (northing) digits are given, then $x_{\text{digits}} = 0$.

For the **UPS portion of MGRS**, the northing y is obtained by:

$$y = 100\,000\, y_{\text{letter}} + 10^{5-n}\, y_{\text{digits}}$$

where y_{letter} is the number listed in the appropriate lettering-scheme table for the given northing letter and y_{digits} is the number defined by the n given northing digits. If no easting (northing) digits are given, then $y_{\text{digits}} = 0$. This concludes the MGRS to UPS conversion.

If the first character of the MGRS string is a digit, the string belongs to the **UTM portion of MGRS**, as has been said. The UTM Zone number Z is the leading digit(s) of the MGRS string, taken as a positive number if the MGRS latitude band letter is in the range N-X and taken as a negative number if the latitude band letter is in the range C-M.

Obtaining the UTM northing y requires several steps. A preliminary northing y_{prelim} is obtained by:

$$y_{\text{prelim}} = 100\,000\, y_{\text{letter}} + 10^{5-n}\, y_{\text{digits}}$$

where, like above, y_{letter} is the number listed in the appropriate lettering-scheme table for the given northing letter and y_{digits} is the number defined by the n given northing digits. If no easting (northing) digits are given, then $y_{\text{digits}} = 0$. Then the northing y is calculated:

$$y = 2\,000\,000\, y_{\text{band}} + y_{\text{prelim}}$$

where y_{band} is the choice among 0,1,2,3 and 4 that satisfies the requirement that converting the obtained UTM coordinates $\{x,\ y\}$ back to $\{\lambda,\ \phi\}$ yields a latitude ϕ lying in the given MGRS latitude band (see Subsection 11.7). To help choose among 0,1,2,3 and 4, a trial value may be obtained from row 2 of the following table. The first row is the MGRS latitude band letter; the other rows give the possible values of y_{band}. For some columns (*e.g.* column "E"), there is only one possibility and the trial value is the actual value. In such cases, a UTM-to-Lon/Lat calculation is not needed.

C	D	E	F	G	H	J	K	L	M	N	P	Q	R	S	T	U	V	W	X
1	1	1	2	2	3	3	4	4	4	0	0	0	1	1	2	2	3	3	3
0	0		1		2		3					1		2		3		4	4

11.14 MGRS to UTM conversion example

An example of a MGRS-to-UTM conversion is now given. Consider the MGRS string 06STB1980012345 for a point in the central Pacific referred to the WGS 84 ellipsoid. Picking it apart, in order, gives UTM absolute zone 06, latitude band S, easting letter T, northing letter B, easting digits 19800, and northing digits 12345. The UTM zone is $Z = 6$, (rather than $Z = -6$), because S falls in the sequence N–X. The easting is considered first. Since

Mod (|Z|, 3) = Mod (6, 3) = 0, entering the Section 11.2 tables with T yields $x_{\text{letter}} = 2$. Note that the precision is one meter using $n = 5$ digits. Combining $x_{\text{letter}} = 2$ with $x_{\text{digits}} = 19\,800$ gives $x = 219\,800$ meters for the UTM easting. The northing is considered next. Since $|Z| = 6$ is even, entering the Section 11.2 tables with B yields $y_{\text{letter}} = 16$. Combining $y_{\text{letter}} = 16$ with $y_{\text{digits}} = 12\,345$ gives $y_{\text{prelim}} = 1\,612\,345$ meters. The Section 11.13 table under S is consulted to obtain the possible values $y_{\text{band}} = 1, 2$. They generate the possibilities $y_1 = 3\,612\,345$ or $y_2 = 5\,612\,345$ for the UTM northing y. The coordinates $(x,\ y_1)$ convert to $\lambda = -149.98596$ deg and $\phi = 32.61320$ deg. The Section 11.7 table is entered with the calculation Floor $(\phi/(8\text{ deg})) = 4$ to re-obtain S as the MGRS latitude band letter. This decides in favor of y_1 over y_2. Therefore, the UTM coordinates are $x = 219\,800$ and $y = 3\,612\,345$ in zone $Z = 6$.

■ 11.15 Legacy tables for the lettering schemes

The methods of this Section to find the easting/northing letters given the numerical $(x,\ y)$ coordinates employed the one-dimensional tables found in Sections 11.2, 11.3, 11.10, and 11.11. This provided succinct logic for the software developer. The equivalent and familiar two dimensional tables for lettering schemes “AA” and “AL” are provided (newly printed) on the next two pages. For the two dimensional version of the UPS-related lettering scheme tables, see the plots in Section 15.

	Zones 1, 7, 13, 19, 25, 31, 37, 43, 49, 55							
2000	AV	BV	CV	DV	EV	FV	GV	HV
	AU	BU	CU	DU	EU	FU	GU	HU
	AT	BT	CT	DT	ET	FT	GT	HT
	AS	BS	CS	DS	ES	FS	GS	HS
	AR	BR	CR	DR	ER	FR	GR	HR
1500	AQ	BQ	CQ	DQ	EQ	FQ	GQ	HQ
	AP	BP	CP	DP	EP	FP	GP	HP
	AN	BN	CN	DN	EN	FN	GN	HN
	AM	BM	CM	DM	EM	FM	GM	HM
	AL	BL	CL	DL	EL	FL	GL	HL
1000	AK	BK	CK	DK	EK	FK	GK	HK
	AJ	BJ	CJ	DJ	EJ	FJ	GJ	HJ
	AH	BH	CH	DH	EH	FH	GH	HH
	AG	BG	CG	DG	EG	FG	GG	HG
	AF	BF	CF	DF	EF	FF	GF	HF
500	AE	BE	CE	DE	EE	FE	GE	HE
	AD	BD	CD	DD	ED	FD	GD	HD
	AC	BC	CC	DC	EC	FC	GC	HC
	AB	BB	CB	DB	EB	FB	GB	HB
	AA	BA	CA	DA	EA	FA	GA	HA
	200	300	400	500	600	700	800	

	Zones 2, 8, 14, 20, 26, 32, 38, 44, 50, 56							
2000	JE	KE	LE	ME	NE	PE	QE	RE
	JD	KD	LD	MD	ND	PD	QD	RD
	JC	KC	LC	MC	NC	PC	QC	RC
	JB	KB	LB	MB	NB	PB	QB	RB
	JA	KA	LA	MA	NA	PA	QA	RA
1500	JV	KV	LV	MV	NV	PV	QV	RV
	JU	KU	LU	MU	NU	PU	QU	RU
	JT	KT	LT	MT	NT	PT	QT	RT
	JS	KS	LS	MS	NS	PS	QS	RS
	JR	KR	LR	MR	NR	PR	QR	RR
1000	JQ	KQ	LQ	MQ	NQ	PQ	QQ	RQ
	JP	KP	LP	MP	NP	PP	QP	RP
	JN	KN	LN	MN	NN	PN	QN	RN
	JM	KM	LM	MM	NM	PM	QM	RM
	JL	KL	LL	ML	NL	PL	QL	RL
500	JK	KK	LK	MK	NK	PK	QK	RK
	JJ	KJ	LJ	MJ	NJ	PJ	QJ	RJ
	JH	KH	LH	MH	NH	PH	QH	RH
	JG	KG	LG	MG	NG	PG	QG	RG
	JF	KF	LF	MF	NF	PF	QF	RF
	200	300	400	500	600	700	800	

	Zones 3, 9, 15, 21, 27, 33, 39, 45, 51, 57							
2000	SV	TV	UV	VV	WV	XV	YV	ZV
	SU	TU	UU	VU	WU	XU	YU	ZU
	ST	TT	UT	VT	WT	XT	YT	ZT
	SS	TS	US	VS	WS	XS	YS	ZS
	SR	TR	UR	VR	WR	XR	YR	ZR
1500	SQ	TQ	UQ	VQ	WQ	XQ	YQ	ZQ
	SP	TP	UP	VP	WP	XP	YP	ZP
	SN	TN	UN	VN	WN	XN	YN	ZN
	SM	TM	UM	VM	WM	XM	YM	ZM
	SL	TL	UL	VL	WL	XL	YL	ZL
1000	SK	TK	UK	VK	WK	XK	YK	ZK
	SJ	TJ	UJ	VJ	WJ	XJ	YJ	ZJ
	SH	TH	UH	VH	WH	XH	YH	ZH
	SG	TG	UG	VG	WG	XG	YG	ZG
	SF	TF	UF	VF	WF	XF	YF	ZF
500	SE	TE	UE	VE	WE	XE	YE	ZE
	SD	TD	UD	VD	WD	XD	YD	ZD
	SC	TC	UC	VC	WC	XC	YC	ZC
	SB	TB	UB	VB	WB	XB	YB	ZB
	SA	TA	UA	VA	WA	XA	YA	ZA
	200	300	400	500	600	700	800	

	Zones 4, 10, 16, 22, 28, 34, 40, 46, 52, 58							
2000	AE	BE	CE	DE	EE	FE	GE	HE
	AD	BD	CD	DD	ED	FD	GD	HD
	AC	BC	CC	DC	EC	FC	GC	HC
	AB	BB	CB	DB	EB	FB	GB	HB
	AA	BA	CA	DA	EA	FA	GA	HA
1500	AV	BV	CV	DV	EV	FV	GV	HV
	AU	BU	CU	DU	EU	FU	GU	HU
	AT	BT	CT	DT	ET	FT	GT	HT
	AS	BS	CS	DS	ES	FS	GS	HS
	AR	BR	CR	DR	ER	FR	GR	HR
1000	AQ	BQ	CQ	DQ	EQ	FQ	GQ	HQ
	AP	BP	CP	DP	EP	FP	GP	HP
	AN	BN	CN	DN	EN	FN	GN	HN
	AM	BM	CM	DM	EM	FM	GM	HM
	AL	BL	CL	DL	EL	FL	GL	HL
500	AK	BK	CK	DK	EK	FK	GK	HK
	AJ	BJ	CJ	DJ	EJ	FJ	GJ	HJ
	AH	BH	CH	DH	EH	FH	GH	HH
	AG	BG	CG	DG	EG	FG	GG	HG
	AF	BF	CF	DF	EF	FF	GF	HF
	200	300	400	500	600	700	800	

	Zones 5, 11, 17, 23, 29, 35, 41, 47, 53, 59							
2000	JV	KV	LV	MV	NV	PV	QV	RV
	JU	KU	LU	MU	NU	PU	QU	RU
	JT	KT	LT	MT	NT	PT	QT	RT
	JS	KS	LS	MS	NS	PS	QS	RS
	JR	KR	LR	MR	NR	PR	QR	RR
1500	JQ	KQ	LQ	MQ	NQ	PQ	QQ	RQ
	JP	KP	LP	MP	NP	PP	QP	RP
	JN	KN	LN	MN	NN	PN	QN	RN
	JM	KM	LM	MM	NM	PM	QM	RM
	JL	KL	LL	ML	NL	PL	QL	RL
1000	JK	KK	LK	MK	NK	PK	QK	RK
	JJ	KJ	LJ	MJ	NJ	PJ	QJ	RJ
	JH	KH	LH	MH	NH	PH	QH	RH
	JG	KG	LG	MG	NG	PG	QG	RG
	JF	KF	LF	MF	NF	PF	QF	RF
500	JE	KE	LE	ME	NE	PE	QE	RE
	JD	KD	LD	MD	ND	PD	QD	RD
	JC	KC	LC	MC	NC	PC	QC	RC
	JB	KB	LB	MB	NB	PB	QB	RB
	JA	KA	LA	MA	NA	PA	QA	RA
	200	300	400	500	600	700	800	

	Zones 6, 12, 18, 24, 30, 36, 42, 48, 54, 60							
2000	SE	TE	UE	VE	WE	XE	YE	ZE
	SD	TD	UD	VD	WD	XD	YD	ZD
	SC	TC	UC	VC	WC	XC	YC	ZC
	SB	TB	UB	VB	WB	XB	YB	ZB
	SA	TA	UA	VA	WA	XA	YA	ZA
1500	SV	TV	UV	VV	WV	XV	YV	ZV
	SU	TU	UU	VU	WU	XU	YU	ZU
	ST	TT	UT	VT	WT	XT	YT	ZT
	SS	TS	US	VS	WS	XS	YS	ZS
	SR	TR	UR	VR	WR	XR	YR	ZR
1000	SQ	TQ	UQ	VQ	WQ	XQ	YQ	ZQ
	SP	TP	UP	VP	WP	XP	YP	ZP
	SN	TN	UN	VN	WN	XN	YN	ZN
	SM	TM	UM	VM	WM	XM	YM	ZM
	SL	TL	UL	VL	WL	XL	YL	ZL
500	SK	TK	UK	VK	WK	XK	YK	ZK
	SJ	TJ	UJ	VJ	WJ	XJ	YJ	ZJ
	SH	TH	UH	VH	WH	XH	YH	ZH
	SG	TG	UG	VG	WG	XG	YG	ZG
	SF	TF	UF	VF	WF	XF	YF	ZF
	200	300	400	500	600	700	800	

Easting–letter / northing–letter combinations for lettering scheme "AA" for the UTM portion of MGRS to be used for the Geodetic Reference System 1980 ellipsoid or the World Geodetic System 1984 ellipsoid or as otherwise explained in the manual. Easting and northing labels are given in kilometers. The pattern repeats in the northing direction by multiples of 2000 km.

Northing (km)	Zones 1, 7, 13, 19, 25, 31, 37, 43, 49, 55							
2000	AK	BK	CK	DK	EK	FK	GK	HK
	AJ	BJ	CJ	DJ	EJ	FJ	GJ	HJ
	AH	BH	CH	DH	EH	FH	GH	HH
	AG	BG	CG	DG	EG	FG	GG	HG
	AF	BF	CF	DF	EF	FF	GF	HF
1500	AE	BE	CE	DE	EE	FE	GE	HE
	AD	BD	CD	DD	ED	FD	GD	HD
	AC	BC	CC	DC	EC	FC	GC	HC
	AB	BB	CB	DB	EB	FB	GB	HB
	AA	BA	CA	DA	EA	FA	GA	HA
1000	AV	BV	CV	DV	EV	FV	GV	HV
	AU	BU	CU	DU	EU	FU	GU	HU
	AT	BT	CT	DT	ET	FT	GT	HT
	AS	BS	CS	DS	ES	FS	GS	HS
	AR	BR	CR	DR	ER	FR	GR	HR
500	AQ	BQ	CQ	DQ	EQ	FQ	GQ	HQ
	AP	BP	CP	DP	EP	FP	GP	HP
	AN	BN	CN	DN	EN	FN	GN	HN
	AM	BM	CM	DM	EM	FM	GM	HM
	AL	BL	CL	DL	EL	FL	GL	HL
Easting (km)		200	300	400	500	600	700	800

Northing (km)	Zones 2, 8, 14, 20, 26, 32, 38, 44, 50, 56							
2000	JQ	KQ	LQ	MQ	NQ	PQ	QQ	RQ
	JP	KP	LP	MP	NP	PP	QP	RP
	JN	KN	LN	MN	NN	PN	QN	RN
	JM	KM	LM	MM	NM	PM	QM	RM
	JL	KL	LL	ML	NL	PL	QL	RL
1500	JK	KK	LK	MK	NK	PK	QK	RK
	JJ	KJ	LJ	MJ	NJ	PJ	QJ	RJ
	JH	KH	LH	MH	NH	PH	QH	RH
	JG	KG	LG	MG	NG	PG	QG	RG
	JF	KF	LF	MF	NF	PF	QF	RF
1000	JE	KE	LE	ME	NE	PE	QE	RE
	JD	KD	LD	MD	ND	PD	QD	RD
	JC	KC	LC	MC	NC	PC	QC	RC
	JB	KB	LB	MB	NB	PB	QB	RB
	JA	KA	LA	MA	NA	PA	QA	RA
500	JV	KV	LV	MV	NV	PV	QV	RV
	JU	KU	LU	MU	NU	PU	QU	RU
	JT	KT	LT	MT	NT	PT	QT	RT
	JS	KS	LS	MS	NS	PS	QS	RS
	JR	KR	LR	MR	NR	PR	QR	RR
Easting (km)		200	300	400	500	600	700	800

Northing (km)	Zones 3, 9, 15, 21, 27, 33, 39, 45, 51, 57							
2000	SK	TK	UK	VK	WK	XK	YK	ZK
	SJ	TJ	UJ	VJ	WJ	XJ	YJ	ZJ
	SH	TH	UH	VH	WH	XH	YH	ZH
	SG	TG	UG	VG	WG	XG	YG	ZG
	SF	TF	UF	VF	WF	XF	YF	ZF
1500	SE	TE	UE	VE	WE	XE	YE	ZE
	SD	TD	UD	VD	WD	XD	YD	ZD
	SC	TC	UC	VC	WC	XC	YC	ZC
	SB	TB	UB	VB	WB	XB	YB	ZB
	SA	TA	UA	VA	WA	XA	YA	ZA
1000	SV	TV	UV	VV	WV	XV	YV	ZV
	SU	TU	UU	VU	WU	XU	YU	ZU
	ST	TT	UT	VT	WT	XT	YT	ZT
	SS	TS	US	VS	WS	XS	YS	ZS
	SR	TR	UR	VR	WR	XR	YR	ZR
500	SQ	TQ	UQ	VQ	WQ	XQ	YQ	ZQ
	SP	TP	UP	VP	WP	XP	YP	ZP
	SN	TN	UN	VN	WN	XN	YN	ZN
	SM	TM	UM	VM	WM	XM	YM	ZM
	SL	TL	UL	VL	WL	XL	YL	ZL
Easting (km)		200	300	400	500	600	700	800

Northing (km)	Zones 4, 10, 16, 22, 28, 34, 40, 46, 52, 58							
2000	AQ	BQ	CQ	DQ	EQ	FQ	GQ	HQ
	AP	BP	CP	DP	EP	FP	GP	HP
	AN	BN	CN	DN	EN	FN	GN	HN
	AM	BM	CM	DM	EM	FM	GM	HM
	AL	BL	CL	DL	EL	FL	GL	HL
1500	AK	BK	CK	DK	EK	FK	GK	HK
	AJ	BJ	CJ	DJ	EJ	FJ	GJ	HJ
	AH	BH	CH	DH	EH	FH	GH	HH
	AG	BG	CG	DG	EG	FG	GG	HG
	AF	BF	CF	DF	EF	FF	GF	HF
1000	AE	BE	CE	DE	EE	FE	GE	HE
	AD	BD	CD	DD	ED	FD	GD	HD
	AC	BC	CC	DC	EC	FC	GC	HC
	AB	BB	CB	DB	EB	FB	GB	HB
	AA	BA	CA	DA	EA	FA	GA	HA
500	AV	BV	CV	DV	EV	FV	GV	HV
	AU	BU	CU	DU	EU	FU	GU	HU
	AT	BT	CT	DT	ET	FT	GT	HT
	AS	BS	CS	DS	ES	FS	GS	HS
	AR	BR	CR	DR	ER	FR	GR	HR
Easting (km)		200	300	400	500	600	700	800

Northing (km)	Zones 5, 11, 17, 23, 29, 35, 41, 47, 53, 59							
2000	JK	KK	LK	MK	NK	PK	QK	RK
	JJ	KJ	LJ	MJ	NJ	PJ	QJ	RJ
	JH	KH	LH	MH	NH	PH	QH	RH
	JG	KG	LG	MG	NG	PG	QG	RG
	JF	KF	LF	MF	NF	PF	QF	RF
1500	JE	KE	LE	ME	NE	PE	QE	RE
	JD	KD	LD	MD	ND	PD	QD	RD
	JC	KC	LC	MC	NC	PC	QC	RC
	JB	KB	LB	MB	NB	PB	QB	RB
	JA	KA	LA	MA	NA	PA	QA	RA
1000	JV	KV	LV	MV	NV	PV	QV	RV
	JU	KU	LU	MU	NU	PU	QU	RU
	JT	KT	LT	MT	NT	PT	QT	RT
	JS	KS	LS	MS	NS	PS	QS	RS
	JR	KR	LR	MR	NR	PR	QR	RR
500	JQ	KQ	LQ	MQ	NQ	PQ	QQ	RQ
	JP	KP	LP	MP	NP	PP	QP	RP
	JN	KN	LN	MN	NN	PN	QN	RN
	JM	KM	LM	MM	NM	PM	QM	RM
	JL	KL	LL	ML	NL	PL	QL	RL
Easting (km)		200	300	400	500	600	700	800

Northing (km)	Zones 6, 12, 18, 24, 30, 36, 42, 48, 54, 60							
2000	SQ	TQ	UQ	VQ	WQ	XQ	YQ	ZQ
	SP	TP	UP	VP	WP	XP	YP	ZP
	SN	TN	UN	VN	WN	XN	YN	ZN
	SM	TM	UM	VM	WM	XM	YM	ZM
	SL	TL	UL	VL	WL	XL	YL	ZL
1500	SK	TK	UK	VK	WK	XK	YK	ZK
	SJ	TJ	UJ	VJ	WJ	XJ	YJ	ZJ
	SH	TH	UH	VH	WH	XH	YH	ZH
	SG	TG	UG	VG	WG	XG	YG	ZG
	SF	TF	UF	VF	WF	XF	YF	ZF
1000	SE	TE	UE	VE	WE	XE	YE	ZE
	SD	TD	UD	VD	WD	XD	YD	ZD
	SC	TC	UC	VC	WC	XC	YC	ZC
	SB	TB	UB	VB	WB	XB	YB	ZB
	SA	TA	UA	VA	WA	XA	YA	ZA
500	SV	TV	UV	VV	WV	XV	YV	ZV
	SU	TU	UU	VU	WU	XU	YU	ZU
	ST	TT	UT	VT	WT	XT	YT	ZT
	SS	TS	US	VS	WS	XS	YS	ZS
	SR	TR	UR	VR	WR	XR	YR	ZR
Easting (km)		200	300	400	500	600	700	800

Easting–letter / northing–letter combinations for lettering scheme "AL" for the UTM portion of MGRS to be used as explained in the manual. Easting and northing labels are given in kilometers. The pattern repeats in the northing direction by multiples of 2000 km.

12. Topics in MGRS

The agenda of this document is the programming logic needed by the software developer. For MGRS, this is covered in Section 11 — in one sense, covered completely. But it is prudent to take notice of some related issues, and this is done here.

12.1 Formal definition of MGRS

Section 11 adopts a formal point of view, *i.e.* MGRS is merely a respelling of UTM or UPS coordinates truncated to the desired precision. If the administrative rules (Subsections 7.4 or 10.4) were in effect when the UTM or UPS coordinates were produced, they remain in effect when these coordinates are converted to MGRS. If the UTM or UPS coordinates were produced outside the administrative rules, they can yet be converted to MGRS provided they satisfy the inequalities for x and y given for the relevant lettering scheme, *i.e.* the inequalities in Subsections 11.2, 11.3 (implied), 11.10 and 11.11.

If the UTM or UPS coordinates $\{x, y\}$ are both multiples of the desired MGRS precision, 10^{5-n} (see Subsection 11.6), then the double conversion UTM → MGRS → UTM yields the original coordinates $\{x, y\}$ exactly. With no change in the desired precision, the double conversion MGRS → UTM → MGRS yields the original MGRS string. Likewise for UPS in place of UTM. All this is true when keeping to the principles of Section 11.

12.2 Administrative rules

The intended usage of MGRS is meant to comply with the administrative rules of Subsections 7.4 and 10.4.

At some level of the software hierarchy, the MGRS conversion routines should be written in accordance with Section 11. This will allow the crossing of an administrative-rule boundary when necessary or when convenient and allowed. At a higher level, the administrative rules may be enforced in software. The goal is to keep UTM/UPS synchronized with MGRS. In any situation, the administrative rules should be applied to both or neither; they should not be applied to only one.

12.3 Rounding *v.* truncating

The intended usage of UTM and UPS coordinates for the calculating or recording of positions complies with the usual rounding rules of science and engineering. When a precise coordinate, *e.g.* $x = 512\,378$ m, is to be converted to a less precise coordinate, *e.g.* $x = 512\,380$ m or $x = 512\,400$ m, the operation is rounding, not dropping of digits (truncating).

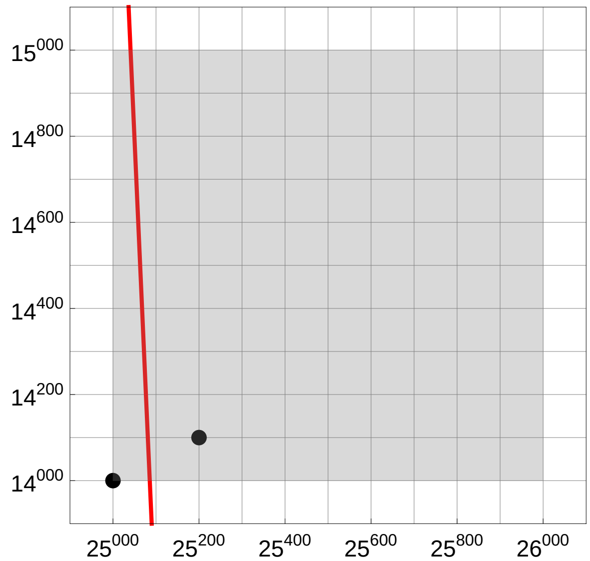
For UTM (and UPS) conversions to MGRS, the operation is truncating, not rounding (see Section 11). Continuing the above example, $x = 512\,378$ m becomes $x = 512\,370$ m (easting digits 1237) or $x = 512\,300$ m (easting digits 123).

For the reverse conversion, *i.e.* MGRS to UTM or MGRS to UPS, if the requirement is for the best UTM or UPS position rather than for a defined area's bottom-left corner (discussed next), one-half the precision should be added to the result of the Section 11 conversion. For example, if the given easting digits are 1237, the meaning of those digits is a 10 meter interval from (say) $x = 512\,370$ m to $x = 512\,380$ m and the appropriate value of x would be $x = 512\,375$ m.

12.4 Point *v.* area

MGRS is also an area identification scheme. If there are n easting digits (with the same number of northing digits), the MGRS string defines a square in the UTM or UPS plane whose side is 10^{5-n} meters and whose bottom left corner is the UTM or UPS equivalent of the MGRS string (using Section 11 for the conversion). For non-polar areas, the bottom left corner is the southwest corner.

The administrative rules are amended to allow some MGRS strings as area identifiers that would not be allowed as point identifiers. Point *vice* area is an important distinction. Here is an example: The administrative limits for UTM zone (−53) are $132\,\text{deg} \le \lambda < 138\,\text{deg}$. Point 53ELR2520014100 lies east of 132°E and is compliant. It belongs to area 53ELR2514, whose southwest corner is point 53ELR2514 or point 53ELR2500014000 (to use a consistent precision for points, in this example). Almost all of area 53ELR2514 lies east of 132°E. But the corner point 53EL-R2500014000 lies west of 132°E and is therefore non-compliant. (See the figure in this subsection). Zone (−53) should not be used for this point. Its administratively correct specification is point 52EFA7482914007 in the next zone westward, i.e. zone (−52). But no good will come of this conversion. String 52EFA7482914007 has the wrong precision to identify the desired area (1000-meter square) and truncating it to 52EFA7414 (the desired precision) defines a different area. (This is an example of a general principle: it is impossible to simultaneously specify an area of the earth by UTM zone (−52) grid-lines and by UTM zone (−53) grid-lines, even if the administrative rules are completely abandoned). Therefore the administrative rules are amended to say that although "53ELR2514" is not allowed as the specification of a point, *area* 53ELR2514 shall mean the portion of this 1000-meter square east of 132°E.



1000-meter square 53ELR2514 (gray) with intersection by meridian 132E (red)

12.5 Latitude band letter — efficiency — northern hemisphere

Because various characteristics of MGRS are unhelpful to analytical work (see Subsection 11.8), this document suggests (but does not mandate) the following division of labor between MGRS and UTM/UPS when both are under consideration. UTM/UPS should be be used for calculations, analytical work, and storage & retrieval of geographic information; MGRS should be limited to notations on maps and charts, displays on end-user devices and person-to-person or person-to-machine communication. Therefore, there would not seem to be a great need for efficiency in the conversion algorithms between UTM/UPS and MGRS, as large data sets that would consume computer resources should already be stored in UTM or UPS coordinates.

The above notwithstanding, there could be occasions where these conversion algorithms need to be efficient. The UPS-to-MGRS algorithm and its inverse present no issues. The UTM-to-MGRS algorithm and it inverse, however, could be improved for efficiency. The issue is the latitude band letter.

For the UTM-to-MGRS conversion, rather than *always* execute the UTM-to-Lon./Lat. algorithm to obtain the latitude and thus the latitude band letter, the software should invoke the following table for the northern hemisphere. For each parallel circle, the table provides two staircase-like functions that envelope the parallel — one on its north side; the other on its south side. This allows a table look-up to complete the latitude band letter determination for the vast majority of cases. All x and y values in the table are kilometers on the UTM plane. For the MGRS-to-UTM conversion, this table obviates the need for an execution of the UTM-to-Lon./Lat. algorithm. (See the examples in Subsection 12.7). The table is valid for any reference ellipsoid listed in Section 4.

Lat. (deg)	y-value for 500 ≤ x < 600 (km)	y-value for 600 ≤ x < 700 (km)	y-value for 700 ≤ x < 800 (km)	y-value for 800 ≤ x < 900 (km)
		Latitude band X		
72	7992	7999	8011	8029
72	7988	7990	7997	8009
		Latitude band W		
64	7099	7104	7112	7123
64	7096	7097	7102	7110
		Latitude band V		
56	6208	6211	6217	6225
56	6205	6206	6210	6215
		Latitude band U		
48	5318	5320	5325	5331
48	5315	5316	5319	5323
		Latitude band T		
40	4429	4431	4434	4439
40	4427	4427	4429	4433
		Latitude band S		
32	3541	3543	3545	3549
32	3540	3540	3542	3544
		Latitude band R		
24	2655	2656	2658	2660
24	2653	2654	2655	2657
		Latitude band Q		
16	1770	1770	1771	1773
16	1768	1768	1769	1770
		Latitude band P		
8	885	885	886	887
8	884	884	884	885
		Latitude band N		
0	0	0	0	0

(A subroutine to convert Lon./Lat. to MGRS by combining the guidance in several sections of this document will not need efficiency improvements like the above. The latitude is a given input item; the latitude band letter is easily determined by Subsection 11.7).

12.6 Latitude band letter — efficiency — symmetry of tables

On the other side of the line $x = 500\,000$, symmetry is applied as if the headings of the tables in Subsections 12.5 and 12.8 were:

Lat. (deg)	y-value for 400 ≤ x < 500 (km)	y-value for 300 ≤ x < 400 (km)	y-value for 200 ≤ x < 300 (km)	y-value for 100 ≤ x < 200 (km)

12.7 Latitude band letter — efficiency — examples

To convert $\{x, y\} = \{705\,000, 1\,765\,123\}$, use the column 700 km $\le x < 800$ km and find that $y = 1\,765\,123$ is safely in band P because it is south of $y = 1769$ km and north of $y = 886$ km.

To convert $\{x, y\} = \{705\,000, 1\,769\,123\}$, which is the point displaced 4000 meters more in northing, the UTM-to-Lon./Lat. algorithm will have to be executed because $y = 1\,769\,123$ lies between $y = 1769$ km and $y = 1771$ km.

Let 31SFR1500042887 be given as an MGRS string for an ellipsoid that uses lettering scheme "AA". This example finds the corresponding UTM coordinates. The UTM zone is $Z = +31$. The easting letter is “F” which by the tables of Subsection 11.2 represents 6, i.e. $6 \times 10^5 = 600\,000$ meters. Add the easting digits to get $x = 615\,000$. The northing letter is “R”, which by the same tables of Subsection 11.2 represents 15, i.e. 15×10^5meters, which is understood to stand for $1\,500\,000 + 2\,000\,000\,k$ meters for $k = 0, 1, 2, 3,$ or 4 (to be determined). Add the northing digits to get $y = 1\,542\,887 + 2\,000\,000\,k$. In other words, the candidates for y are 1 542 887, 3 542 887, 5 542 887, 7 542 887 and 9 542 887, which ambiguity is to be resolved by the latitude band letter “S”. Consulting the table in Subsection 12.5 under column $600\text{ km} \le x < 700\text{ km}$, we see that $y = 3\,542\,887$ lies inside the expanded limits of band S, i.e. $y = 3540$ km to $y = 4431$ km. Therefore, $y = 3\,542\,887$.

12.8 Latitude band letter — efficiency — southern hemisphere

The table for the southern hemisphere follows. It is valid for any reference ellipsoid listed in Section 4.

Lat. (deg)	y-value for 500 ≤ x < 600 (km)	y-value for 600 ≤ x < 700 (km)	y-value for 700 ≤ x < 800 (km)	y-value for 800 ≤ x < 900 (km)
0	10000	10000	10000	10000
		Latitude band M		
-8	9116	9116	9116	9115
-8	9115	9115	9114	9113
		Latitude band L		
-16	8232	8232	8231	8230
-16	8230	8230	8229	8227
		Latitude band K		
-24	7347	7346	7345	7343
-24	7345	7344	7342	7340
		Latitude band J		
-32	6460	6460	6458	6456
-32	6459	6457	6455	6451
		Latitude band H		
-40	5573	5573	5571	5567
-40	5571	5569	5566	5561
		Latitude band G		
-48	4685	4684	4681	4677
-48	4682	4680	4675	4669
		Latitude band F		
-56	3795	3794	3790	3785
-56	3792	3789	3783	3775
		Latitude band E		
-64	2904	2903	2898	2890
-64	2901	2896	2888	2877
		Latitude band D		
-72	2012	2010	2003	1991
-72	2008	2001	1989	1971
		Latitude band C		

12.9 Latitude band letter — leniency

Many software programs allow some leniency in the latitude band letter during the MGRS-to-UTM conversion process. The example of Subsection 11.8 is a case in point. In that example, the string 13VFC4967108679 is invalid by the rules of Section 11, and would have to be rejected and not converted. (An error message would be helpful). Opposed to this, both 13UFC4967108679 (valid) and 13VFC4967108679 (invalid) convert to $Z = +13$, $\{x, y\} = \{649\,671, 6\,208\,679\}$ by the application of the latitude-band-letter efficiency table of Subsection 12.5 (and see Sub-subsection 12.7.3). So, it would seem that the error in the Latitude band letter is recoverable in this case and maybe shouldn't be called an error. Again, this leniency is outside the definition of MGRS in Section 11.

Of the choice to be lenient or not, many system developers adopt the more generous view and apply it to cases more aggressively in the wrong latitude band than the above example. If this practice is to be allowed, this document should offer some guidance. The purpose of the latitude band letter — the only purpose with respect to the algorithms at issue — is to resolve the 2 000 000 k meters ambiguity in the northing where k is one of the integers 0, 1, 2, 3, or 4. The design of a leniency rule has to include the requirement that a candidate MGRS string that is off by one latitude band letter but otherwise valid converts to the *intended* UTM coordinates.

■ 12.10 Latitude band letter — leniency rule

For the UTM to MGRS conversion, there is no leniency — the latitude band letter is to be computed correctly by the foregoing principles. For the MGRS to UTM conversion, the following leniency rule is to be applied to decipher a candidate MGRS string: Give each "latitude band" (hereafter, bloated latitude band) a much larger area. Refer to the tables in Subsections 12.5 and 12.8. For each latitude band other than C and X, start with the pair of staircase functions immediately above and below it. Modify these to create new limits for the band. Modify the y-values to move the northern limit of each band another 400 000 meters further north and to move the southern limit 400 000 meters further south. For latitude bands C and X, expand 200 000 meters in the direction toward the Equator. Then if none of the 5 choices for value of y (see above, where k equals 0,1,2,3 or 4) falls into the bloated latitude band corresponding to the given letter, the candidate MGRS string is invalid and cannot be converted.

The above leniency rule is quite lax, while yet retaining the ability to resolve the 2 000 000 k meters ambiguity in the northings. For quality assurance of imported geographic data, analysts may devise and perform more stringent tests to filter-out candidate MGRS data for further review before acceptance.

■ 12.11 MGRS–UTM hybrid

Nothing in this document prohibits DoD components and their contractors from employing a mixture of UTM and MGRS information for displays on end-user equipment or for margin notes on printed maps, *etc*. Prominent in this category is the following MGRS–UTM hybrid:

(i) UTM zone number in absolute value
(ii) MGRS latitude band letter
(iii) UTM x-coordinate (Easting) to precision 1 meter
(iv) UTM y-coordinate (Northing) to precision 1 meter

As an example, here is a point specified three ways:

UTM (stored internally):	`Zone = +31, x = 345 009, y = 6 700 123`
MGRS:	`31VCH4500900123`
MGRS–UTM hybrid:	`31V, 345009mE, 6700123mN`

Details of the format and wording of margin notes and device displays are outside the scope of this document. See [11] for guidance and these remarks: From the information-content point-of-view, the MGRS latitude band letter belongs to MGRS, not UTM. For greater readability the above example appends "`mE`" for meters east and "`mN`" for meters north, as shown. Also for readability, UTM may show "31 north" in place of "+31", but the capital letters "N" and "S" may not be used as abbreviations for north and south.

Here is another example. It is 10, 000, 000 meters less in northing, and on the other side of the Equator:

UTM (stored internally):	`Zone = +31, x = 345 009, y = −3 299 877`
UTM (stored internally):	`Zone = −31, x = 345 009, y = 6 700 123`
MGRS:	`31JCH4500900123`
MGRS–UTM hybrid:	`31J, 345009mE, 6700123mN`

The suffix "mN" for "meters North" is to be used for points on both sides of the Equator.

13. MGRS Quick-Start

The guidance given to this point has assumed that the routines for processing MGRS are part of a larger package of map projection and coordinate conversion software to include UTM and UPS and, more generally, transverse Mercator and polar stereographic routines. When these are available, the additional code to implement MGRS is merely a few table look-ups (see Sections 11 and 12; note some exceptions), and MGRS is efficiently linked to UTM and UPS.

Modern positioning (*e.g.* GPS technology) is in pursuit of centimeter accuracy. This manual's conversions between Lon./Lat. and UTM support such a goal but MGRS does not. Consequently and for other reasons, the development of transverse Mercator and UTM in this document is more extensive than needed for MGRS. UTM is recommended for serious analytical work with grid coordinates but some software developers might need only MGRS. For them, this section provides some short-cuts. Some short cuts are for the reader; some are for the machine.

This section provides guidance for converting directly between longitude/latitude and MGRS. Only the UTM portion of MGRS is considered. Only the WGS 84 ellipsoid is considered. The administrative rules apply. The chosen precision for MGRS will be 1 meter. For aspects of MGRS outside this agenda, see the full treatment in Sections 11 and 12 and the earlier sections to which they refer. Sections 1, 2, and 3 are prerequisite.

■ 13.1 Given Lon./Lat. compute MGRS

The procedure to compute MGRS from longitude and latitude is given as a series of steps to be followed in the order given.

13.1.1) Let `LonD` and `LatD` be the given longitude and latitude in decimal degrees of the point to be converted. Points north of the Equator have positive latitudes; points south have negative. Points east of the nominal Greenwich meridian have positive longitudes; points west have negative.
If `LatD` < −`80` or `LatD` > `84`, the point cannot be converted to the UTM portion of MGRS and an error message should be issued.

13.1.2) The set of allowed central meridians in degrees is the list −`177` to +`177` by increment of `6`. Find the member of this list closest to `LonD` and call it `CMdeg`. The UTM absolute zone number is calculated `absZ` = (`CMdeg` + `183`) / `6`.

13.1.3) If `LatD` ≥ `56` and `0` ≤ `LonD` < `42`, an adjustment to `CMdeg` from Step 13.1.2 might be required. (See Subsection 7.5).

13.1.4) Divide `LatD` by `8` and discard the remainder, *i.e.* compute `Floor[LatD / 8]`. Enter the following table with the result to find the latitude band letter.

−10	−9	−8	−7	−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7	8	9	10
C	D	E	F	G	H	J	K	L	M	N	P	Q	R	S	T	U	V	W	X	X

13.1.5) Convert the angles in Steps 13.1.1 and 13.1.2 to radians. In other words, compute `Lon` = `LonD * Pi / 180` and `Lat` = `LatD * Pi / 180` and `CM` = `CMdeg * Pi / 180`.

13.1.6) Next is needed the conformal latitude χ or, rather, its cosine and sine. See Subsection 2.8 and use the formulas there. The value to use for e, the eccentricity is $e = \sqrt{f\,(2-f)}$ where $f = 1/298.257223563$.

13.1.7) Perform the computations of Eq. (3.8) to obtain the quantities u and v using λ = `Lon` − `CM` for the value of λ in those equations.

13.1.8) Compute $\cos(2\,v)$, $\cos(4\,v)$, $\sin(2\,v)$, $\sin(4\,v)$, directly or with help from Eq. (3.10)

13.1.9) Compute $\cosh(2\,u)$, $\cosh(4\,u)$, $\sinh(2\,u)$, $\sinh(4\,u)$, directly or with help from Eq. (3.12)

13.1.10) Perform the computations of Eq. (3.7) in the following abbreviated way. (This is possible because the computational accuracy required in this situation is merely one meter.)

$$x = R_4\,(\,u + a_2 \sinh(2\,u)\cos(2\,v) + a_4 \sinh(4\,u)\cos(4\,v)\,)$$
$$y = R_4\,(\,v + a_2 \cosh(2\,u)\sin(2\,v) + a_4 \cosh(4\,u)\sin(4\,v)\,)$$

13.1.11) Compute the UTM easting and northing as follows, where y_{eq} is 0 if `Lat ≥ 0` and is 10 000 000 otherwise:

$$x_{utm} = (0.9996)\,x + 500\,000 \qquad \text{truncated to 1 meter}$$
$$y_{utm} = (0.9996)\,y + y_{eq} \qquad \text{truncated to 1 meter}$$

13.1.12) Apply lettering scheme "AA" (see Subsection 11.2) to the numbers $\{x_{utm}, y_{utm}\}$ found in Step 13.1.11 with $|Z|$ there equal to `absZ` here.

13.1.13) The MGRS string consists of the absolute zone number `absZ` from Step 13.1.2, followed by the latitude band letter from Step 13.1.4, followed by the easting-letter obtained in Step 13.1.12, followed by the northing letter obtained also in Step 13.1.12, followed the 5 least significant digits of x_{utm} obtained in Step 13.1.11, followed finally by the 5 least significant digits of y_{utm} obtained also in Step 13.1.11.

13.2 Given MGRS, compute Lon./Lat.

The procedure to compute the longitude and latitude from MGRS is given as a series of steps to be followed in the order given.

13.2.1) Check that the given MGRS string consists of 1 or 2 digits (the UTM absolute zone number `absZ`) followed by a letter in the range C-X (the latitude band letter) followed by another letter (easting-letter) followed by another letter (northing-letter) followed by 5 digits (easting-digits x_{digits}) followed finally by 5 more digits (northing-digits y_{digits}). None of the letters may be "I" or "O". Other checks will arise in what follows.

13.2.2) The central meridian in degrees is computed `CMdeg = −183 + 6 * absZ`. Its radian equivalent is `CM = CMdeg * Pi / 180`.

13.2.3) Apply lettering scheme "AA" (see Subsection 11.2) in reverse to the easting-letter and northing-letter from Step 13.2.1 to obtain their numerical equivalents x_{letter} and y_{letter}. Successful table look-ups should yield answers in the ranges $1 \le x_{letter} \le 8$ and $0 \le y_{letter} \le 19$. Take $|Z|$ there to be equal to `absZ` here.

13.2.4) Combine the above pieces of information according to the following equations to obtain the UTM easting x_{utm} and the UTM northing y_{utm}.

$$x_{utm} = 100\,000\,x_{letter} + x_{digits}$$
$$y_{prelim} = 100\,000\,y_{letter} + y_{digits}$$
$$y_{utm} = 2\,000\,000\,y_{band} + y_{prelim}$$

where y_{band} is one of the numbers 0,1,2,3 or 4 to be determined. (The five candidates for y_{band} yield five candidates for y_{utm}.)

13.2.5) Determine y_{band} by one of these two methods. (i) Enter the latitude band efficiency tables of Subsections 12.5 and 12.8 with x_{utm} and the latitude band letter and the 5 candidate values of y_{utm} to see which one fits. Or, (ii) consult the following table (from Subsection 11.13) to obtain the one or two possible values of y_{band} , compute y_{utm} for each value and apply the remaining steps of this subsection to each y_{utm} candidate to see which latitude (final answer) fits the given latitude band.

C	D	E	F	G	H	J	K	L	M	N	P	Q	R	S	T	U	V	W	X
1	1	1	2	2	3	3	4	4	4	0	0	0	1	1	2	2	3	3	3
0	0		1		2		3					1		2		3		4	4

13.2.6) Compute the transverse Mercator coordinates x and y as follows, where y_{eq} is 10 000 000 if the latitude band letter is among C-M and is 0 if it is among N-X:

$$x = (x_{\mathrm{utm}} - 500\,000)/(0.9996)$$
$$y = \left(y_{\mathrm{utm}} - y_{\mathrm{eq}}\right)\Big/(0.9996)$$

13.2.7) Apply the formulas and logic of Subsection 3.5 to the values for $\{x,\ y\}$ from Step 13.2.6. The formulas for u and v may be shortened as follows:

$$u = \frac{x}{R_4} + b_2 \sinh\left(\frac{2\,x}{R_4}\right)\cos\left(\frac{2\,y}{R_4}\right) + b_4 \sinh\left(\frac{4\,x}{R_4}\right)\cos\left(\frac{4\,y}{R_4}\right)$$
$$v = \frac{y}{R_4} + b_2 \cosh\left(\frac{2\,x}{R_4}\right)\sin\left(\frac{2\,y}{R_4}\right) + b_4 \cosh\left(\frac{4\,x}{R_4}\right)\sin\left(\frac{4\,y}{R_4}\right)$$

14. United States National Grid

This section explains the United States National Grid (USNG). It is included in this document because it is almost the same as MGRS.

14.1 Definition of USNG

Like MGRS, the United States National Grid (USNG) [5] is built on UTM coordinates (eastings and northings), a lettering scheme for multiples of 100 000 meters, and latitude bands. It adopted almost all of the rules of the UTM portion of MGRS given in Section 11. The sole exception concerns the choice between lettering schemes “AA” and “AL” in a particular circumstance. The following table tells which scheme is used for which ellipsoid/datum:

Ellipsoid	MGRS	USNG
GRS 80 ellipsoid (used by the NAD 83 datum)	AA	AA
Clark 1866 ellipsoid (used by the NAD 27 datum)	AL	AA

For NAD 83, the MGRS and USNG systems are the same. For NAD 27, they are not.

14.2 USNG example

It is a goal of the U.S. federal government to convert all the land maps of the U.S. from NAD 27 to NAD 83. When that happens, USNG will be identical to MGRS in usage because NAD 27 will be obsolete. In the meantime, a point in Nevada at 117°W, 39°N (NAD 27) has these competing representations, differing at the northing letter. Note “P” v. “D”.

```
MGRS:    11SNP0000016568   (NAD 27)
USNG:    11SND0000016568   (NAD 27)
            ^
```

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15. Diagrams for UTM, UPS and MGRS

The following pages are some plots that illustrate principles in this document. The depictions are informative for this purpose only. For guidance on the portrayal of grids and graticules on DoD standard products, see [11].

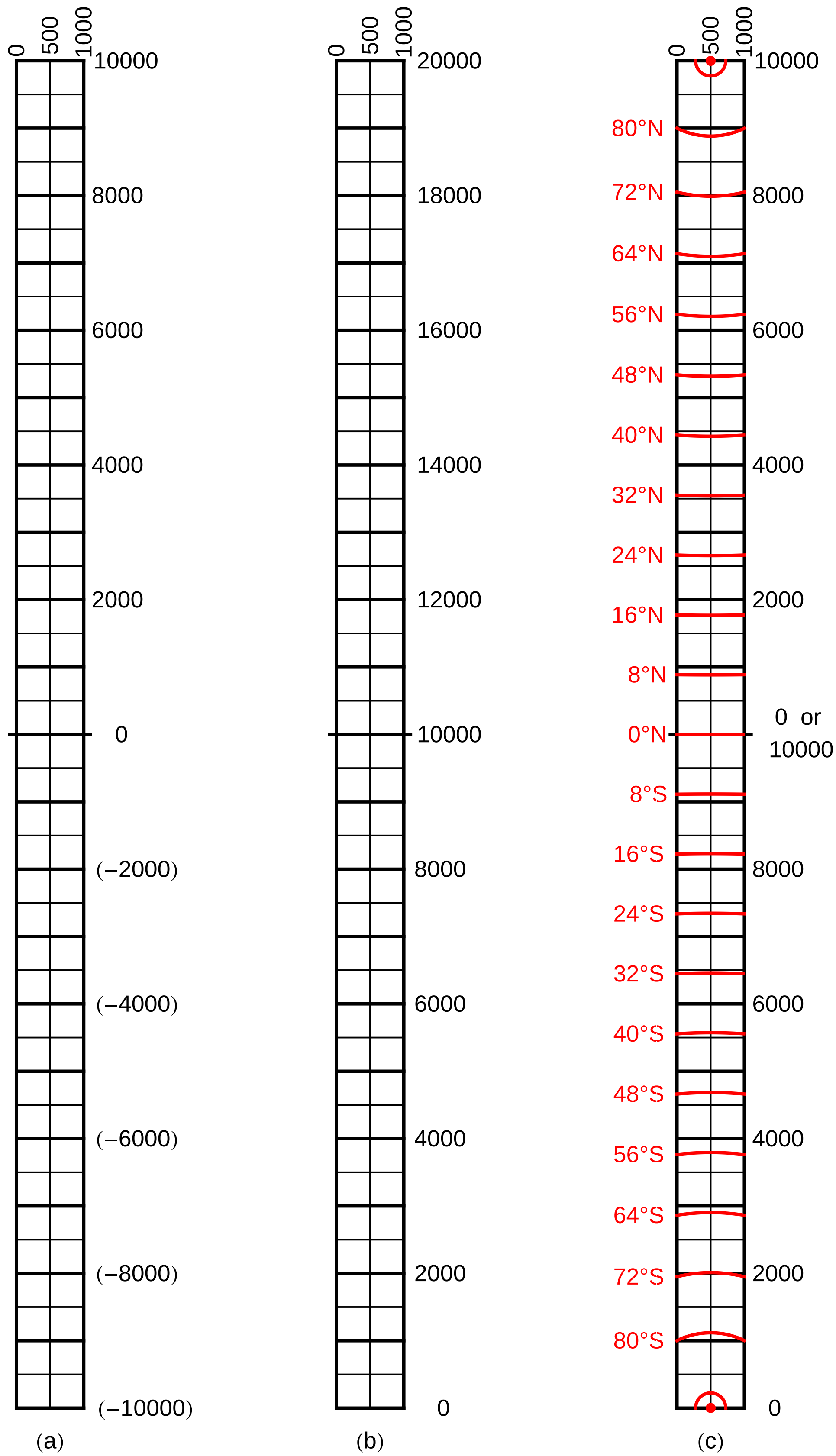

Figure 1. UTM plane for generic zone *Z*. All eastings and northings are in kilometers. (a) Northings if `Z > 0`. (b) Northings if `Z < 0`. (c) Northings in common practice with parallels every 8° and the north and south poles.

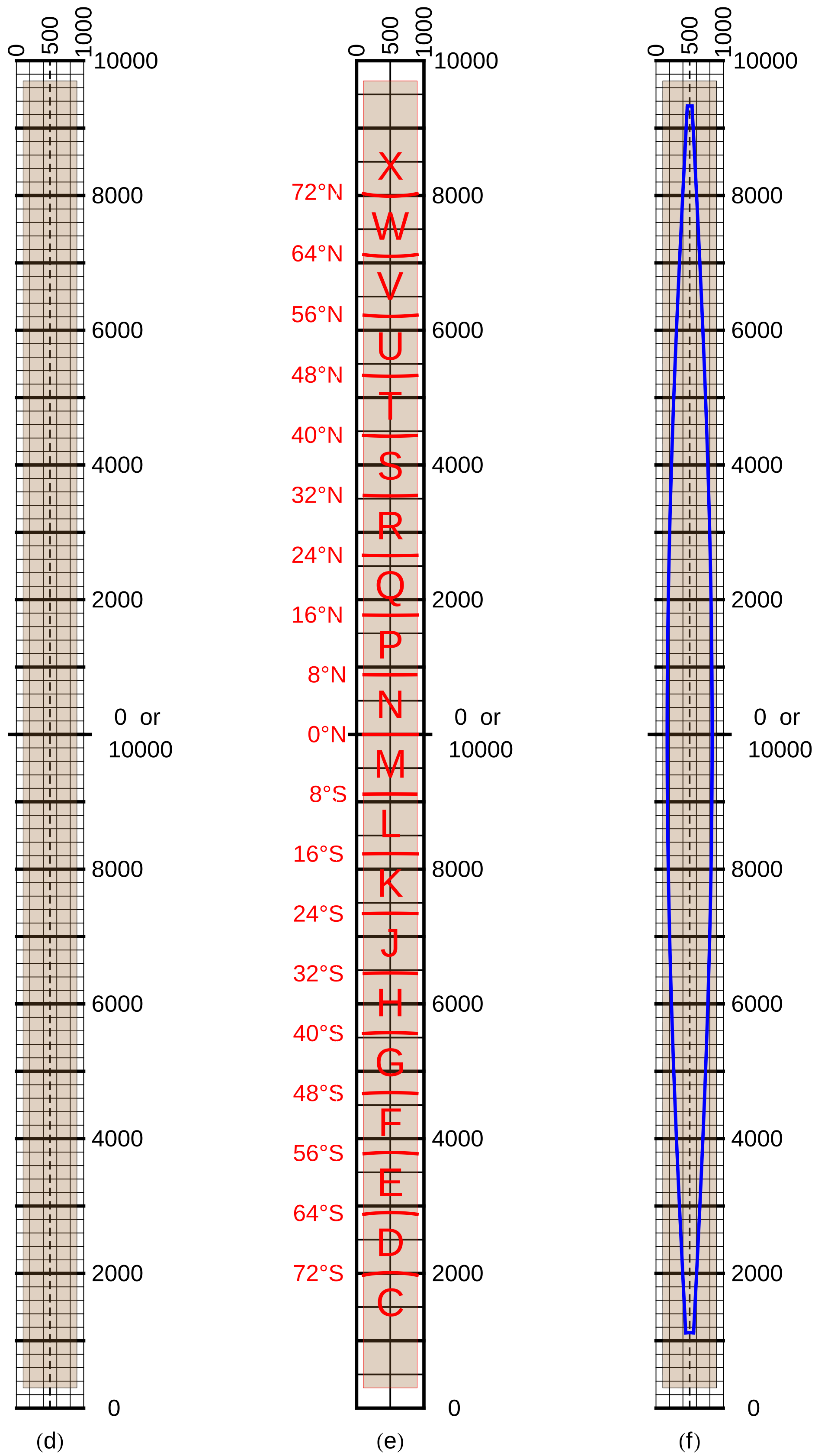

Figure 1 (continued). (d) Portion of the UTM plane representable in MGRS, (e) MGRS latitude bands with their bounding parallels, (f) Meridians at ±3° of the central meridian and parallels at 80°S and 84°N, which are basic to the administrative rules for UTM and MGRS.

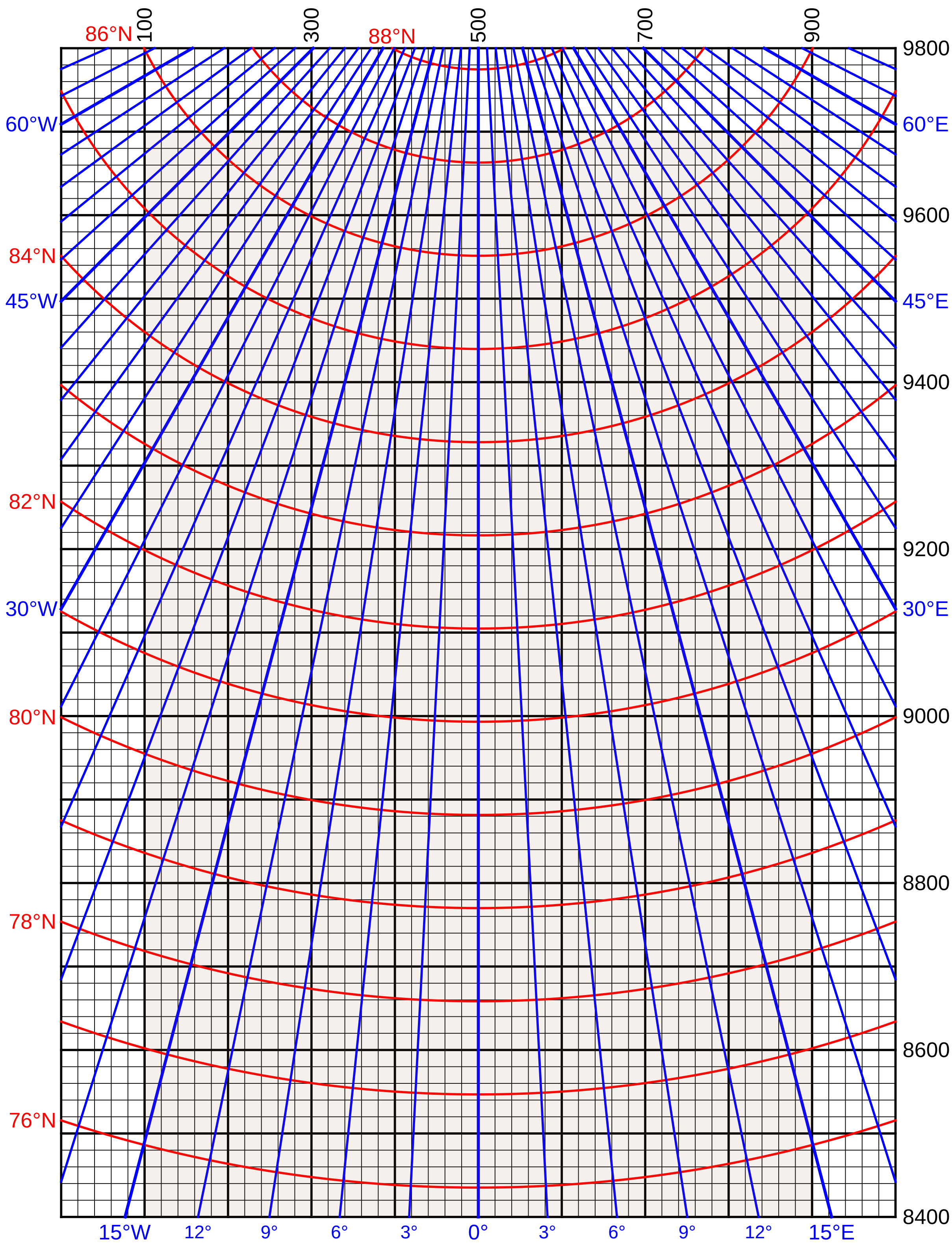

Fig. 2. UTM plane for arbitrary zone `Z > 0` showing grid-lines, meridians, and parallels. All eastings and northings are in kilometers. Longitudes are relative to the unspecified central meridian. The region representable in MGRS is shaded.

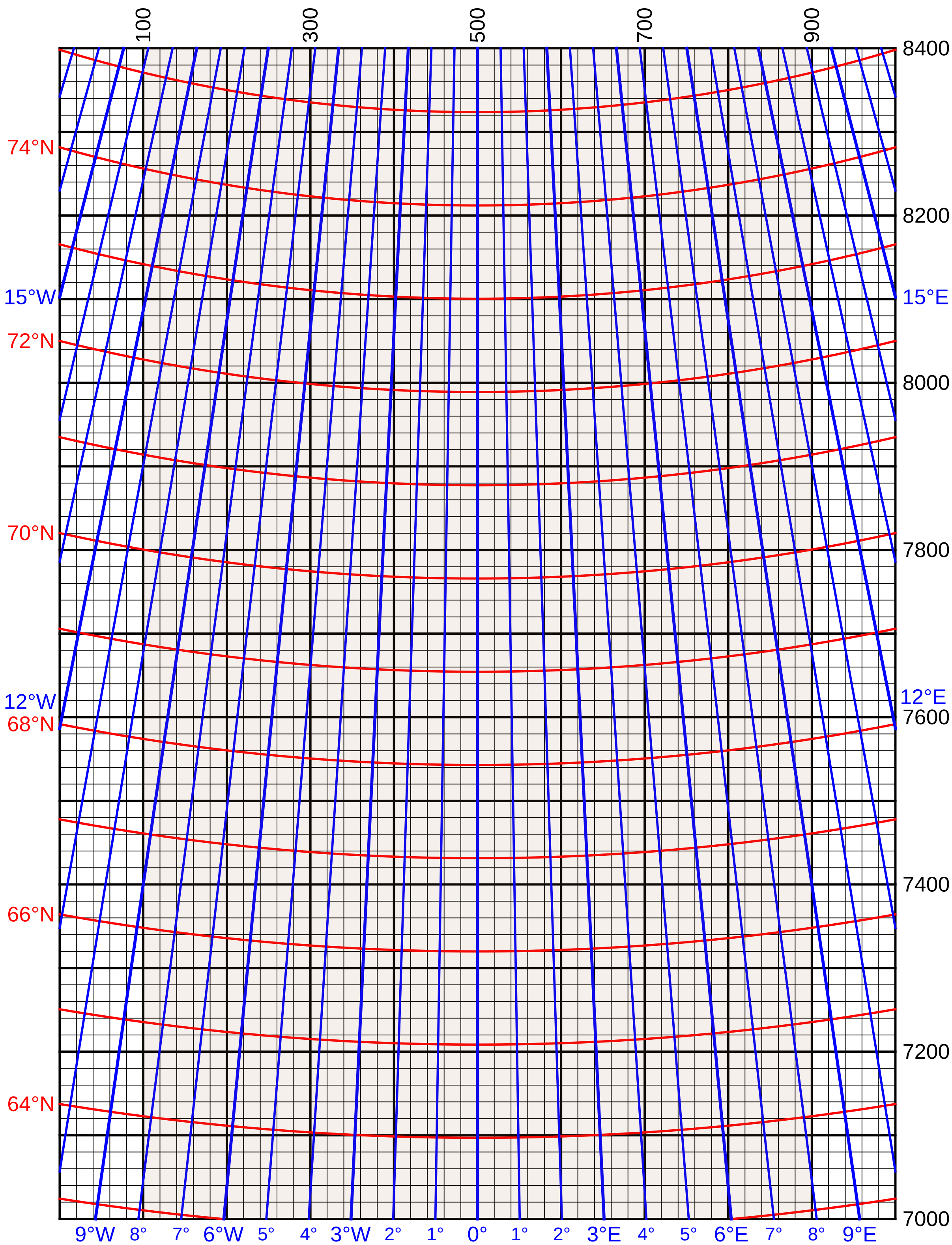

Fig. 3. UTM plane for arbitrary zone $Z > 0$ showing grid-lines, meridians, and parallels. All eastings and northings are in kilometers. Longitudes are relative to the unspecified central meridian. The region representable in MGRS is shaded.

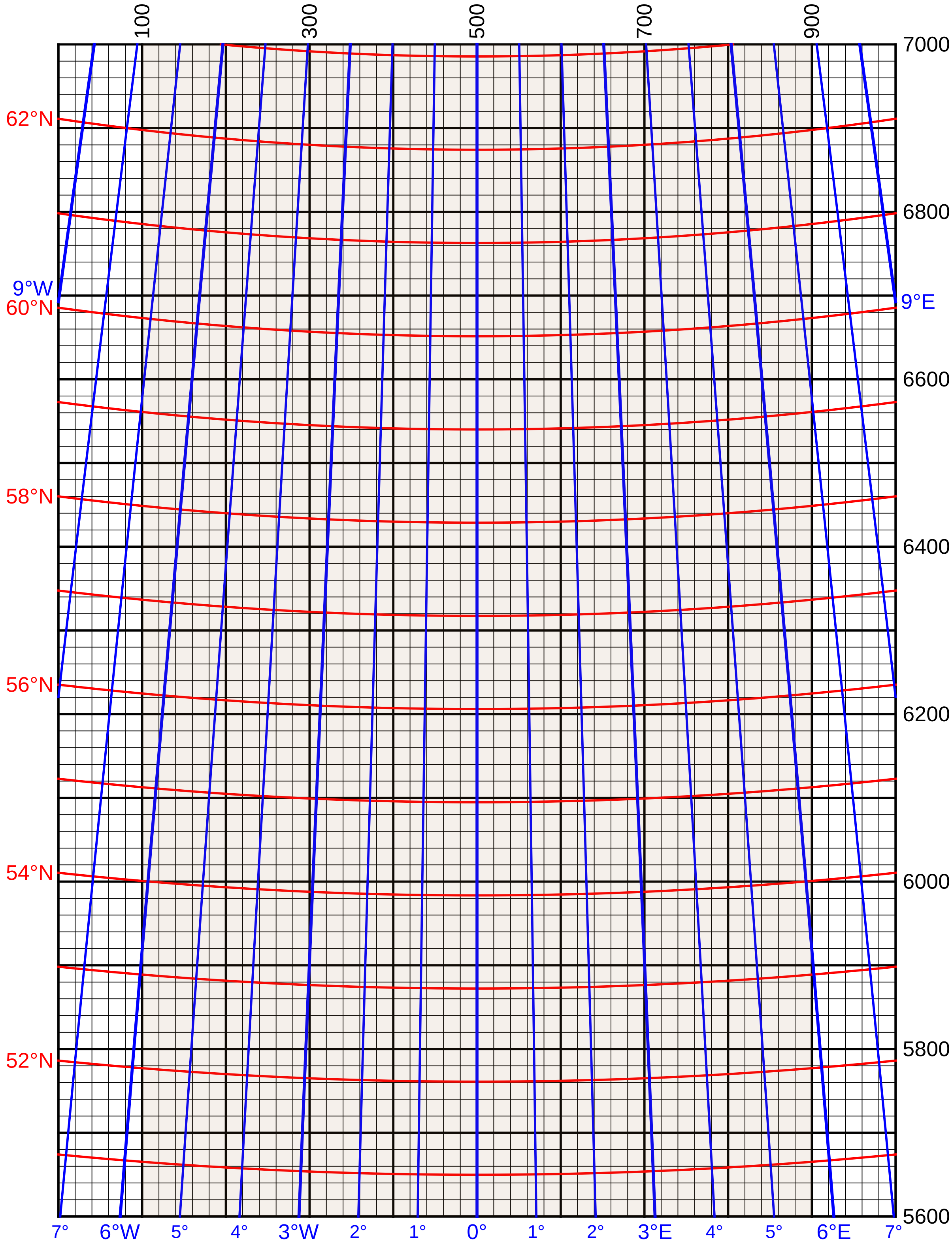

Fig. 4. UTM plane for arbitrary zone $Z > 0$ showing grid-lines, meridians, and parallels. All eastings and northings are in kilometers. Longitudes are relative to the unspecified central meridian. The region representable in MGRS is shaded.

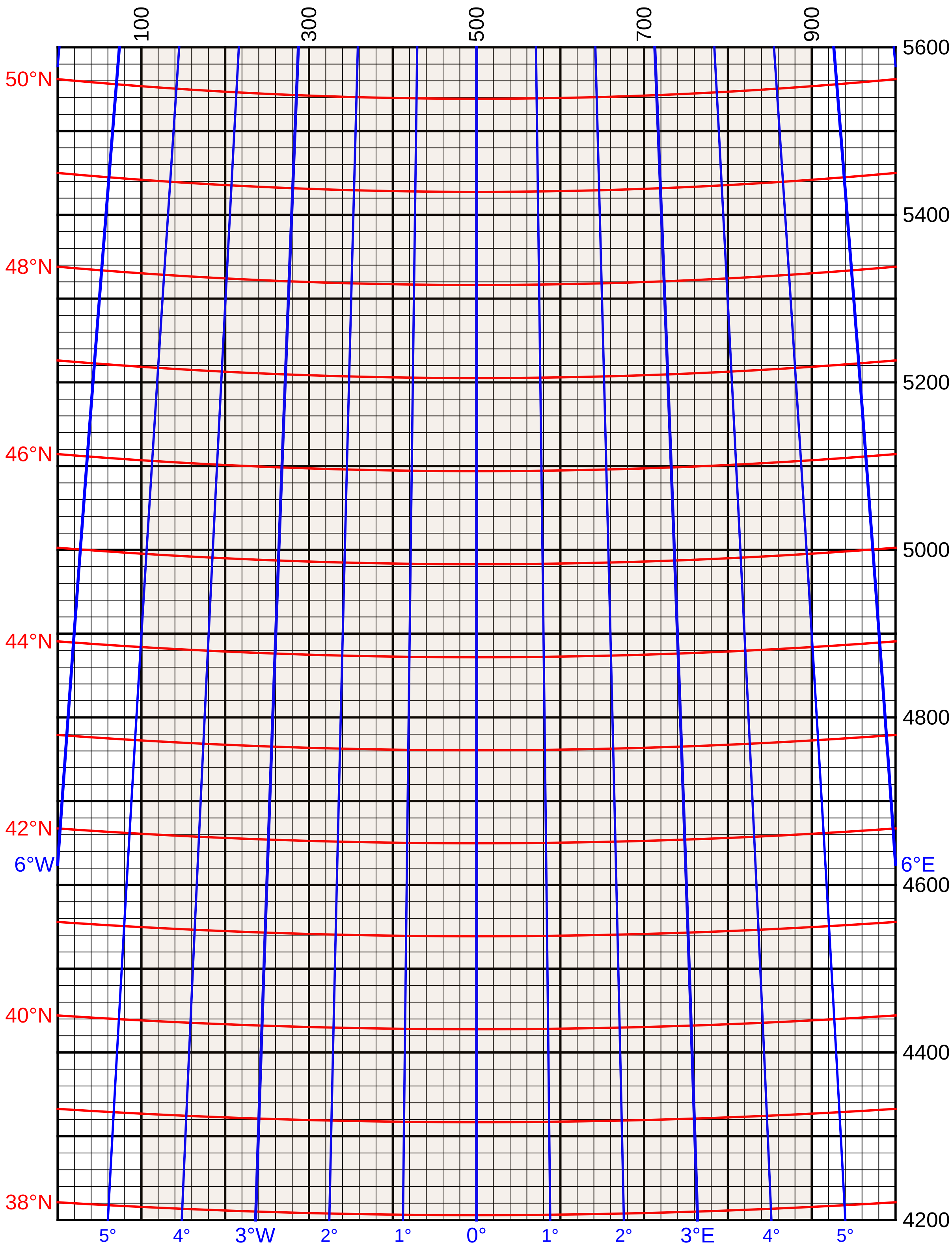

Fig. 5. UTM plane for arbitrary zone Z > 0 showing grid-lines, meridians, and parallels. All eastings and northings are in kilometers. Longitudes are relative to the unspecified central meridian. The region representable in MGRS is shaded.

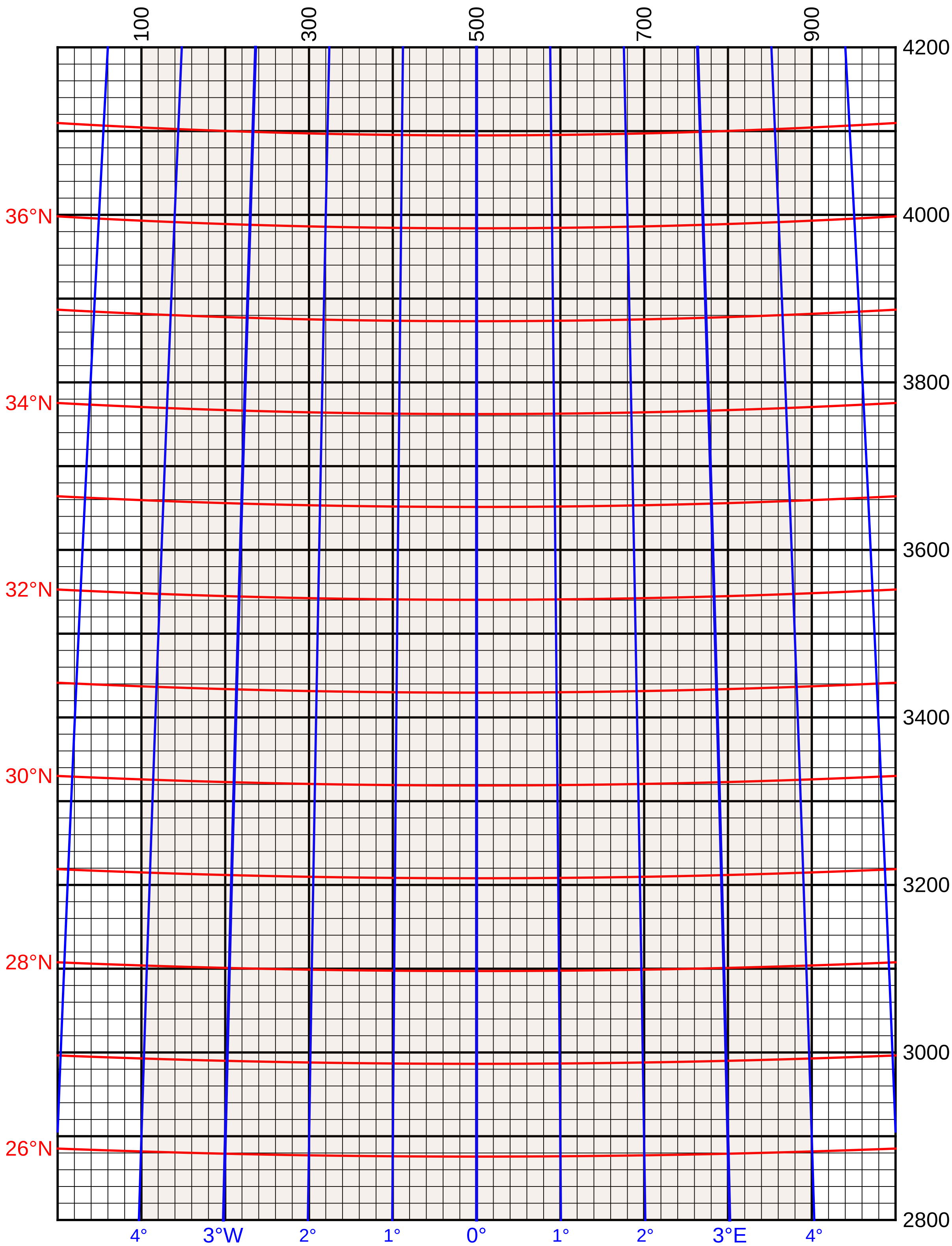

Fig. 6. UTM plane for arbitrary zone $Z > 0$ showing grid-lines, meridians, and parallels. All eastings and northings are in kilometers. Longitudes are relative to the unspecified central meridian. The region representable in MGRS is shaded.

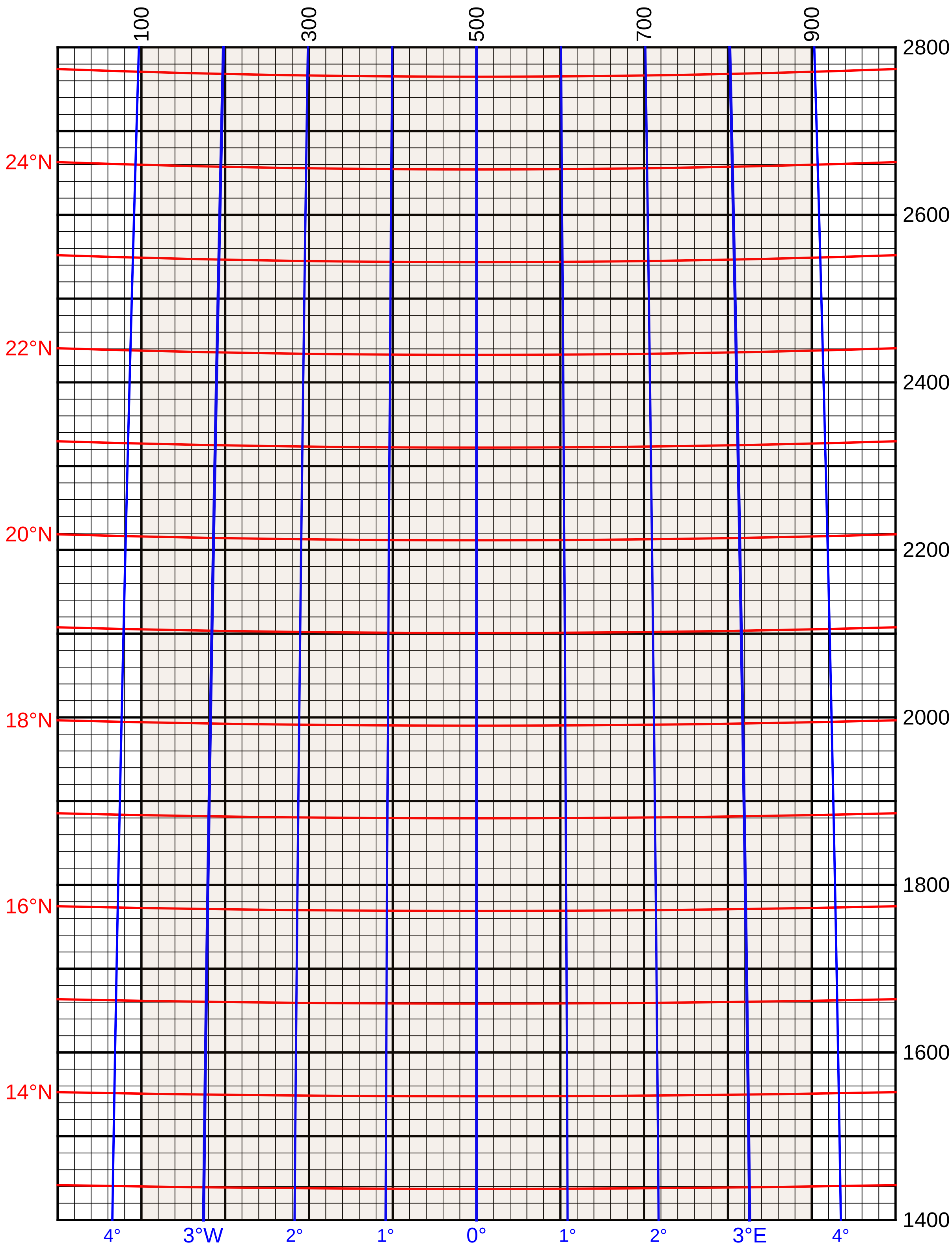

Fig. 7. UTM plane for arbitrary zone `Z > 0` showing grid-lines, meridians, and parallels. All eastings and northings are in kilometers. Longitudes are relative to the unspecified central meridian. The region representable in MGRS is shaded.

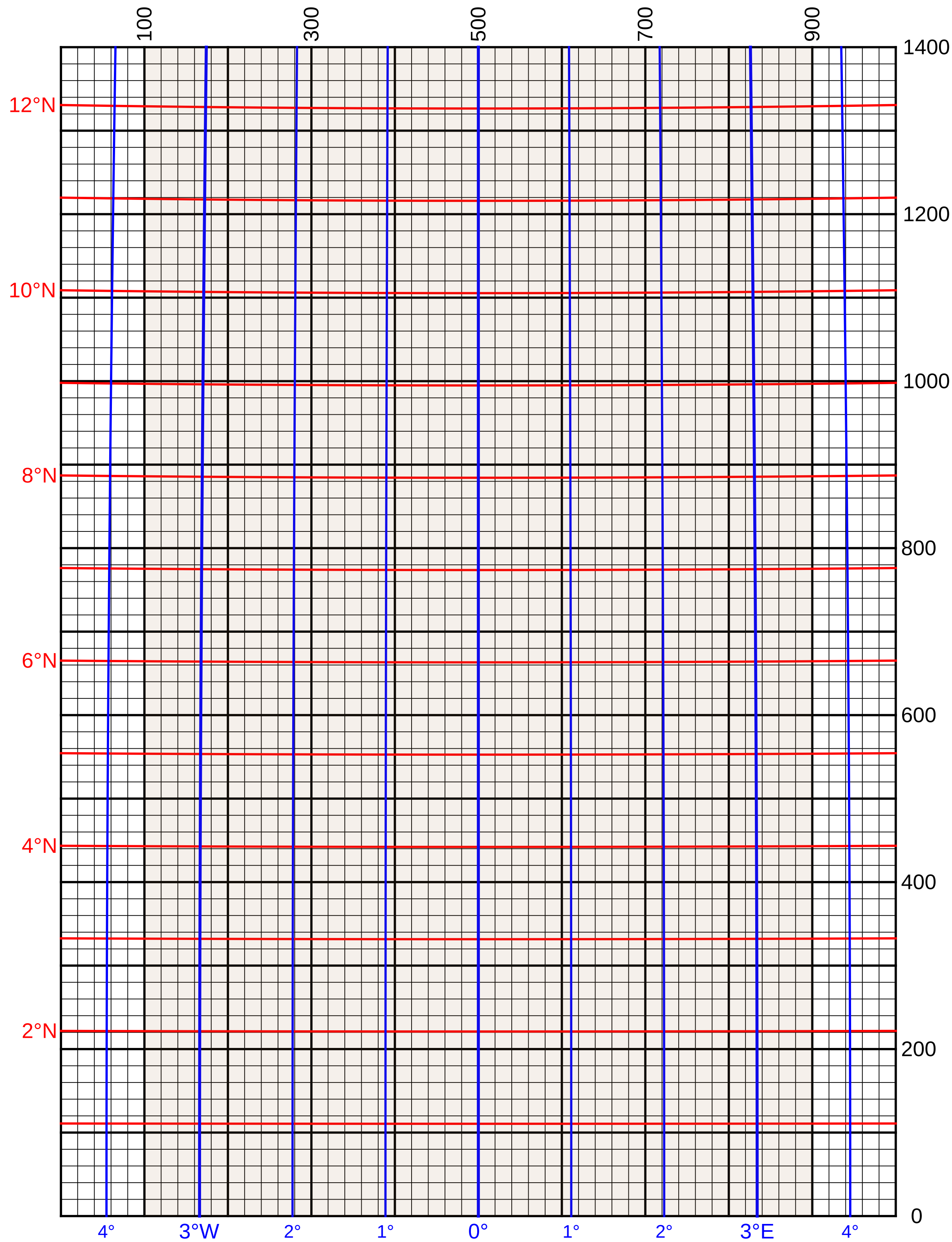

Fig. 8. UTM plane for arbitrary zone $Z > 0$ showing grid-lines, meridians, and parallels. All eastings and northings are in kilometers. Longitudes are relative to the unspecified central meridian. The region representable in MGRS is shaded.

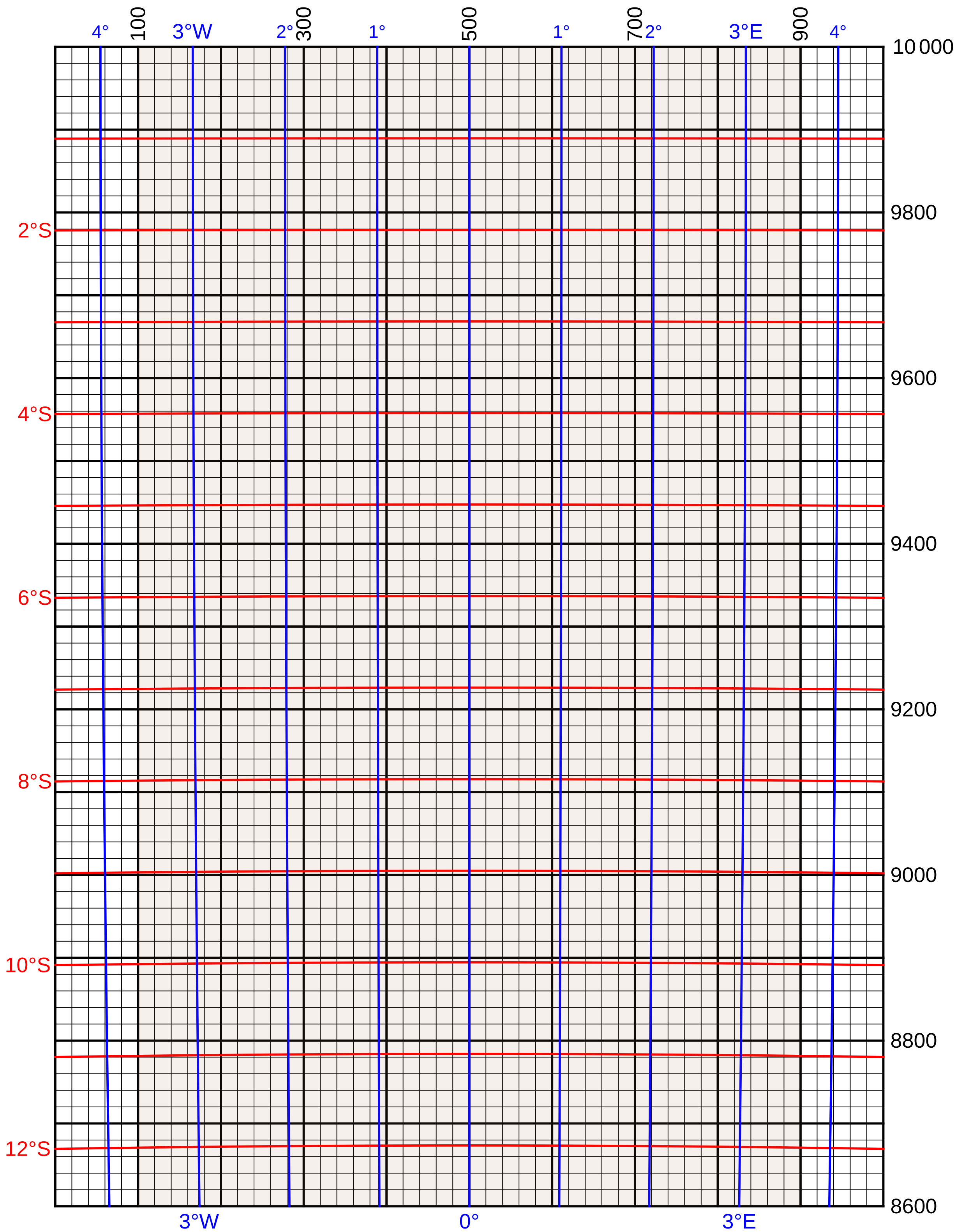

Fig. 9. UTM plane for arbitrary zone `Z < 0` showing grid-lines, meridians, and parallels. All eastings and northings are in kilometers. Longitudes are relative to the unspecified central meridian. The region representable in MGRS is shaded.

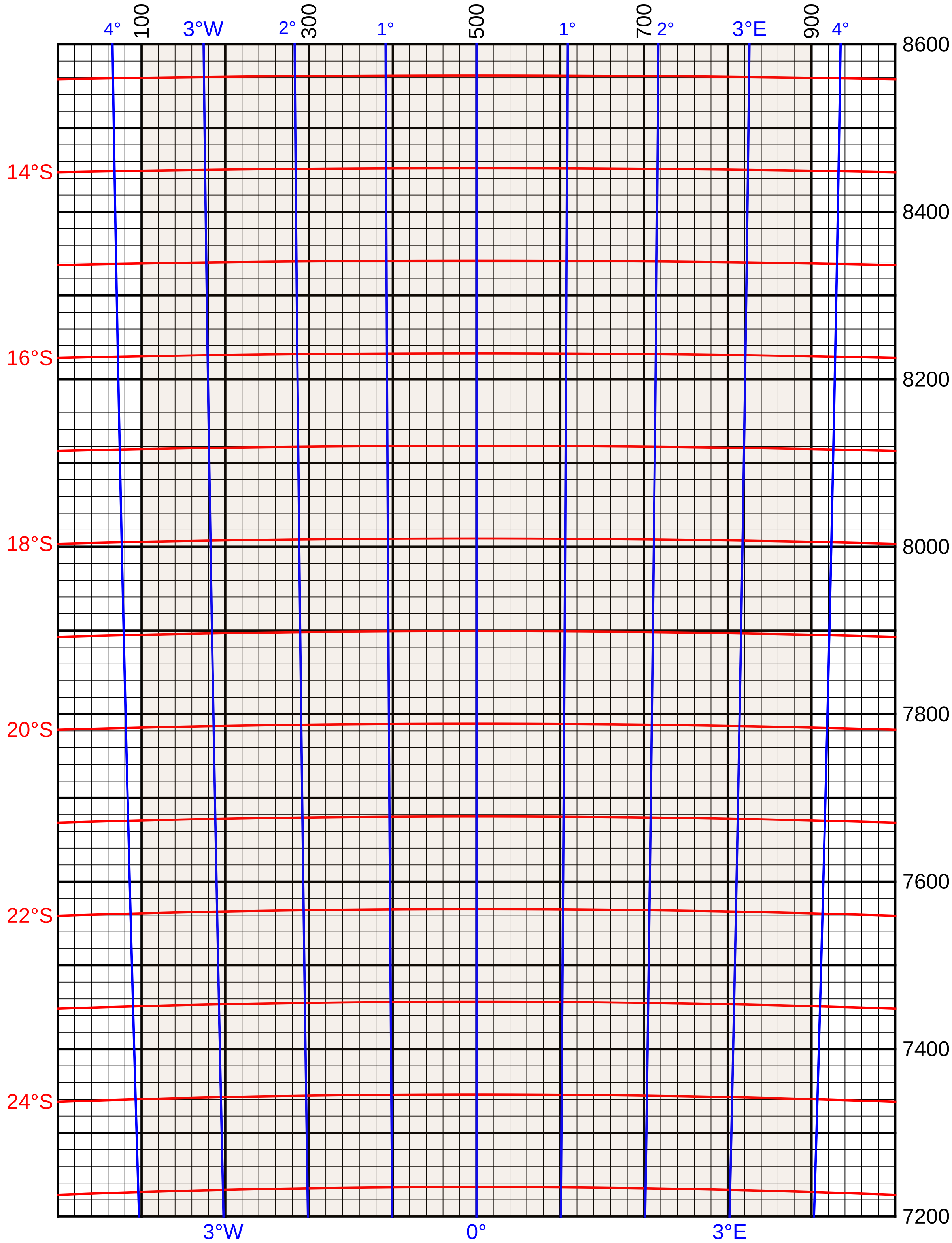

Fig. 10. UTM plane for arbitrary zone $Z < 0$ showing grid-lines, meridians, and parallels. All eastings and northings are in kilometers. Longitudes are relative to the unspecified central meridian. The region representable in MGRS is shaded.

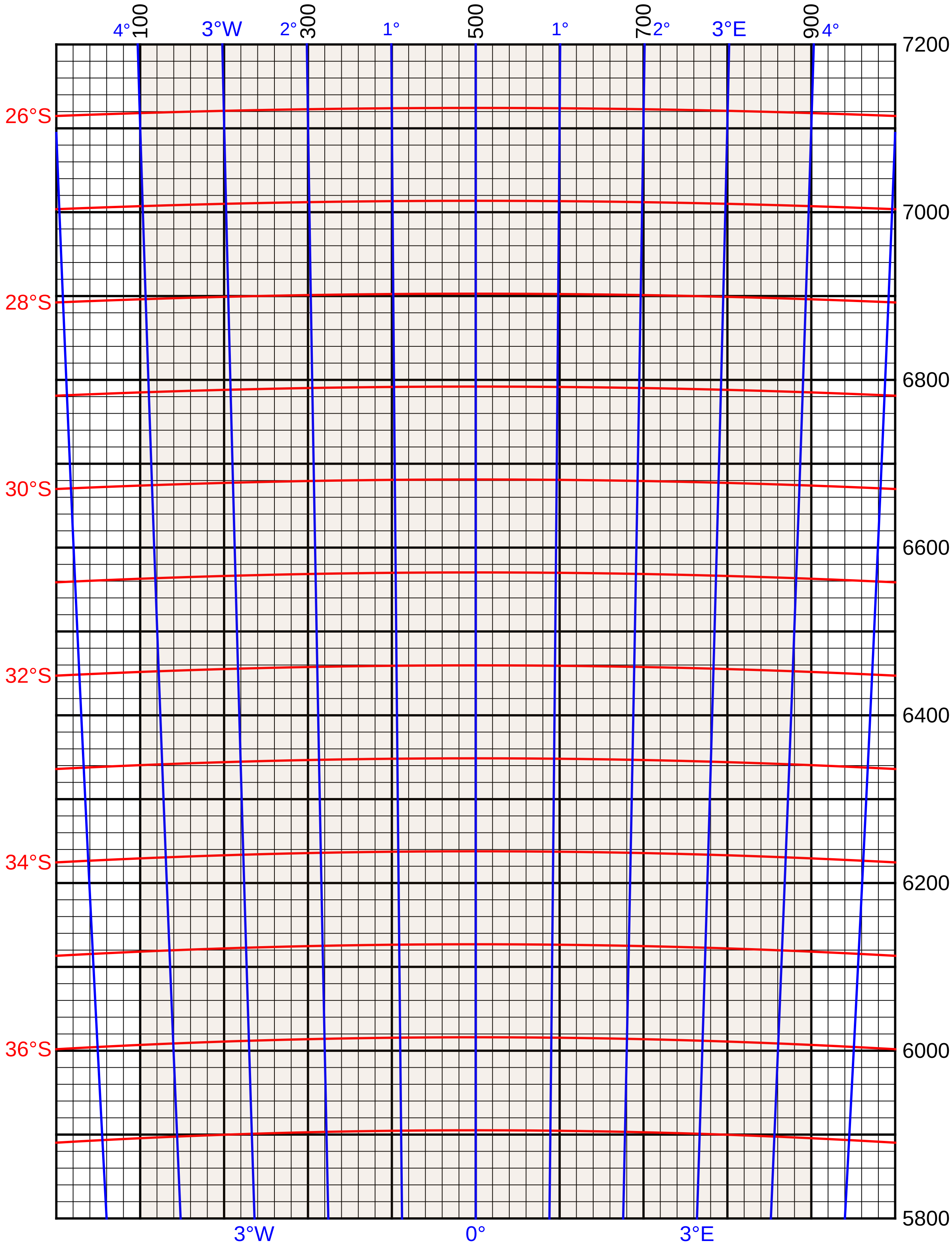

Fig. 11. UTM plane for arbitrary zone `Z < 0` showing grid-lines, meridians, and parallels. All eastings and northings are in kilometers. Longitudes are relative to the unspecified central meridian. The region representable in MGRS is shaded.

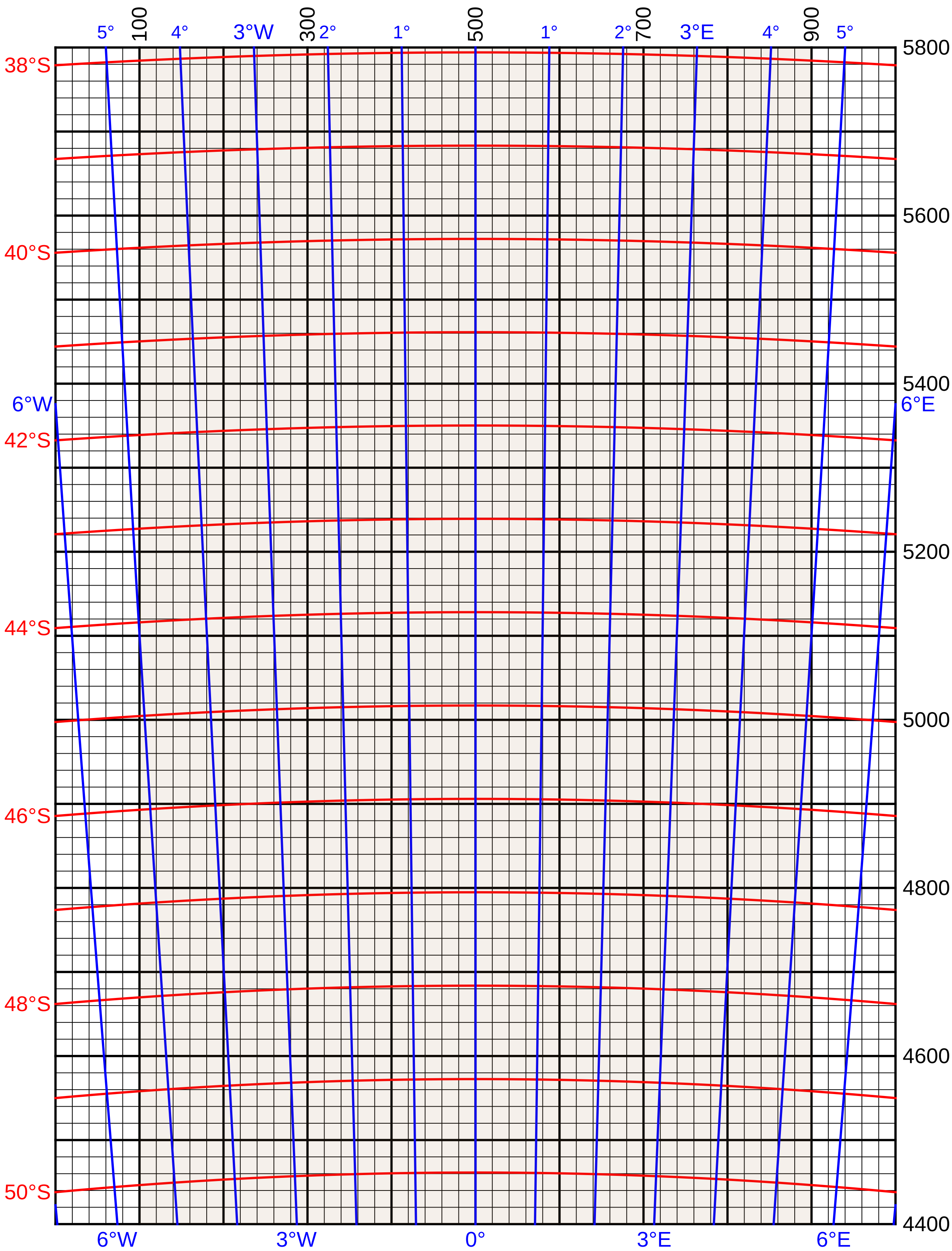

Fig. 12. UTM plane for arbitrary zone Z < 0 showing grid-lines, meridians, and parallels. All eastings and northings are in kilometers. Longitudes are relative to the unspecified central meridian. The region representable in MGRS is shaded.

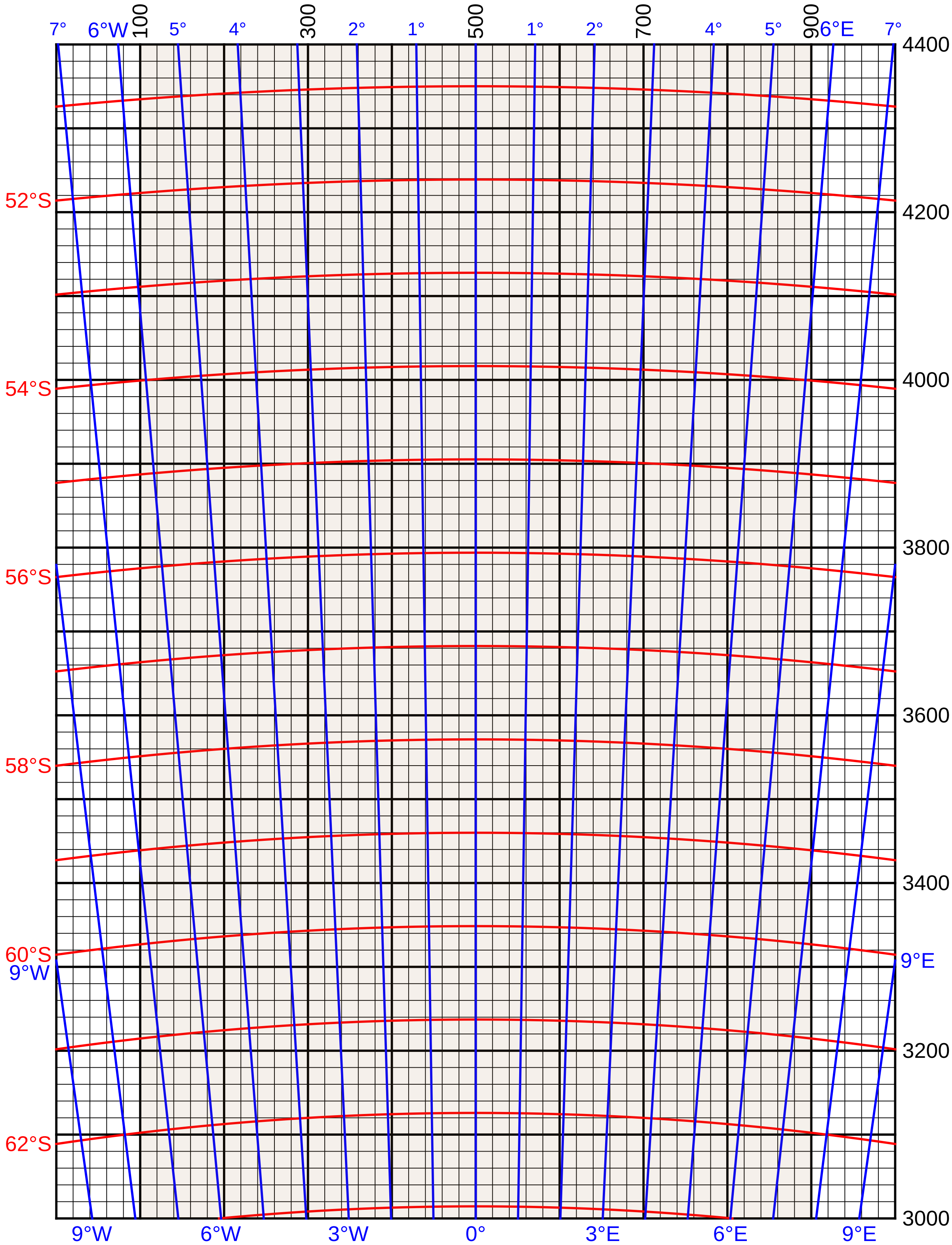

Fig. 13. UTM plane for arbitrary zone `Z < 0` showing grid-lines, meridians, and parallels. All eastings and northings are in kilometers. Longitudes are relative to the unspecified central meridian. The region representable in MGRS is shaded.

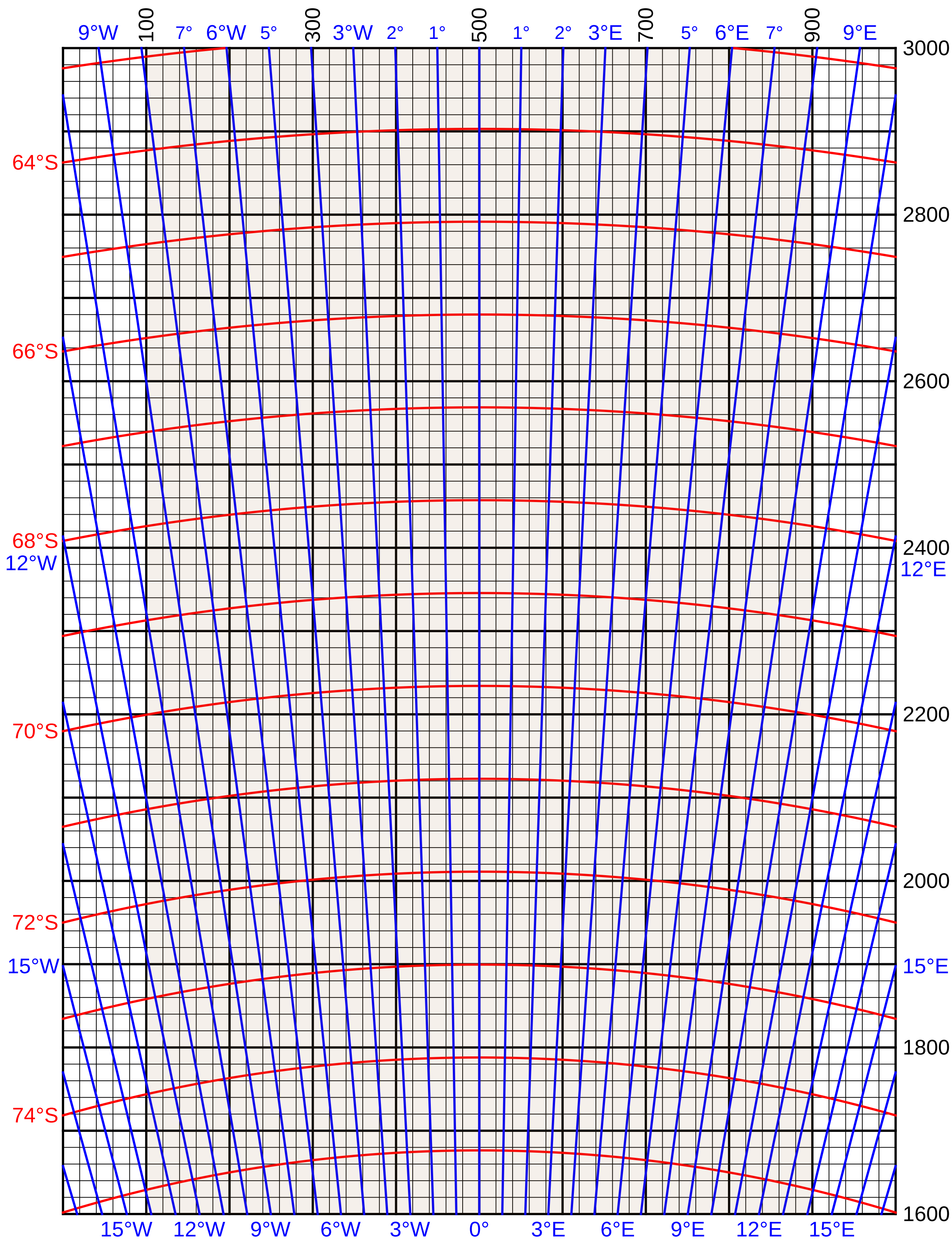

Fig. 14. UTM plane for arbitrary zone Z < 0 showing grid-lines, meridians, and parallels. All eastings and northings are in kilometers. Longitudes are relative to the unspecified central meridian. The region representable in MGRS is shaded.

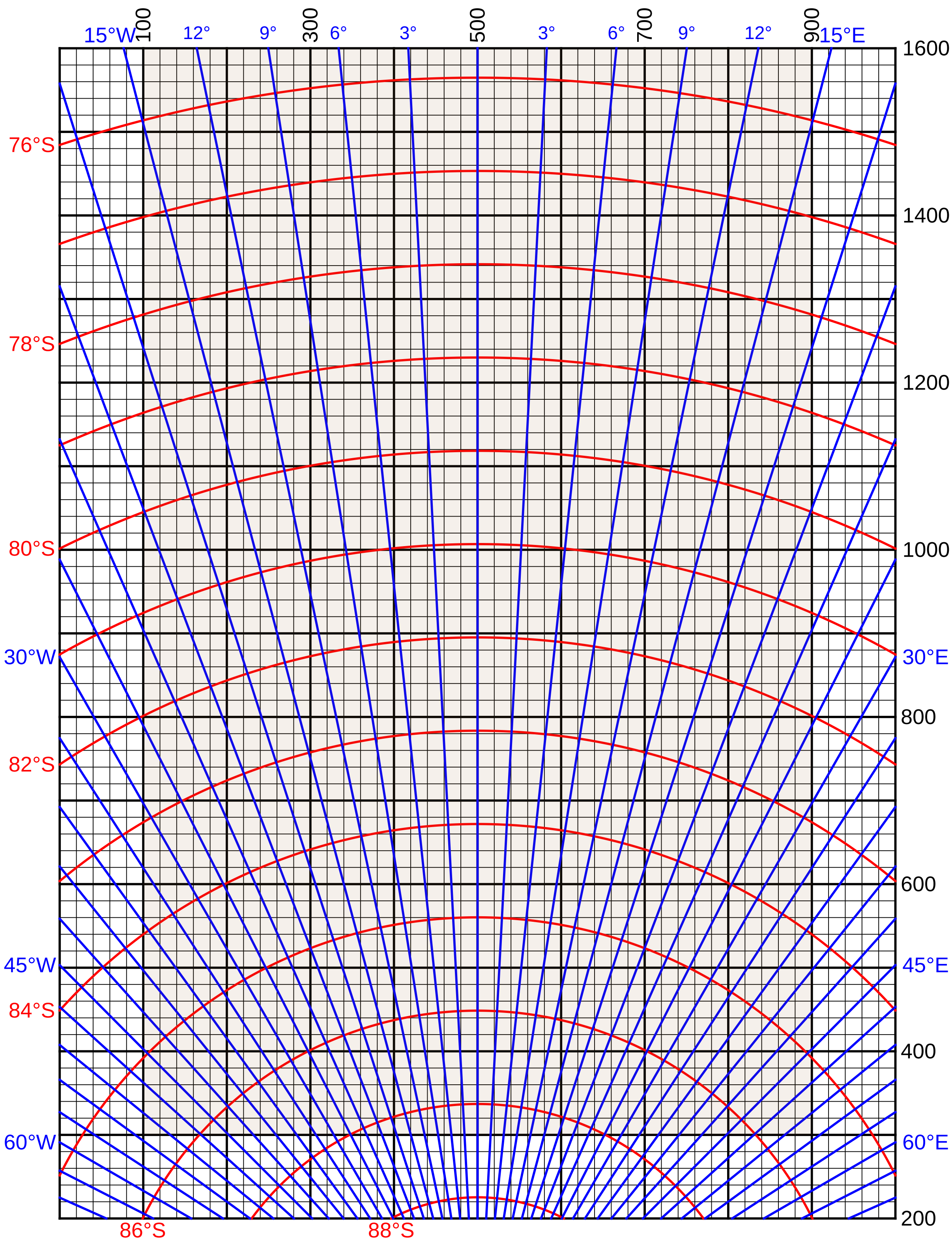

Fig. 15. UTM plane for arbitrary zone Z < 0 showing grid-lines, meridians, and parallels. All eastings and northings are in kilometers. Longitudes are relative to the unspecified central meridian. The region representable in MGRS is shaded.

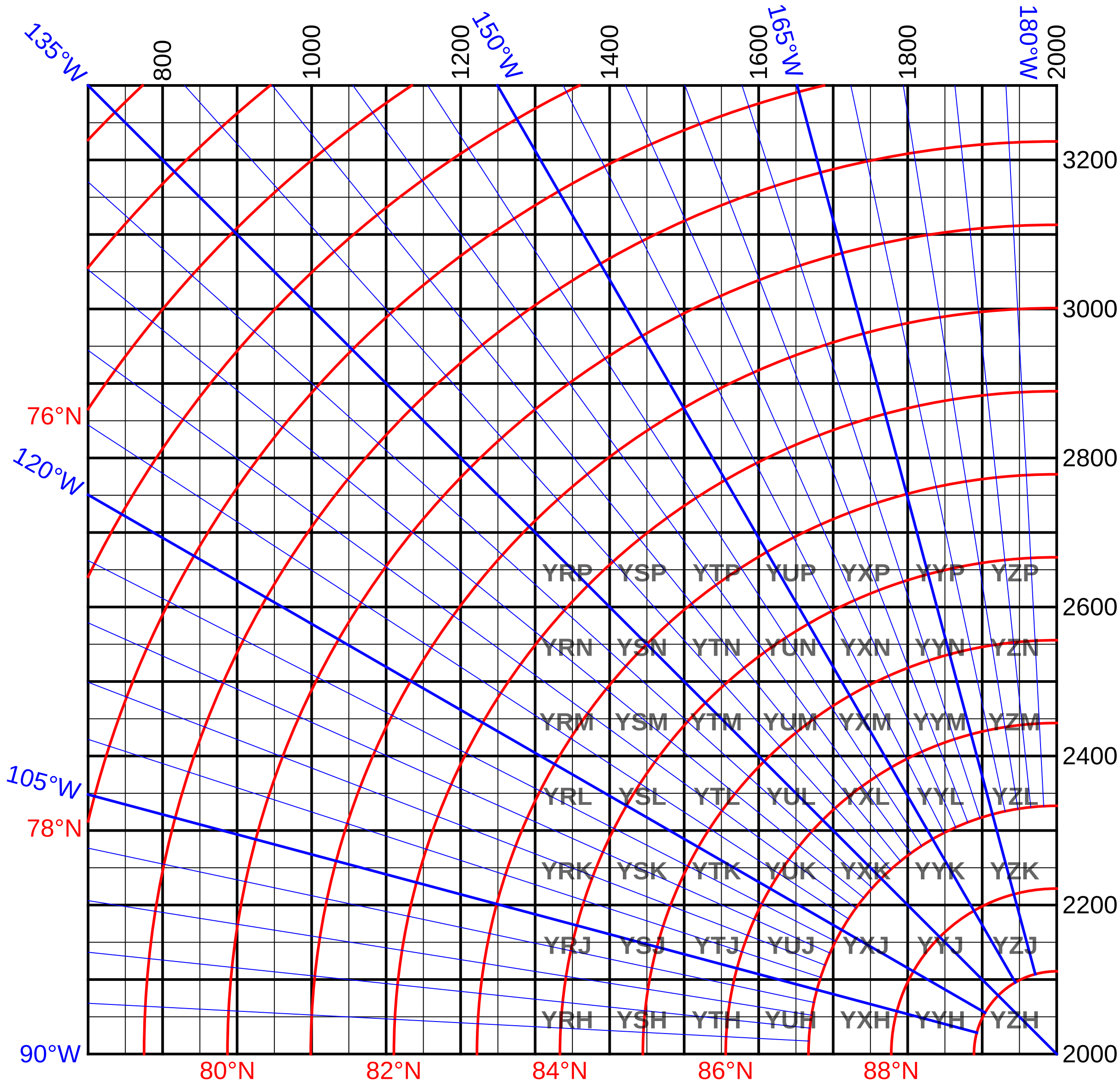

Fig. 16. UPS plane for $Z = +1$ (north zone) showing grid-lines, meridians, parallels and MGRS lettering. All eastings and northings are in kilometers.

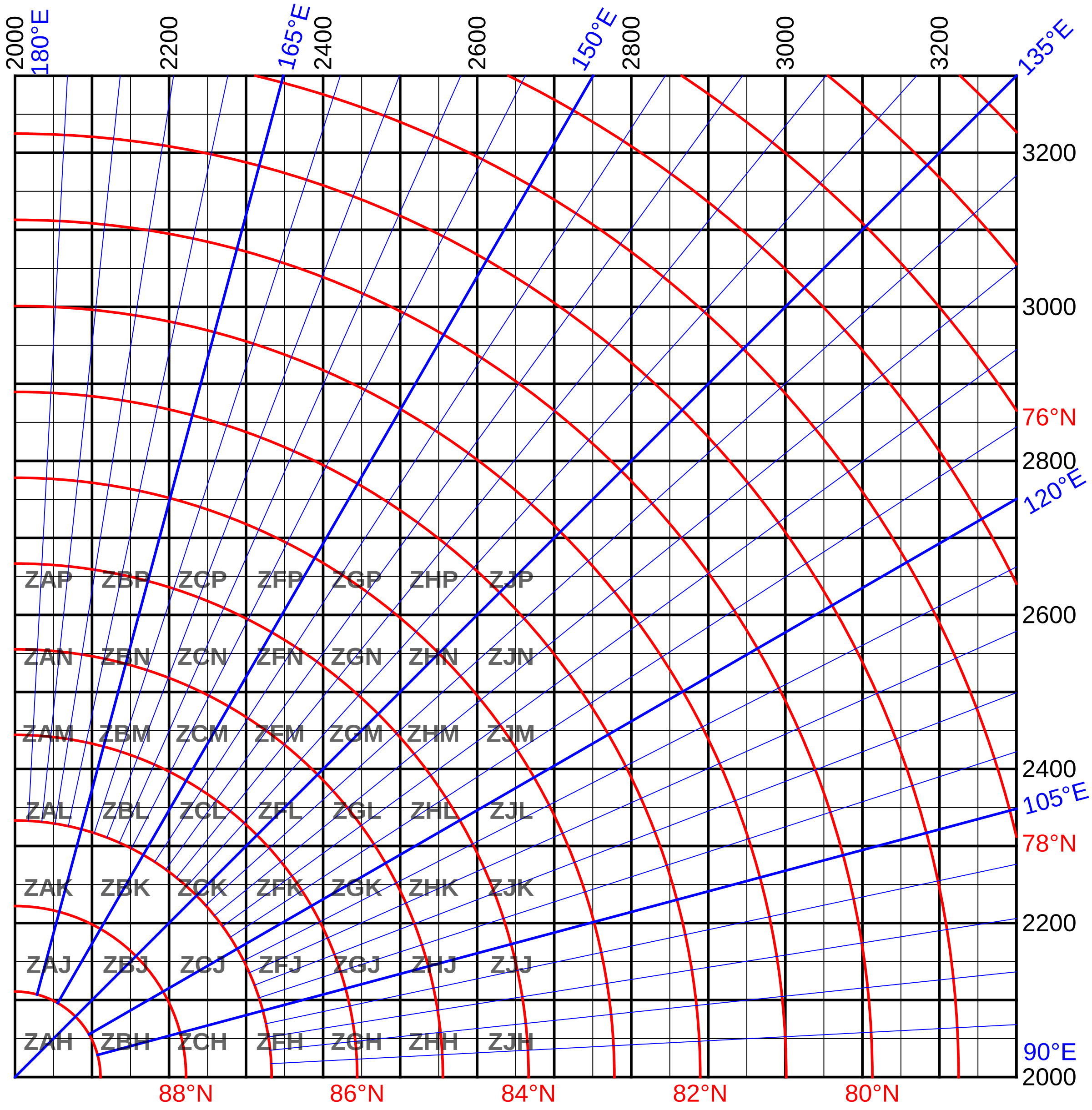

Fig. 17. UPS plane for $Z = +1$ (north zone) showing grid-lines, meridians, parallels and MGRS lettering. All eastings and northings are in kilometers.

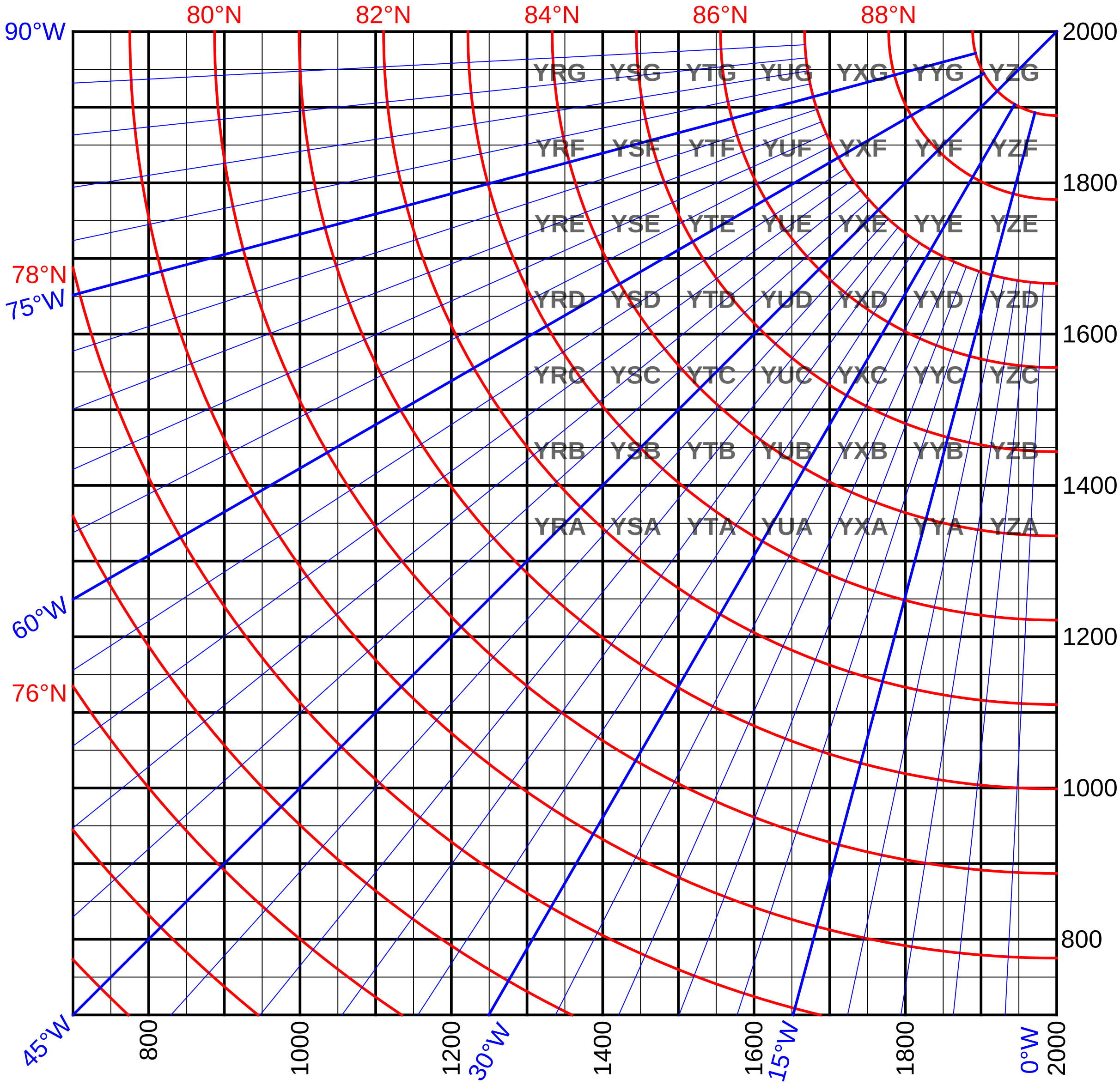

Fig. 18. UPS plane for $Z = +1$ (north zone) showing grid-lines, meridians, parallels and MGRS lettering. All eastings and northings are in kilometers.

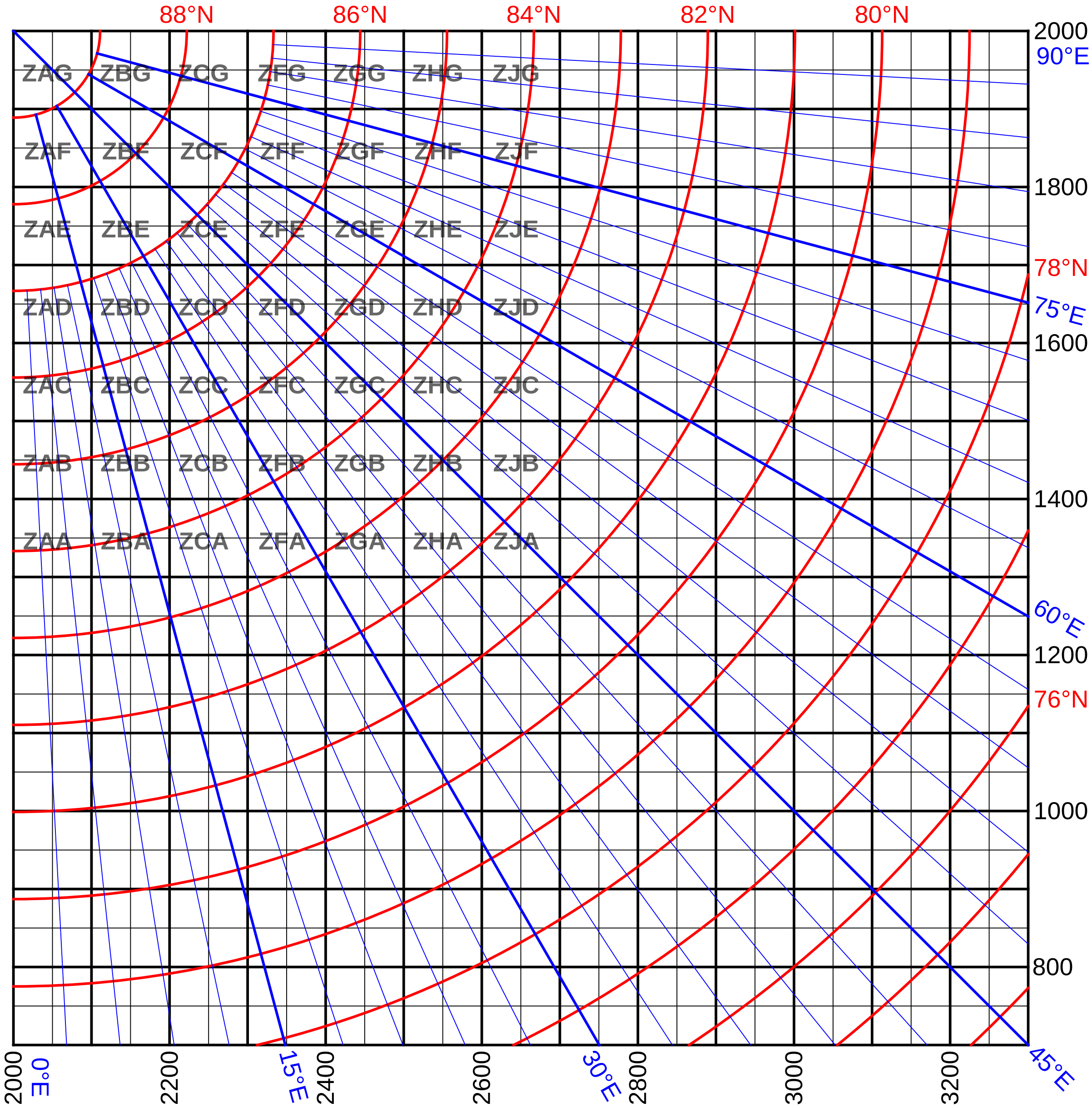

Fig. 19. UPS plane for $Z = +1$ (north zone) showing grid-lines, meridians, parallels and MGRS lettering. All eastings and northings are in kilometers.

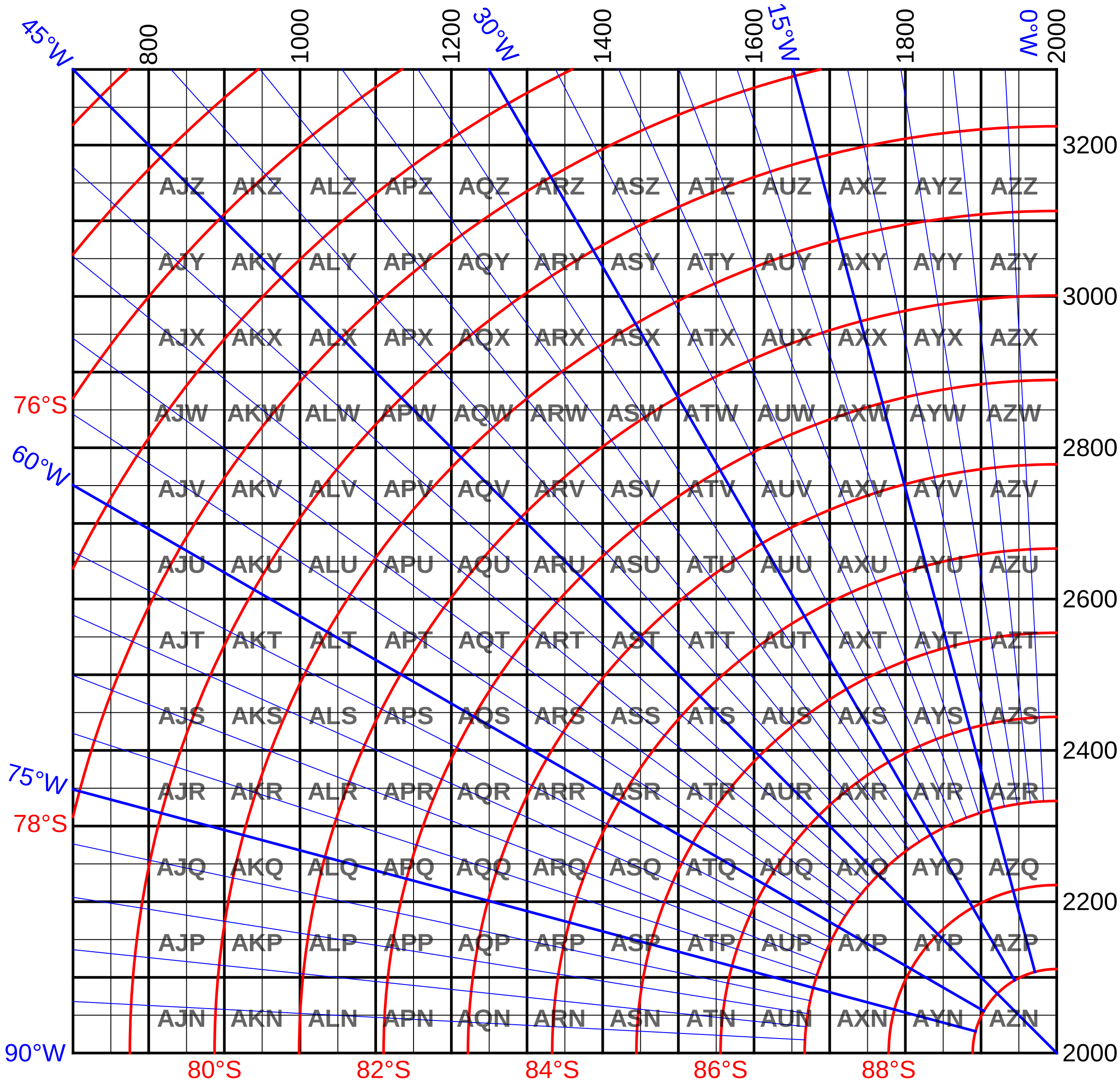

Fig. 20. UPS plane for $Z = -1$ (south zone) showing grid-lines, meridians, parallels and MGRS lettering. All eastings and northings are in kilometers.

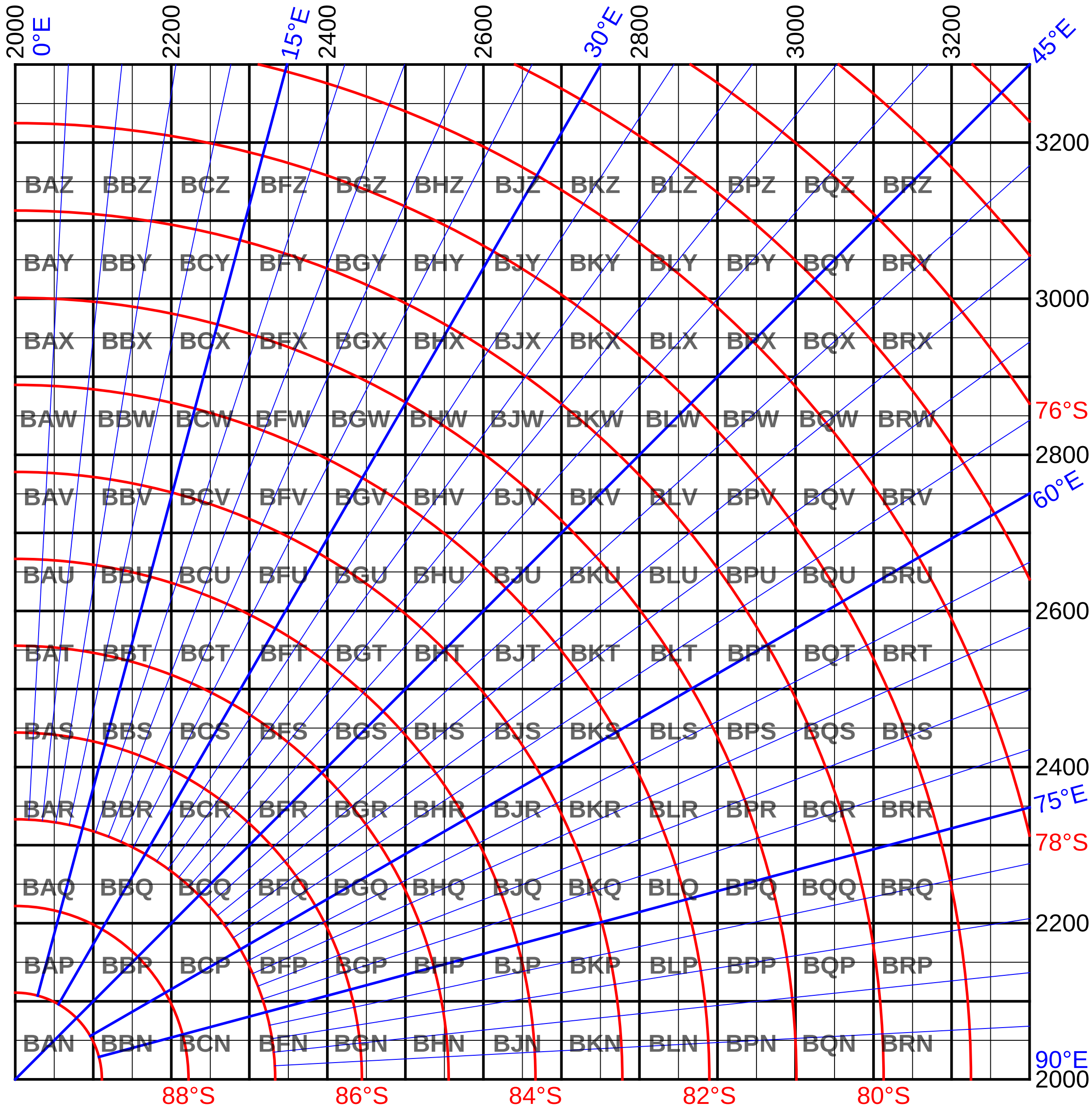

Fig. 21. UPS plane for $Z = -1$ (south zone) showing grid-lines, meridians, parallels and MGRS lettering. All eastings and northings are in kilometers.

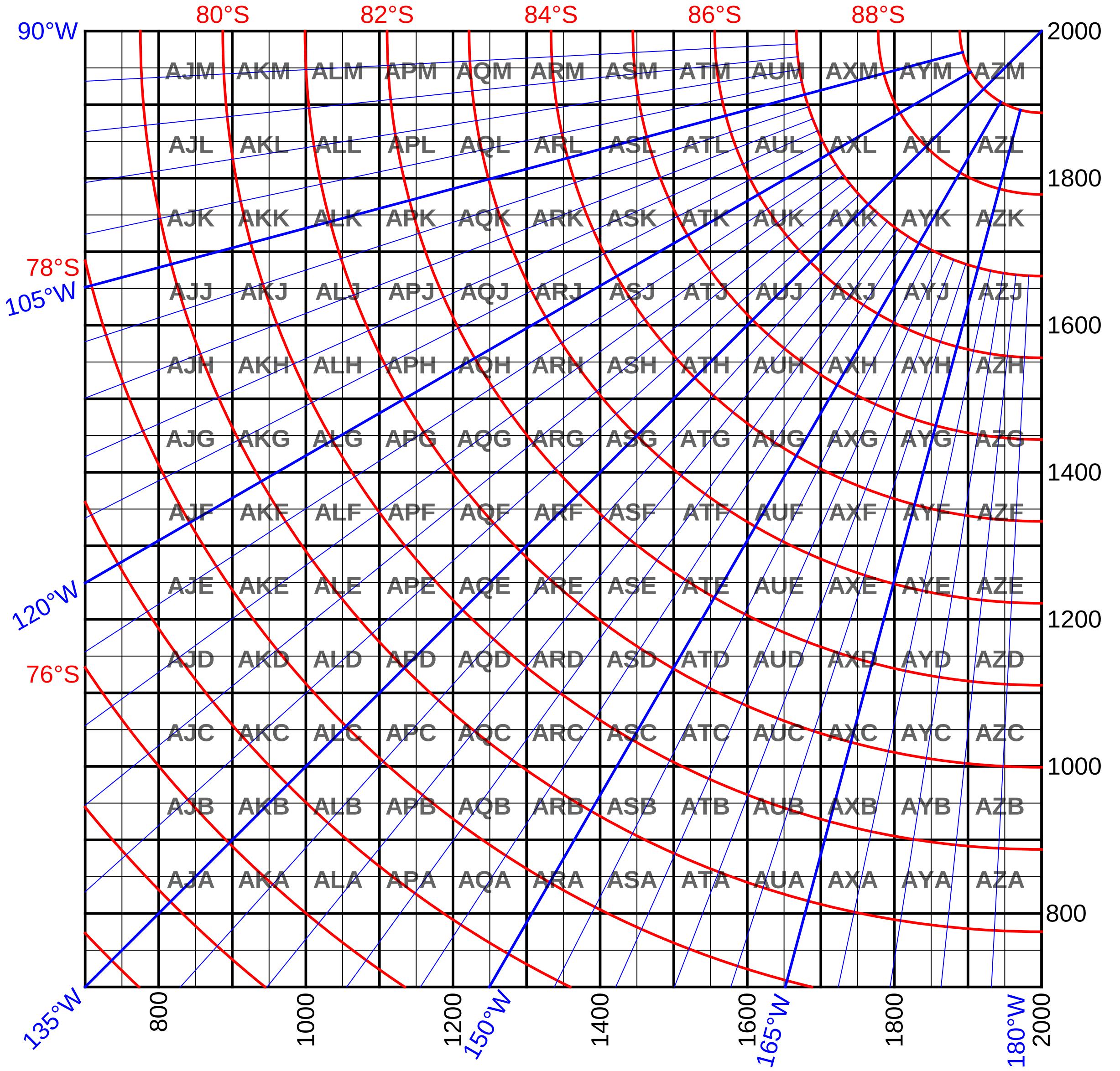

Fig. 22. UPS plane for $Z = -1$ (south zone) showing grid-lines, meridians, parallels and MGRS lettering. All eastings and northings are in kilometers.

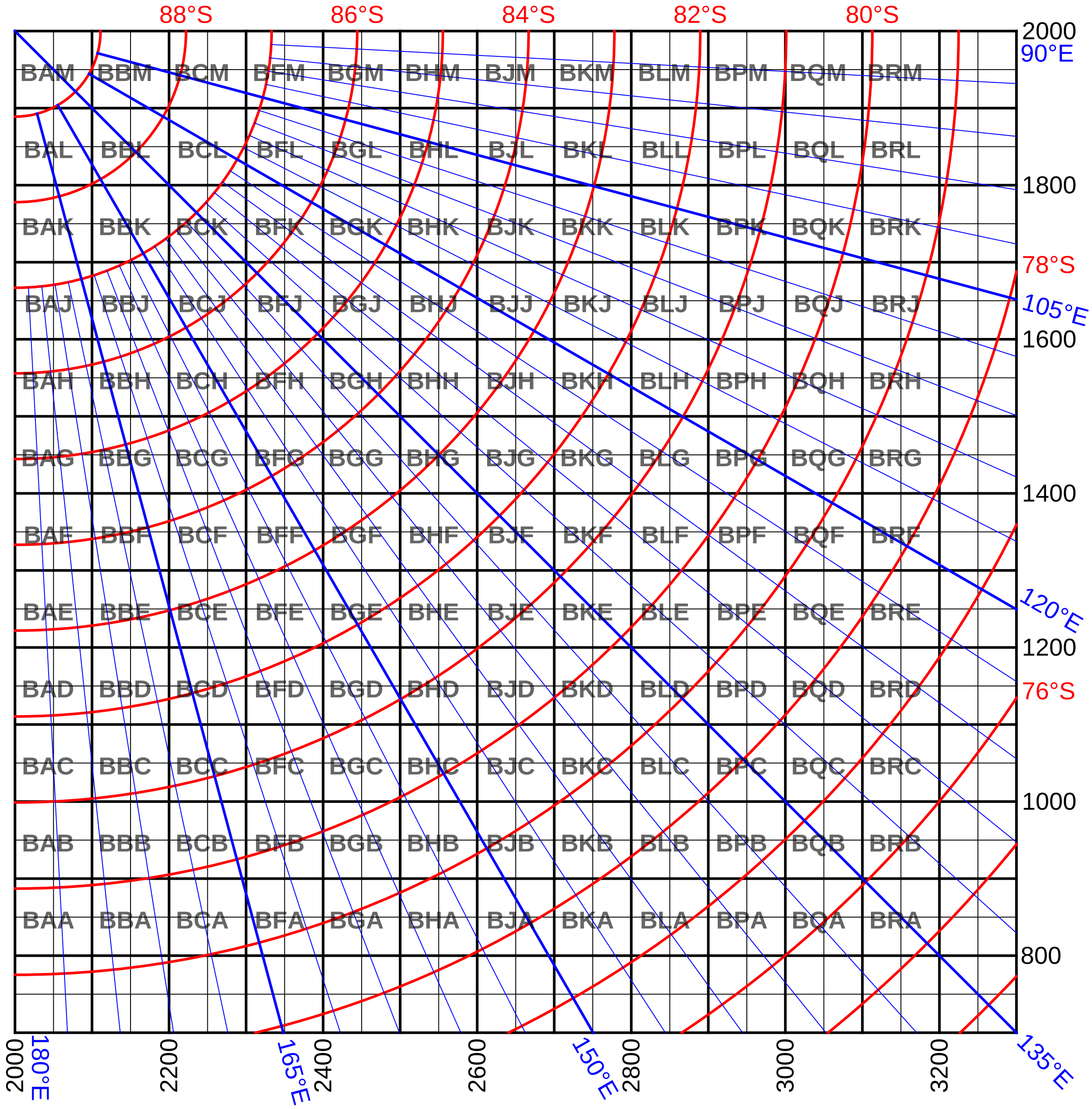

Fig. 23. UPS plane for $Z = -1$ (south zone) showing grid-lines, meridians, parallels and MGRS lettering. All eastings and northings are in kilometers.

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