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AIRCRAFT NAVIGATION

by S. S. Fedchin

"Transport" Press
Moscow, 1966

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TABLE OF CONTENTS

ABSTRACT: The theory and practice of aircraft navigation at the modern level of aviation technology are summarized in this book; the most important practical problems of the utilization of general, radio-engineering, and astronomical means of aircraft navigation are set forth; the procedure of the pilot's preparation for flight, the means of calculating the distance and duration of a flight, and the carrying out of pre-landing maneuvering and landing of the aircraft under complex meteorological conditions during the day or at night are elucidated. /2

The basic material of the book, sufficient for the practical mastery of the means and methods of aircraft navigation, is presented with the application of mathematics within the limits of a secondary school course. The problems which are necessary for a deeper study of the material are discussed in terms of principles of higher mathematics.

The book is intended for pilots and navigators. It can be used as a textbook for students of civil aviation educational institutions.

INTRODUCTION /3

Aircraft navigation or aerial navigation is a science which studies the theory and practical methods of the safe navigation of airplanes as well as other aircraft (helicopters, dirigibles, etc.) in the airspace above the Earth's surface.

By the process of aircraft navigation, we mean the complex of activities of the aircraft crew and the ground traffic control, which are directed toward a constant knowledge of the aircraft's location and which ensure safe and accurate flight along a set course as well as arrival at the point of destination at a set altitude and at an established time.

During the initial period of the development of aviation, aircraft did not have equipment for piloting when the natural horizon was not visible and for orientation when the ground was not visible, so that visual orientation was the basic method of aircraft navigation. The position of the aircraft was determined by comparing visible landmarks in the area over which the aircraft was flying, with their representation on a map.

However, at this time the necessity for instrumental methods of aircraft navigation was already felt. The most simple devices for measuring airspeed, flight altitude, the aircraft's course, and several other flight parameters were installed on aircraft. This period saw the appearance of the first navigator's calculating instruments (wind-speed indicators and navigational slide rules).

At the beginning of the 1920's, the first hydroscopic devices appeared on aircraft; they were turn and glide indicators which (in combination with indicators of airspeed and vertical velocity) indicators (variometers) made it possible to judge in a rather primitive way the position of the aircraft in space when the natural horizon was not visible. By means of these devices, the aircraft crews (after special training) were already able to carry out flights in the clouds and above the clouds.

At the end of the 20's and the beginning of the 30's, more refined pilotage devices were developed: gyrohorizons and gyrosemicompasses, which at this time reliably ensured pilotage of aircraft /4
with adequate safety when the ground was not visible. Later, on the basis of these devices, devices for automatic aircraft pilotage (automatic pilots) were created.

Achievements in the area of piloting aircraft when the Earth was not visible, as well as the growth by that time of the speed, altitude, and distance of aircraft flights, required the creation of means to ensure aircraft navigation independent of the visibility of terrestrial landmarks.

During these years, zone radio beacons which allowed the aircraft's flight direction to be maintained along a narrowly directed radial line which coincides with the direction of the straight part of an aerial route began to appear. Ground radiogoniometers also appeared, by means of which direction was determined in an aircraft, as well as the position of the aircraft along two intersecting directions.

Another aspect of the development of aircraft navigation at this time was astronomical orientation. To determine the location of an aircraft, various sextants were constructed and special computation tables and graphs of the movement of heavenly bodies were compiled for use with the sextants. In the mid-30's, devices appeared for determining the course of an aircraft according to the heavenly bodies.

At the same time, optical sighting devices were used, by means of which (during visibility of the terrestrial landmarks) the groundspeed, flight direction and drift angle of the aircraft were measured, all of which were later used for some time as constants for calculating the path of an aircraft according to flight time and direction.

A very important stage in the development of means of aircraft navigation of the mid-30's was the appearance of aircraft radiogoniometers (radiosemicompasses), a further modification of which were the automatic aircraft radiocompasses. Radiosemicompasses and radiocompasses were, for a period of more than 20 years, the basic means of aircraft navigation in aircraft with piston engines.

During World War II and especially in the postwar years, radio-engineering systems of long and short-distance navigation of a different kind as well as radio-navigation landing systems became widespread. Essentially, these were not autonomous means of aerial navigation but systems which included both ground-based facilities for the security of aircraft navigation and aircraft equipment.

Radical changes in the area of means and methods of aircraft navigation occurred (and are occurring at the present time) in connection with the development of jet aviation technology.

The sharply growing speed, altitude, and distance of flights have required automation of the most laborious processes of aircraft navigation. Magnetic course devices and non-automatic radio navi- /5
gational systems were of little use for ensuring the automation of aircraft navigation and the piloting of high-speed aircraft. There

arose a necessity for developing highly stable gyroscopic compasses, autonomous speed and flight direction meters, and stricter consideration of the aircraft's flight dynamics to ensure the rapid and accurate solution of navigational problems by computers.

The science of "aircraft navigation" grew and developed along with the development of aviation and navigation technology. The works of the outstanding Russian scientists and inventors, M. V. Lomonosov, N. Ye. Zhukovskiy, K. E. Tsiolkovskiy, and A. S. Popov were the basis of aircraft navigation theory.

A large contribution to the science of aircraft navigation was made by the following Soviet navigators and scientists: B. V. Sterligov, S. A. Danilin, I. T. Spirin, G. S. Frenkel', A. V. Belyakov, L. P. Sergeyev, R. V. Kunitskiy, G. O. Fridlender, G. F. Molokanov, B. G. Rats, V. Yu. Polyak, et al.

The successes achieved in the development of aircraft navigation as a science made it possible, even in 1925-1929, to accomplish long flights by Soviet aircraft along the routes: Moscow-Peking (M. M. Gromov), Moscow-Tokyo and Moscow-New York (S. A. Shestakov).

Further nonstop flights by Soviet aviators, organized from 1936-1939 (V. P. Chkalov, M. M. Gromov, and V. K. Kokkinaki) both over the territory of the Soviet Union and especially over the North Pole to the USA, were like a great school, in which the examinations were the successes achieved by Soviet scientists in the area of aircraft navigation.

World War II was a verification of all the achievements in the theory and practice of aircraft navigation, especially in the field of long-distance aviation, with the carrying out of long-distance night flights. During this period, a rich store of experience was accumulated and further improvements in aircraft navigation methods were carried out.

In the postwar period, the science of aircraft navigation under-ent an especially vigorous development in connection with the ap-earance of high-speed jet aircraft, and also in connection with he great achievements of the radio and electronics industry.

Long-distance flights of high-speed aircraft along aerial utes which include international and intercontinental flights, as ll as flights to the Arctic and Antarctic, are becoming routine r civil aviation crews.

At the present time, aircraft navigation science has been dis-guished as an independent and orderly science in which the ievements of a number of the general and special branches of wledge are employed: physics, mathematics, geodesy, astronomy, hysics, aerodynamics, radio engineering, radio electronics, etc.

Navigation technology is developing at a rapid pace; aircraft and ground facilities for aircraft navigation are continually being perfected and the professional training and navigational preparation of flight and ground personnel has improved. All this has radically raised the reliability of aircraft navigation, its accuracy, and its chief criterion, safety. /6

Modern technical means of aircraft navigation are divided into four basic groups according to the principle of operation.

1. *Geotechnical means of aircraft navigation,* which are based on the principle of measuring different parameters of the Earth's fields. They include: magnetic compasses, gyroscopic navigation and piloting devices, gyromagnetic and gyroinduction telecompasses, course systems, airspeed indicators, barometric altimeters, external air thermometers, navigation indicators, inertial indicators, mechanical clocks, etc.

2. *Radio-engineering means of aircraft navigation,* which are based on the operating principle of radio-electronic technology. These include goniometer radio-engineering systems (radio compasses with ground transmitting radio stations, ground radiogoniometers with aircraft receiving-transmitting radio stations, and radio beacons with aircraft receiving radio equipment), rangefinding systems, goniometer-rangefinding systems, ground and aircraft radar, Doppler meters and systems, radio altimeters, course-landing beam systems with their ground and aircraft equipment, etc.

3. *Astronomical (radio astronomical) means of aircraft navigation,* which are based on the principle of measuring the motion parameters of heavenly bodies. These include aviation sextants, astrocompasses, astronomical orientators, etc.

4. *Light engineering means of aircraft navigation,* which are based on the principle of using light energy radiation. These include ground light beams, light and pulse-light equipment for take-off and landing strips as well as aircraft, enclosures for the lighting equipment of the routes and airports (housings for ground installations), various pyrotechnic devices, etc.

At the heart of a safe and accurate flight according to a set route, in the vicinity of the airport, or during take-off and landing, lies the principle of the overall usage of all the available technical means of aircraft navigation, both ground facilities and those aboard the aircraft.

CHAPTER ONE /7*

COORDINATE SYSTEMS AND ELEMENTS OF AIRCRAFT NAVIGATION

1. Elements of Aircraft Movement in Space

The fundamental problem of aircraft navigation in all stages of flight is maintaining a given trajectory of aircraft movement in altitude, direction and time by means of a complex utilization of navigational means and methods. A successful solution to these problems depends on constant and accurate information concerning the position of the craft relative to a given flight trajectory, the nature of the aircraft movement, and the actions of the crew.

As a result of the curvature of the Earth's surface, any given flight trajectory of an aircraft is curvilinear. However, by taking into account the large radius of curvature of the Earth's surface, a small area can always be delineated on it whose surface can be assumed to be plane (Fig. 1.1).

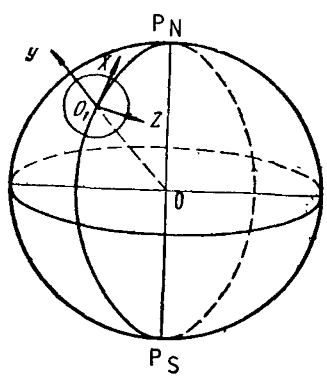

Fig. 1.1. Rectangular Coordinate System on the Earth's Surface.

Let us erect a perpendicular O_1Y from the center of the small area which we have chosen and continue it until it intersects the center of the Earth. Obviously, this will be a perpendicular line, which we can call the *vertical of the locus*.

In the plane of the small area which we have chosen, let us draw a straight line through the point O_1 and take it as the X axis; then let us draw another straight line through the point O_1 in the plane of the area, perpendicular to the first, and call it the Z axis.

Thus, at point O_1 on the Earth's surface, we will obtain a rectangular system of space coordinates X, Y, Z.

* Numbers in the margin indicate pagination in the foreign text.

The travel of an aircraft over the Earth's surface will involve both a shift in the point O_1 (origin of the coordinates) and the rotation of the axes of the coordinate system around the center of the Earth (point O). /8

However, the system of coordinates which we have obtained can be used for determining the directions of the aircraft axes and the component flight speed vectors. Since the origin of this system is being continuously shifted, let us designate it as a *gliding rectangular system of coordinates.*

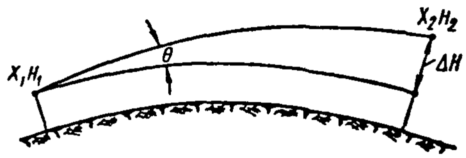

Fig. 1.2. Dip Angle of the Trajectory of Altitude Gain.

In this coordinate system, the following elements can be distinguished:

(a) Position of the longitudinal axis of the aircraft in the horizontal plane (*aircraft course*),

(b) Position of the longitudinal axis of the aircraft in the vertical plane (*angle of pitch of the aircraft*),

(c) Position of the lateral axis of the aircraft in the vertical plane (*lateral banking*),

(d) Distance along the vertical from the Earth's surface (the area which we have chosen) to the aircraft (*flight altitude*),

(e) Vertical speed (*altitude gain and loss*),

(f) Component flight speed along the X and Z axes or the vector of groundspeed and its direction (*groundspeed and flight angle*),

(g) *Angular velocity* of aircraft roll,

(h) Component wind speed along the X and Z axes of the system, or the wind vector and its direction (*wind speed and direction*).

Usually the position of the craft on the Earth's surface is treated in surface-coordinate systems, the most widely used of which are the geographic system and the reference system whose major axis coincides with a given flight trajectory on the Earth's surface.

The position of the aircraft in surface-coordinate systems is assumed to be the position of the origin of the gliding system. To analyze the elements of aircraft navigation, let us combine the X axis of the gliding-coordinate system with a given flight trajectory of the aircraft.

In order to keep the aircraft in the rectilinear horizontal segment of this trajectory, the crew must maintain a flight condition in which the aircraft will not be shifted along the vertical (altitude gain and loss), there will be no lateral deviation (to the right or left), i.e., the vertical velocity V_y and the lateral component of the velocity V_z, will be equal to zero, and the longitudinal flight velocity V_x (along the X axis) will be as given.

If the flight trajectory is inclined (segments of altitude gain and loss), the crew must hold this trajectory by maintaining the vertical and longitudinal flight velocities (V_y and V_x), i.e., maintain a given dip angle of the trajectory Θ (Fig. 1.2). /9

Obviously, at a constant dip angle of the flight trajectory, the latter will have a curvature in the vertical plane just as in horizontal flight. Therefore, if we neglect the curvature of the horizontal flight trajectory, we may assume

$$\mathrm{tg}\,\theta = \frac{H_2 - H_1}{X_2 - X_1}, \qquad (1.1)$$

where Θ is the dip angle of the flight trajectory; X_1, X_2 are the coordinates of the initial and final points of the sloping segment of the trajectory; H_1, H_2 represent a given altitude at the initial and final points.

When the aircraft travels from the initial point X_1 to the moving point X, the flight altitude is changed by the value

$$\Delta H = (X - X_1)\,\mathrm{tg}\,\theta, \qquad (1.2)$$

and the value of the moving flight altitude is

$$H = H_1 + \Delta H = H_1 + (X - X_1)\,\mathrm{tg}\,\theta \qquad (1.3)$$

or if we take Formula (1.1) into account,

$$H = H_1 + (X - X_1)\frac{H_2 - H_1}{X_2 - X_1}. \qquad (1.4)$$

Since the altitude during a sloping trajectory is a variable value, a given flight trajectory is maintained at a constant value of the vertical velocity

$$V_y = V_x\,\mathrm{tg}\,\theta \text{ or } V_y = V_x\frac{H_2 - H_1}{X_2 - X_1}. \qquad (1.5)$$

Checking of the position of the aircraft at given values of the varying flight altitude is carried out only at specific points on the sloping trajectory.

Translator's note: tg = tan.

2. Concepts of Stable and Unstable Flight Conditions

A navigational flight condition is determined by the motion parameters of an aircraft along a trajectory or by navigational elements of flight: course, speed, and altitude.

The motion parameters of an aircraft are usually measured relative to airspace. However, considering that the airspace also shifts, they are selected in such a way as to ensure retaining the given flight trajectory relative to the Earth's surface.

Based on the nature of the trajectory and the conditions of aircraft navigation, four main flight conditions are distinguished: /10
horizontal rectilinear flight, altitude gain, altitude loss, and roll.

Horizontal rectilinear flight is characterized by two constant parameters: height and flight direction.

Altitude gain and loss conditions each have two constant parameters: flight direction, and vertical velocity or dip angle of the trajectory.

The condition of roll is always combined with one of the first three flight conditions, so that the flight direction becomes variable and can be replaced by a parameter which characterizes the curvature of the roll trajectory through the radius of roll or the angular velocity.

A flight condition is stable if its parameters acquire constant values, and *unstable* if its parameters are variable.

Flight practice shows that flight conditions, strictly speaking, are never fixed for any prolonged time, since there are always factors changing the aircraft's motion parameters.

The main sign of a stable flight condition is the equality to zero of the first derivative of the given parameter with time $\left(\frac{dV}{dt}\right)$ or of the second derivative path with time $\frac{d^2S}{dt^2}$.

For example, for the velocity parameter V = const, if

$$\frac{dV}{dt}=0 \text{ or } \frac{d^2S}{dt^2}=0.$$

Analogously, for the flight direction parameter (ψ) and the altitude parameter (H):

$$\psi=\text{const, if } \frac{d\psi}{dt}=0, \quad H=\text{const, if } \frac{dH}{dt}=0.$$

If forces arise during flight which change the aircraft's motion parameters, the extreme values of the motion parameters (i.e., the points of the maxima and minima on the curve which characterizes the change of the given parameter with time) indicate equilibrium of these forces.

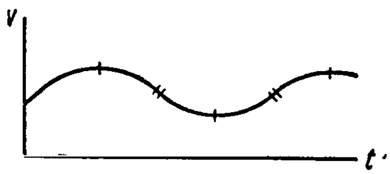

Fig. 1.3. Graph of the Changes of a Navigational Parameter and Points with a Stable Flight Condition.

A stable flight condition based on a given parameter exists only at the extreme points, since the first derivative parameters based on time at these points are equal to zero while the disturbing forces are absent.

The disturbing forces acquire a maximum value at points of inflection, i.e., when the second derivative parameters based on time are equal to zero (Fig. 1.3). On a curve constructed for the velocity parameter, the points of a stable condition are designated by one line, while points of maximum disturbing forces are designated by two lines.

From aerodynamics, we know that in horizontal flight at a velocity significantly less than the speed of sound, the drag of an aircraft in a counterflow is /11

$$Q_x = c_x S \frac{\rho V^2}{2},$$

where c_x is the coefficient of drag of the aircraft, S is the cross-sectional area of the midship section, and ρ is the air density at flight altitude.

It is obvious that the airspeed will be stable if the thrust of the engines (P) is equal to the drag of the aircraft $P = Q_x$.

With a disturbance of this equilibrium, there arises a disturbing force which changes the flight velocity. For example, with an increase in the thrust of the engines the disturbing force will be equal to:

$$\Delta P = P' - c_x S \frac{\rho V^2}{2},$$

which causes an initial acceleration of the aircraft

$$\frac{dV}{dt} = \frac{\Delta P}{m},$$

where m is the mass of the aircraft in kg.

Later, with an increase in velocity, the drag of the aircraft will also increase. The value of this drag will approach the value of the thrust of the engines, i.e., the velocity very slowly approaches a stable value logarithmically.

Changes in airspeed which are analogous in nature arise during changes in the velocity of the headwind or the incident airflow at flight altitude. For example, with an increase in the velocity of the incident airflow, the airspeed diminishes. This provides a surplus of engine thrust. Subsequently, an increase in airspeed occurs logarithmically.

If the lateral component of the wind speed changes, a lateral pressure on the surface of the aircraft arises:

$$Q_z = c_z S_z \frac{\rho V_z^2}{2},$$

where c_z is the coefficient of lateral drag of the aircraft; S_z is the cross-sectional area of the aircraft in the XY plane; V_z is the lateral velocity component equal to u_z.

The initial lateral acceleration of the aircraft is:

$$\frac{dV_z}{dt} = \frac{Q_z}{dt}.$$

Subsequently, the lateral velocity of the aircraft will logarithmically approach the lateral component of the wind velocity, i.e., the flight condition will approach a condition which is stable in direction.

Usually, during navigational calculations for each parameter, its mean value for a definite length of time is called a stable flight condition: mean velocity, mean vertical velocity, mean direction, etc.

From the point of view of maintaining flight direction, aircraft roll is an unstable condition. If a given trajectory is curvilinear, the roll condition is also examined as stable or unstable. /12
The entrance or exit of an aircraft from roll, as well as roll with variable banking, can serve as examples of unstable roll conditions.

The rolling of an aircraft is considered to be coordinated if the longitudinal axis of the aircraft constantly coincides with the tangent to the trajectory of its movement, i.e., external or internal aircraft glide is absent. This is achieved by tilting the rudder of the aircraft for banking in a roll.

During banking of an aircraft, its lift (Y) is directed not along the vertical plane but along the axis of the aircraft, which is deflected from it (Fig. 1.4).

Rolling of an aircraft without descent or with stable vertical velocity is possible only when the vertical component of the lift (Y_1) is equal to the weight of the aircraft G.

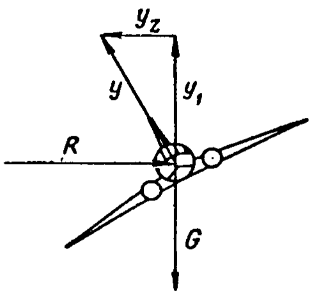

Fig. 1.4. Resolution of Forces During Rolling of an Aircraft.

In this case, the horizontal (centripetal) component of the lift is:

$$Y_z = G \operatorname{tg} \beta.$$

where β is the banking angle of the aircraft.

Since we are examining a coordinate roll (without gliding of the aircraft), the centrifugal force in the roll

$$F_c = \frac{mV^2}{R}$$

will be equal to the centripetal force, i.e.,

$$\frac{mV^2}{R} = G \operatorname{tg} \beta,$$

where m is the mass of the aircraft; and R is the radius of the coordinated roll.

Transforming this equation, taking into account that $m = \frac{G}{g}$, we will obtain formulas for determining both the radius and path of the aircraft with coordinated roll:

$$R = \frac{V^2}{g \operatorname{tg} \beta}; \quad S = 2\pi R. \qquad (1.6)$$

Formulas (1.6) relate the radius of stable coordinated roll of the aircraft with the airspeed and also with banking in rolling, and they are used in calculations of the radius and path of the aircraft along a curvilinear flight trajectory.

3. Form and Dimensions of the Earth /13

In the practice of aircraft navigation, it is necessary first of all to deal with distances and directions on the Earth's surface which are the result of the mutual distribution of objects through which the flight path passes.

The Earth's surface, its relief and mutual distribution of objects can be most accurately expressed on a model of the Earth (a globe). However, a globe with a representation of the Earth's surface that satisfies the demands of aircraft navigation would be so large that its use in flight would be impossible. Therefore, different means of representing the surface of the Earth, which is curved in all directions, on a plane (sheets of paper) are used.

The Earth has a complex form called a *geoid* (without considering the local relief, if we imagine that its entire surface is covered with water at sea level). The surface of a geoid at any point is perpendicular to the direction of the action of gravity. A description of a geoid by mathematical expressions is very complex, and if we consider the folds in the relief of the Earth's surface, then it is practically impossible to express its form mathematically. Therefore, in calculations the form of the Earth is taken as *an ellipsoid of revolution*, the form closest to a geoid.

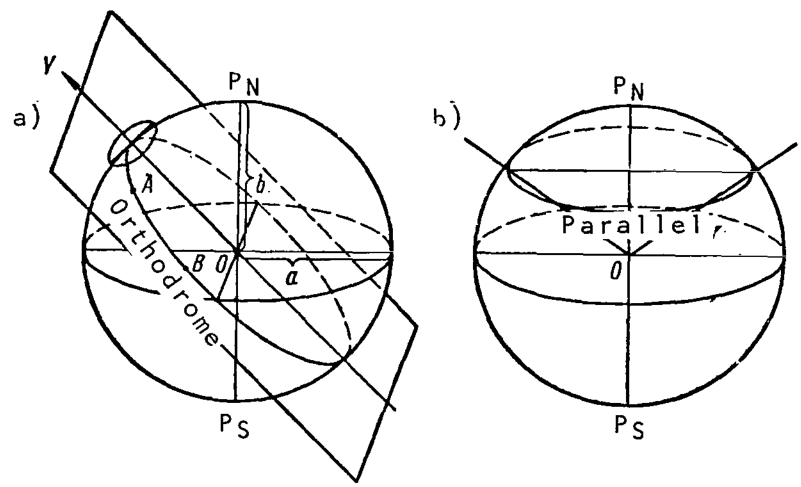

Fig. 1.5. Great and Small Circles on the Earth's Surface. a) Semi-axis of the Earth and Great Circle; b) Small Circle.

According to measurements made by Soviet scientists under the supervision of F. N. Krasovskiy, the major semiaxis of this ellipsoid (a), which coincides with the radius of the equator, is equal to 6,378,245 km. The minor semiaxis of the ellipsoid (b), which coincides with the axis of the Earth's rotation, is equal to 6,356,863 km (Fig. 1.5, a). /14

The flattening of the Earth at the poles is

$$c = \frac{a-b}{a} = \frac{1}{298.3}.$$

These dimensions show that the Earth's ellipsoid of revolution is practically close to a sphere; to simplify the solution of the majority of problems in aircraft navigation, it is taken as a true sphere, equivalent in volume to the Earth's ellipsoid. The radius of such a sphere is equal to 6371 km.

The maximum distortion of distances caused by the replacement of the Earth's ellipsoid by a sphere does not exceed 0.5%, and the distortion of directions is not more than 12 minutes of angle.

In geodesy and cartography, the plotting of maps, as well as in other branches of science where more accurate calculations of distances and directions are necessary, the Earth's surface is taken as an *ellipsoid of revolution.*

4. Elements Which Connect the Earth's Surface with Three-Dimensional Space

Taking the Earth as a true sphere, we will locate a perpendicular (a resting pendulum) at any point above the Earth's surface. Then, disregarding the possible insignificant deviations caused by the varying relief, the irregularity of distribution of the densest masses in the Earth's crust, and the tangential accelerations connected with the Earth's rotation, it is possible to consider that the line of the perpendicular runs in the direction of the center of the Earth.

The perpendicular line (see Fig. 1.5, a) joining the center of the Earth with the point of the observer's position, and continued in the direction of the celestial sphere (Y), is called the *geocentric vertical of the locus.*

The plane on the Earth's surface, tangent to the sphere at the point of the observer and perpendicular to the true vertical of the locus, is called the *plane of the true horizon.*

The direction and velocity of aircraft movement at every point on the Earth's surface are examined in the plane of the true horizon, while the altitude change is examined in the direction of the true vertical.

If we cut the plane of this true horizon in any direction by another plane along the true vertical (through the center of the Earth), the line formed by the intersection of this plane with the Earth's surface forms a closed *great circle,* the mean radius of /15
which will be equal to the radius of the Earth.

The shortest distance between two points *AB* on the Earth's surface or part of the arc of a great circle is called the *orthodrome* (see Fig. 1.5, a).

The mean radius of a great circle is assumed to be equal to 6371 km. The length of the circumference of such a radius is equal to 40,000 km. One degree of arc of a great circle is equal to 111.1 km, while one minute of arc is equal to 1,852 km. The length of a segment of the arc of a great circle at one minute of angle is called a *nautical mile.*

With an intersection of the Earth's sphere by a plane which does not pass through the center of the Earth, the line of intersection of this plane with the Earth's surface forms a closed *small circle*, the radius of which will always be less than the mean radius of the Earth. The small circles parallel to the plane of the equator are called *parallels* (see Fig. 1.5, b).

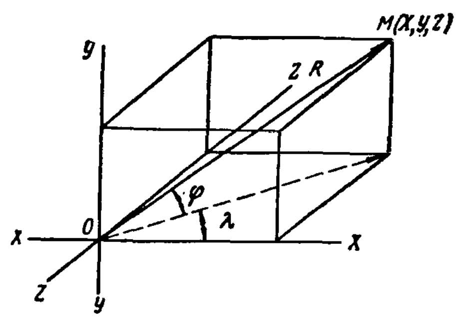

Fig. 1.6. Relationship Between a Spherical System of Coordinates and a Rectangular System.

For the purposes of aircraft navigation, a coordinate system which unequivocally determines the position of an aircraft and objects on the Earth's surface is necessary. Obviously, a spherical coordinate system will be the most convenient (Fig. 1.6).

A spherical coordinate system is distinguished from a rectangular system (Cartesian) by the fact that instead of determining three distances to a point in the directions of the X, Y, and Z axes, we determine the length of the radius-vector R from the center of the coordinate system to a point, and two angles: angle λ between the XY plane and the projection of the radius-vector (R) to the plane XZ, and angle ϕ between the XZ plane and the direction of the radius-vector (R).

There is an obvious relation between spherical and rectangular coordinate systems:

$$\left.\begin{aligned} X &= R\cos\varphi\cos\lambda; \\ Y &= R\sin\varphi; \\ Z &= R\cos\varphi\sin\lambda. \end{aligned}\right\} \qquad (1.7)$$

With a constant length of the radius-vector R, if angles λ and ϕ assume all possible values, the geometric location of the points of the end of the vector radius will be a sphere.

To determine coordinates on the Earth's surface, there is no need to indicate the radius of the Earth (R) each time. This coordinate is considered, once and for all, constant. /16

Thus, the spherical coordinate system is transformed into a two-dimensional surface system which is called a geographic system of coordinates.

The plane of the equator and the plane of the prime (Greenwich) meridian are taken as the initial reference planes in a geographic coordinate system. The point coordinates on the Earth's surface bear the name "longitude of the locus" and "latitude of the locus" (Fig. 1.7).

The dihedral angle between the plane of the prime meridian and the plane of the meridian of a given point is called the *longitude of the point* (λ). Determination of the longitude can be given in arc values: the length of the arc of the equator (or the parallel), expressed in degrees, between the prime meridian and the meridian of a given point is called the longitude of the point.

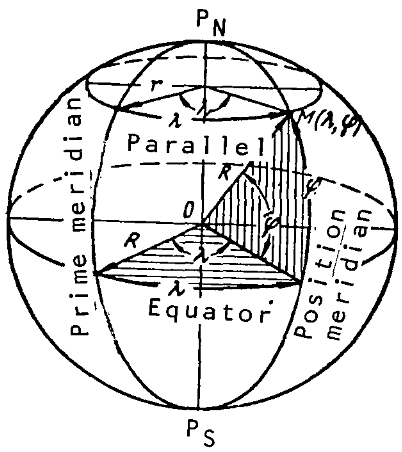

Fig. 1.7. Spherical Coordinate System on the Earth's Surface.

Reading of the longitude is carried out from 0 to 180° east of the rpime meridian (*east longitude*) and from 0 to 180° west of the rpime meridian (*west longitude*). In navigational calculations, east longitude is taken as positive and is designated by a plus sign, while west longitude is negative and is designated by a minus sign. However, in carrying out navigational calculations, it is more convenient to carry out a reading of longitude in the easterly direction from zero to 360°.

The angle between the plane of the equator and the true vertical of a given point (or the length of the meridian arc, expressed in degrees, from the plane of the equator to the parallel of a given point) is called the *latitude of the point* (ϕ). Since a set of true verticals at a constant latitude forms a cone with the vertex in the center of the Earth and an angle at the vertex equal to 90°-ϕ, then in contrast to the dihedral angle between the planes of the meridians, we shall call a similar angle in other spherical systems, the *conic angle*.

Reading of the latitude is carried out from the plane of the equator to the north and south from 0 to 90° (*north and south latitude*). In navigational calculations, north latitude is considered positive and south, negative.

A geographic coordinate system is a surface curvilinear system, i.e., the meridians of the coordinate grid on the Earth are not parallel. However, if we examine the meridians and parallels on any unit area of the Earth's surface, they turn out to be orthogonal (perpendicular in one plane). Two special points on the Earth's surface (the geographic poles) are an exception. /17

A geographic coordinate system is used not only to determine the location of a point (object) on the Earth, but to determine direction from one point to another.

The angle included between the northern direction of the meridian which passes through a given point and the orthodrome direction to a point setting a course is called the *bearing* or *azimuth*. Reading of the angles of bearing or azimuth is done clockwise from 0 to 360°.

Since the meridians on the Earth's surface are generally not parallel, the value of the azimuth changes with a change in the moving longitude along the line which joins the two points; the greater the latitude, the more it changes. Therefore, for the orthodrome direction together with an indication of the azimuth, it is necessary to mention from which meridian thia direction is measured.

The change in azimuth with a change in the moving longitude does not make it possible to use magnetic compasses for moving along the orthodrome without introducing corresponding corrections, especially when the two points are far apart.

If the magnetic declination does not change, following a constant magnetic course will cause the meridians to intersect at identical angles. The line which intersects the meridians at a constant angle is called the *loxodrome*.

In order to proceed to a more detailed examination of the elements of aircraft navigation and their measurement, it is necessary to become acquainted with the making of maps, their scales, and some features of cartographic projections.

5. Charts, Maps, and Cartographic Projections

The representation of a small part of the Earth's surface on a plane is called a *chart*. Distortion as a result of the curvature of the Earth's surface is practically absent on a chart.

The conventional representation of the Earth's surface in a plane is called a *map*.

A *map* is a continuous representation of the surface of the Earth or a part of it without discontinuities and folds, made with a variable scale according to a definite rule. The sphericity of the Earth's surface does not allow it to be represented with complete accuracy on a plane surface. Therefore, there are many ways /18 of projecting the Earth's surface onto a plane which make it possible to represent most accurately on the map only those parameters (elements) which are most necessary under the given conditions of application.

Methods or laws of representing the Earth's surface on a plane are called cartographic projections.

A *common geometrical projection* is the point of intersection of the line of sight (which passes through the eye of the observer and the projected point) with the plane onto which the given point is projected. It is a special case of cartographic projection.

A cartographic projection is set analytically as a function of geographical coordinates on the Earth (sphere) between the coordinates of a point on a plane.

If we call one of the main directions on a map the *X* axis and the perpendicular to it the *Z* axis, then

$$X = F_1(\varphi; \lambda) \text{ and } Z = F_2(\varphi; \lambda);$$

or

$$\rho = F_3(\varphi; \lambda) \text{ and } \delta = F_4(\varphi; \lambda),$$

where ρ and δ are the main directions on maps of conic and azimuthal projections, and ϕ and λ are the geographical coordinates of a point on the Earth (sphere).

The properties of the projections will depend on the properties of these functions (F_1, F_2, F_3, and F_4), which must be continuous and well-defined, since the map is made without discontinuities so that a single point on the map corresponds to every point in the location.

Map Scales

The map-making process is divided into two stages.

a) The Earth is decreased to the definite dimensions of a globe.

b) The globe is unrolled to form a plane.

The extent of the overall decrease in the Earth's dimensions to the fixed dimensions of a globe is called a *principal scale*.

A principal scale is always indicated on the edge of a map and makes it possible to judge the decrease of the length of a segment in transferring it from the Earth's surface to the globe.

A principal scale is numerically equal to the ratio of the distance on the globe to the actual distance at a location:

$$M = \frac{\Delta S_g}{\Delta S_{e.s.}}$$

where *M* is the principal scale, ΔS_g is a segment on the globe, and $\Delta S_{e.s.}$ is a segment on the Earth's surface which corresponds to the segment on the globe.

On maps, the principal scale is usually shown as a fraction (numerical scale) and by means of a special scale (linear scale).

The *numerical scale* is a fraction, the numerator of which is one, while the denominator shows how many such units of measurement fit into the location.

For example, 1:1,000,000 means that if we take 1 cm on a map, then 1,000,000 cm at a location (i.e., 10 km) will correspond to it. /19

A *linear scale* is a scale on a map in which a definite number of kilometers at a location correspond to special segments of the scale.

However, a principal scale (numerical and linear) is insufficient for accurately measuring distances on the entire field of a map. It is necessary to know the laws of distortion of distances and directions. The laws of change in the principal scale along the map field are determined by a special scale.

A *special scale* is the ratio of an infinitely small segment in a given place on the map in a given direction, to an analogous segment in a location (globe). At each point on the map, the special scale is different. It is either somewhat larger or somewhat smaller than the principal scale.

Distortions of Cartographic Projections

Ellipse of Distortions

Let us draw on a sphere (globe), an infinitely small circle with radius r; let us also designate a rectangular coordinate system on the sphere by x and z (Fig. 1.8, a). Then

$$r^2 = x^2 + z^2. \quad (1.8)$$

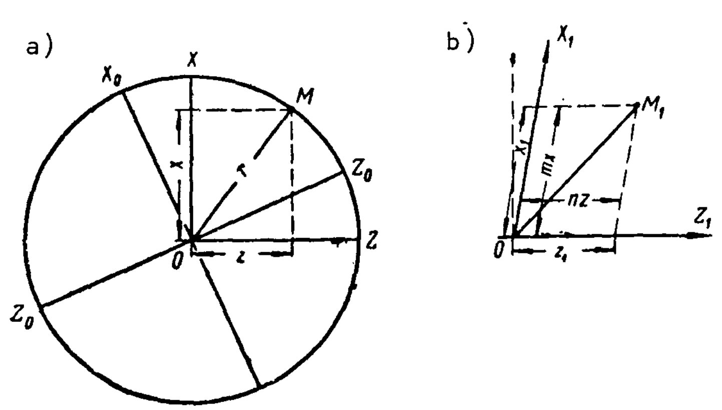

Fig. 1.8. Distortion of Scales on a Plane: (a) Scale on a Globe; (b) Scale on a Plane.

In the transfer of the coordinate system from the sphere (globe) to the plane, the direction of the coordinate axes is distorted (Fig. 1.8, b).

Having designated the special scales on a plane (map) by m in the direction X and n in the direction z, we obtain:

$$x_1 = mx;$$
$$z_1 = nz$$

or

$$x = \frac{x_1}{m} \text{ while } z = \frac{z_1}{n}.$$ /20

Substituting the latter in (1.8),

$$r^2 = \left(\frac{x_1}{m}\right)^2 + \left(\frac{z_1}{n}\right)^2,$$

and then dividing both sides of the equation by r^2, we obtain

$$\left(\frac{x_1}{mr}\right)^2 + \left(\frac{z_1}{nr}\right) = 1 \quad (1.9)$$

From mathematics, it is known that this is the formula of an ellipse with conjugate diameters; therefore:

a) Any infinitely small circle on the surface of the Earth's sphere in any projection is represented by an infinitely small ellipse.

b) On the surface of the Earth's sphere (globe), it is possible to choose two mutually perpendicular directions which will be transferred to a map without any distortions.

These directions are called principal directions.

Knowing the special scales (m and n) in the principal directions, it is always possible to construct an ellipse of distortions which will make it possible to judge the nature of the distortions of the projection as a whole. In the majority of projections, the directions along the meridians and parallels are taken as the principal directions.

Distortion of Lengths

If an infinitely small circle on the Earth is represented by an ellipse (Fig. 1.9, b) with its transfer to a plane, the distortion of the special scale in any direction (ΔS_α) can be expressed as follows: /21

$$\Delta S_\alpha = \frac{O_1M_1}{OM} = \frac{\sqrt{(mx)^2 + (nz)^2}}{r}, \quad (1.10)$$

but from the circle in Figure 1.9, a:

$$x = \sin \alpha r, \text{ while } z = \cos \alpha r,$$

then

$$\Delta S_\alpha = \sqrt{m^2 \sin^2 \alpha + n^2 \cos^2 \alpha}, \tag{1.11}$$

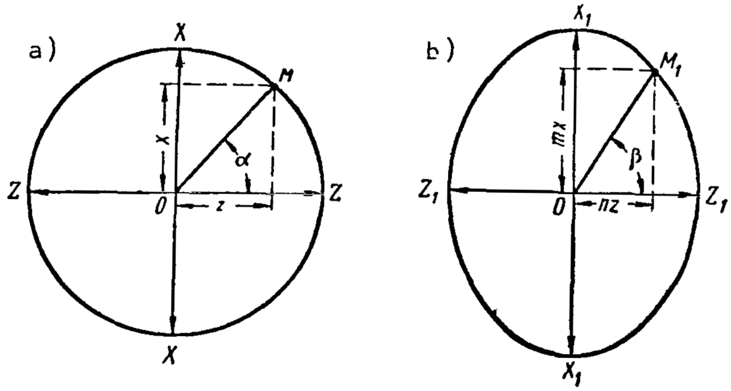

Fig. 1.9. Distortion in a Plane: (a) Length on a Globe; (b) Length on a Plane.

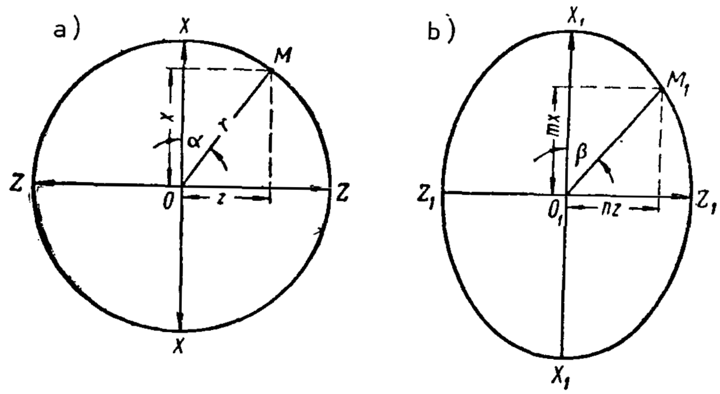

Fig. 1.10. Distortion of Directions on a Map. (a) Direction on a Globe; (b) Direction on a Map

i.e., knowing the special scales for the principal directions, we can always judge the value of the distortion of the special scale in any direction (and therefore, the distortion of the length of the segment as a whole).

Distortion of Directions

Let us take the radius $r = 1$ (Fig. 1.10) of an infinitely small circle on the Earth; then

$$\operatorname{tg} \alpha = \frac{z}{x}, \text{ while } \operatorname{tg} \beta = \frac{nz}{mx}. \tag{1.12}$$

Dividing Equations (1.12) into one another, we obtain:

$$\operatorname{tg}\beta = \frac{n}{m}\operatorname{tg}\alpha. \qquad (1.13)$$

Obviously, knowing the special scales for the principal directions, it is always possible to find an angle β on a map for an angle α in a location, and vice versa.

Distortion of Areas /22

The distortion of areas ΔP can be determined by a comparison or division of the area of the ellipse (S_{el}) by the area of a circle (S_{ci}); see Figure 1.11:

$$\Delta P = \frac{S_{el}}{S_{cl}} = \frac{\pi ab}{\pi r^2} = \frac{ab}{r^2}, \qquad (1.14)$$

but if we take the radius of the circle on the Earth as equal to 1, then

$$\Delta P = ab$$

or, if we express a and b by special scales for the principal directions, we obtain:

$$\Delta P = mn, \qquad (1.15)$$

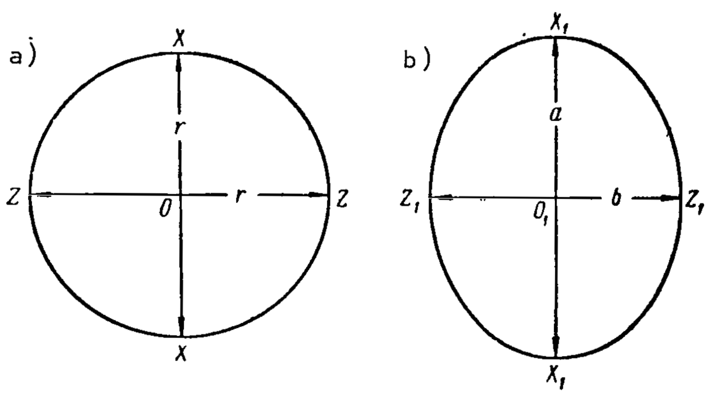

Fig. 1.11. Distortion of Areas on a Map. (a) Area on a Globe; (b) Area on a Map.

The distortion of areas is equal to the product of the special scales for the principal directions.

Hence, we see that if we know the special scales for the principal directions, we can give the complete characteristics of any map projection.

Classification of Cartographic Projections

There are many cartographic projections. They can be divided according to two basic characteristics:

(a) according to the nature of the distortions, and
(b) according to the means of construction or the appearance of the normal grid.

By *normal grid* we mean the coordinate system on a globe which is most simply represented on a map. Obviously, this is a system of meridians and parallels.

Division of Projections by the Nature of the Distortions /23

The choice of cartographic projections depends on the problems for whose solution they are intended. According to the nature of the elements which have the least distortion on a map, cartographic projections are divided into the following groups:

1. Isogonal or conformal projections

These projections must satisfy the requirement of equality of angles and similarity of figures (conformability) within the limits of unit areas of the Earth's surface, i.e., so that in projecting a surface of a globe onto a plane (map), the angles and similar figures do not change.

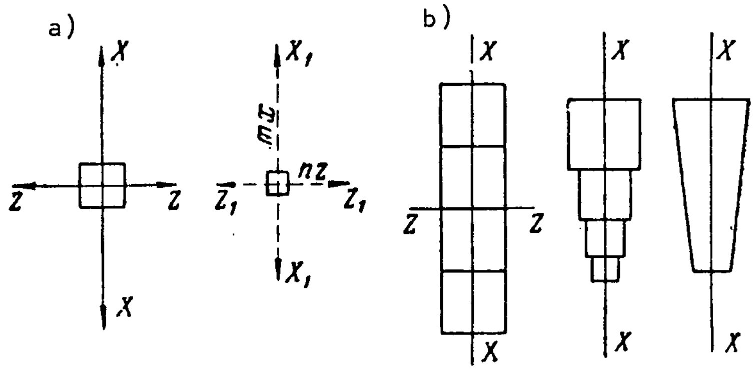

Fig. 1.12. Conformability of Figures on Maps. (a) Preserving the Conformability of a Unit Area; (b) Destroying the Conformability of a Long Strip.

According to the stipulation, the angle on a map must be equal to the angle at the location: $\angle\beta = \angle\alpha$, but from (1.13) it is obvious that in this case $m = n$.

Therefore, the equation of special scales for principal directions is a condition for isogonality.

On large parts of the surface, within the limits of which it is impossible to disregard the change in scale, the conformability (and therefore the isogonality) are not preserved. Figure 1.12 gives an example of preserving the conformability of a unit area and destroying the conformability of a long strip.

The unit area (Fig. 1.12, a) is transferred to the map on a definite scale without distortions. The long strip (Fig. 1.12, b) can be divided into a number of unit areas, each of which will be transferred to the map on a somewhat changed scale. Since the scales mx and nz are increased proportionally in the direction of the strip, each of the small areas is represented on the map with the conformability being preserved, only on a different scale. By equating the lateral limits of the small areas, we do not obtain a conformal /24
figure, i.e., the similarity of small figures in isogonal projections is preserved, while the similarity of large figures (large lakes, seas, etc.) is destroyed.

2. Equally spaced or equidistant projections

The equivalence to unity of the special scales for a principal direction (m = 1 or n = 1) is a necessary condition of this group of projections.

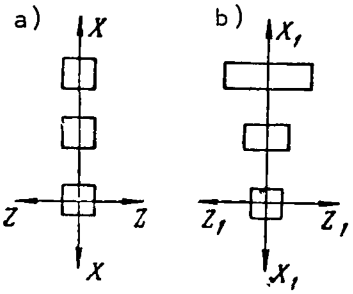

Fig. 1.13. Distortion of Conformability in Equally Spaced Projections: (a) Appearance of a Figure in a Location; (b) Appearance of the Figure on a Map.

This means that the map scale will be preserved in one of the principal directions. Therefore, when using such a map we can measure the distance in one of the directions by means of a scale. The nature of the distortion of conformability in these projections is shown in Fig. 1.13. Here m = const, while n is a function of Z.

3. Equally large or equivalent projections

This group of projections must satisfy the condition of equivalence of areas, i.e., the product of the special scales for the principal directions must equal unity (mn = 1); therefore, the relation between the special scales for the principal directions will be inversely proportional:

$$m=\frac{1}{n};\quad n=\frac{1}{m}.$$

These projections do not have an equivalence of angles and a similarity of figures.

4. Arbitrary projections

Projections of this group do not satisfy any of the conditions mentioned above. However, they are also used when comparatively small portions of the Earth's surface are projected onto a plane where the distortions of the angles and the scales for the principal directions and along the entire map field are insignificant and the similarity of figures and areas which satisfy the needs of their practical application is preserved. This group of projections includes a basic flight map on a scale of 1:1,000,000, which is constructed according to a special law and which has been accepted by international agreement.

For the purposes of aircraft navigation, the most necessary conditions are (obviously) isogonality and equal scale of the maps. Equally large and equally spaced projections of maps are used in aircraft navigation only as survey maps for special applications. They include maps of hour zones, magnetic declinations, composite /25
diagrams of topographical map sheets, climatological and meteorological maps, etc.

Division of Projections According to the Method of Construction (According to the Appearance of the Normal Grid)

Depending on the method of construction, cartographic projections are divided into several groups, the bases of which are the following:

(a) group of cylindrical projections;
(b) group of conic projections and their variants, polyconic projections;
(c) group of azimuthal projections;
(d) group of special projections.

Each of these projections is divided in turn into the following categories: *normal*, if the Earth's axis concides with the axis of the figure onto which the Earth's surface is projected; *transverse*, if the Earth's axis forms an angle of 90° with the axis of the figure, and *oblique*, if the axis of the Earth does not coincide with the axis of the figure and intersects it at an angle which is not equal to 90°.

Cylindrical Projections

Normal (equivalent) cylindrical projection

All cylindrical projections are formed by means of the imaginary transfer of the Earth's surface (globe) to a tangential or intersecting cylinder, with subsequent unrolling.

In Figure 1.14, a simple normal cylindrical projection is given, i.e., a projection of the Earth on a tangential cylinder, the axis of which coincides with the axis of the Earth (globe), while the height of the cylinder is proportional to the length of the axis.

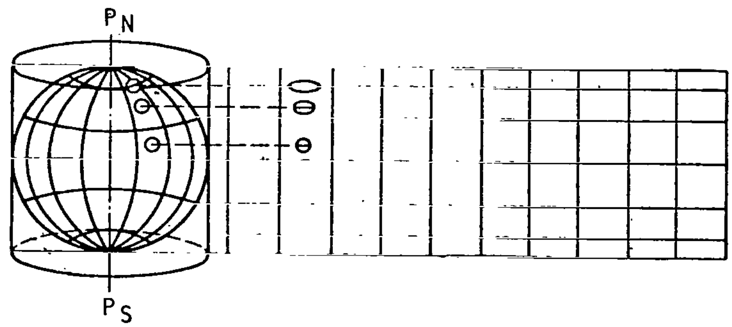

Fig. 1.14. Normal (Equivalent) Cylindrical Projection

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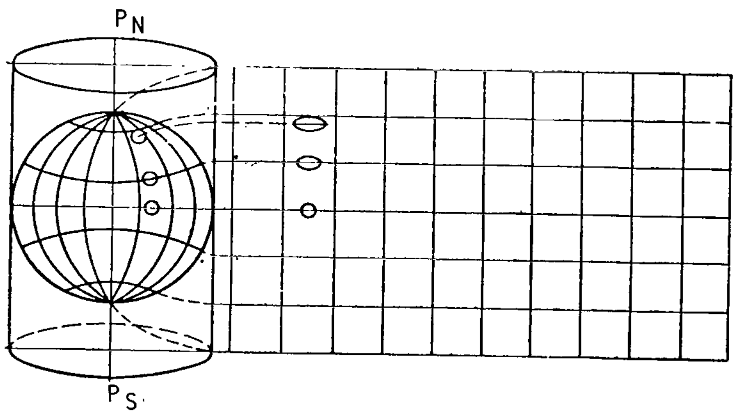

Fig. 1.15. Simple Equally Spaced Cylindrical Projection

In this projection, the meridians are compressed while the parallels are extended to a degree which increases with latitude. The projection includes a category of equally large and equivalent projections, since it satisfies the condition of an equivalence of areas.

Its equation can be written in the following form:

$$X = R\sin\varphi; \quad Z = R\lambda, \tag{1.16}$$

where X represents the coordinates of a point along the meridian; Z represents the coordinates of a point along the equator; and R is the Earth's radius.

Let us determine what the special scales for the directions are equal to in this projection:

$$m = \frac{dS_{\text{map}}}{dS_{\text{globe}}} = \frac{rd\varphi}{Rd\varphi} = \frac{R\cos\varphi d\varphi}{Rd\varphi} = \cos\varphi; \quad (1.17)$$

$$n = \frac{dS_{\text{map}}}{dS_{\text{globe}}} = \frac{Rd\lambda}{rd\lambda} = \frac{Rd\lambda}{R\cos d\lambda} = \frac{1}{\cos\varphi} = \sec\varphi, \quad (1.18)$$

where m is a partial scale along a meridian; n is a partial scale along a parallel; dS_{map} is an increase in distance on the map; dS_{globe} is an increase in distance on the globe.

The product of the special scales is

$$mn = \cos\varphi \frac{1}{\cos\varphi} = 1 \text{ or } m = \frac{1}{n}, \text{ while } n = \frac{1}{m}$$

Therefore, the given projection is equal. Since $m \neq n$; $m \neq 1$ and $n \neq 1$ in the principal directions (meridians and parallels) it is not isogonal and not equally spaced. Only in the equatorial band, in the limits from 0 to ±5° along its latitude, is it practically possible to consider it isogonal and equally spaced. /27

Simple equally spaced cylindrical projection

If we take the height of a cylinder to be proportional not to the length of the Earth's axis, but to the length of a meridian, and instead of simply projecting we unfold the meridians to the cylinder walls, as shown in Fig. 1.15, then a simple, equally spaced cylindrical projection is obtained. It is regarded as normal since the axis of the globe coincides with the axis of the cylinder.

In this projection, the meridians will be transformed to their full size during their transfer from the globe's surface to a map (i.e., $m = 1$), and the equator also will be transformed to full size (at the equator, $n = 1$), while the parallels will be extended just as in a normal (equivalent) projection. The magnitude of the effect increases with latitude.

The coordinate grid of the map of this projection has the appearance of a uniform rectangular ruling. Its equations have the form:

$$X = R\varphi; \quad Z = R\lambda.$$

The special scales are equal to:

along the meridian $$m = \frac{dS_{\text{map}}}{dS_{\text{globe}}} = \frac{Rd\varphi}{Rd\varphi} = 1; \quad (1.19)$$

along the parallel $$n = \frac{1}{\cos\varphi} = \sec\varphi. \quad (1.20)$$

Since $m = 1$, the projection is equally spaced along the meridians and also along the equator. Since $m \neq n$ and $mn \neq 1$, the projection is not isogonal and not equally large, except for the equatorial band in the limits from 0 to ±5° along the latitude, where it is practically possible to consider it isogonal and equally large.

Maps in normal (equivalent) and simple, equally distant cylindrical projections are used in aviation only as references: maps of hour zones, maps of natural light, etc.

Isogonal cylindrical projection

An isogonal cylindrical projection (Mercator projection) is the most valuable of all the cylindrical projections for navigation. It is obtained from a simple, equally spaced cylindrical projection by artificially extending the scale along the latitude (lengthening the meridians), proportional to the change in scale along the longitude. The coordinate grid of the map of this projection is shown in Figure 1.16.

The reason for its use is the fact that the angles measured on the map are equal to the corresponding angles at the location, i.e., $m = n = \sec \phi$. /28

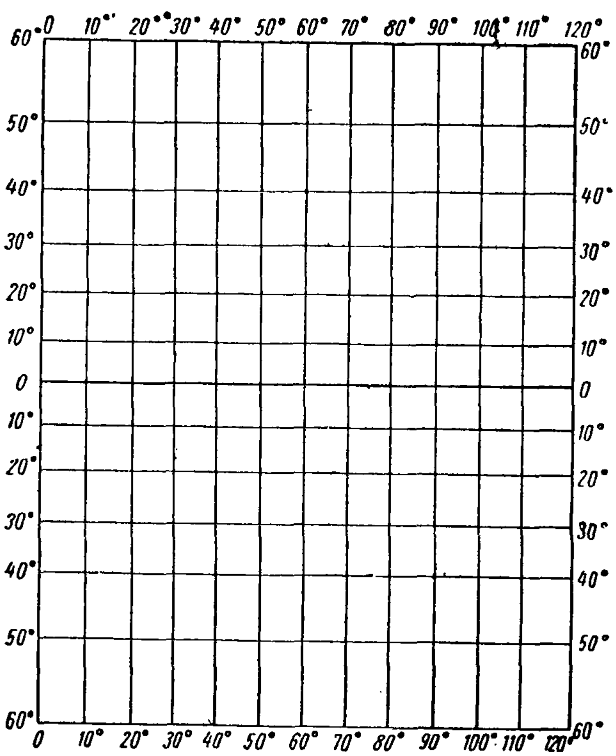

Fig. 1.16. Coordinate Grid of an Isogonal Cylindrical Projection.

Let us write an equation of this map projection along a meridian (X-coordinate) for which we can find m:

$$m = \frac{dS_{\text{map}}}{dS_{\text{globe}}} = \frac{dX}{Rd\varphi},$$

where dS is an increase of distance along the meridian on the map; and $Rd\phi$ is an increase in distance along the meridian at the location.

We must have

$$m = \sec\varphi,$$

We shall then equate the right-hand sides of these equations:

$$\frac{dX}{Rd\varphi} = \sec\varphi, \text{ whence } dX = \frac{Rd\varphi}{\cos\varphi}. \quad (1.21)$$

After integrating (1.21), we will obtain the X-coordinate along the meridian: /29

$$X = R \ln \operatorname{tg}\left(45^\circ + \frac{\varphi}{2}\right), \quad (1.22)$$

while the Z-coordinate along the parallel is determined by the simple equation:

$$Z = R\lambda. \quad (1.22a)$$

Since $m = n$, the projection is isogonal but not equally spaced ($m \neq 1$ and $n \neq 1$) and not equally large ($mn \neq 1$).

The basic advantage of maps in an isogonal cylindrical projection is the simplicity of their use with magnetic compasses for moving from one point on the Earth to another, since the loxodrome in this projection has the appearance of a straight line. Therefore, the isogonal cylindrical projection has been used widely, primarily in marine navigation during the compilation of naval maps.

The change in scale with latitude is a disadvantage of normal cylindrical projections. Here, in normal (equivalent) and simple, equally spaced cylindrical projections, the map scale is not identical in the principal directions (north-south and east-west), so that the distance between two points in directions not parallel to the lines of the grating can be determined only by calculation.

In an isogonal cylindrical projection, the map scale along the latitude is also variable, but at any point on the map it is identical in the principal directions. This makes it possible to measure distances by means of compasses, for which a scale (varying with the latitude) is drawn on the western and eastern edges of the map. Means for measuring distances on maps with such a projection are indicated in manuals for marine navigation.

Isogonal oblique cylindrical projections

The basis for creating maps in an isogonal cylindrical projection is a property of the Mercator projection: its isogonality. Such projections are used in the preparation of special flight maps on scales of 1:1,000,000, 1:2,000,000, and 1:4,000,000 which are used in civil aviation.

The tangential (Fig. 1.17) or intersecting (Fig. 1.18) cylinder is situated at such an angle to the axis of the globe that the tangent of the cylinder's surface to the globe or the intersection runs along the flight path. Usually the strip along the tangent does not extend more than 500-600 km to either side of the route (or the middle line of the route, if it has discontinuities), while on the intersecting cone it does not extend more than 1000-1400 km to either side of the given middle line of the routes.

In practice, such flight maps are isogonal, equally spaced, and equally large; however, since the cylinder is in contact with the globe along the arc of a great circle or cuts the globe comparatively close to the arc of a great circle, the orthodrome on these maps will in practice be represented by a straight line. /30

The distortions of lengths on flight maps of oblique tangential projections do not exceed 0.5%; for intersecting projections they do not exceed 0.8%-1.2%.

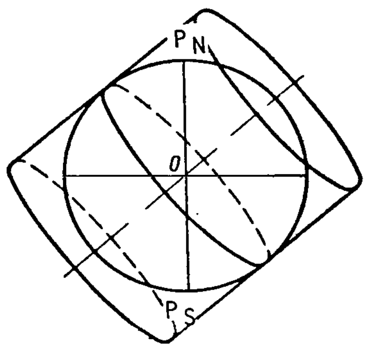

Fig. 1.17. Isogonal Oblique (Tangential) Cylindrical Projection.

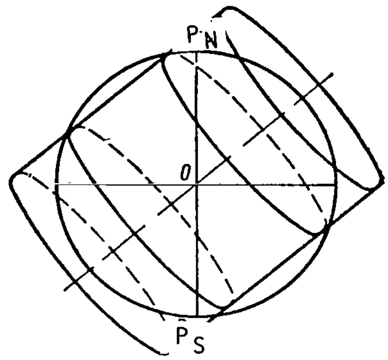

Fig. 1.18. Isogonal Oblique (Intersecting) Cylindrical Projection.

Isogonal transverse and cylindrical Gaussian projection

The axis of the cylinder in Gaussian projections is perpendicular to the axis of rotation of the Earth (globe). The construction of maps with this projection is similar to the construction of maps with oblique cylindrical projections. For example, a flight map on a scale of 1:1,000,000 for Leningrad-Kiev has been compiled on such a projection. However, on the whole, isogonal transverse cylindrical Gaussian projection is used for compiling maps on a large scale, where the special principles of construction are used.

A spheroid (Earth's ellipsoid) is taken as the figure from which the Earth's surface is projected, while the tangential cylinder on which the Earth's surface is projected has an elliptical base according to the form of the Earth's ellipsoid.

The entire Earth's surface is divided by meridians into zones, each of which has a latitude of 6° and is projected onto its own cylinder which is tangential to the Earth's surface along the middle meridian of the given zone.

Thus, in order to project the whole surface of the Earth, it is necessary to turn the elliptical cylinder mentally around the axis of the Earth's ellipsoid through 6° at a time. In Figure 1.19, a, the projection of only one zone for 6° longitude is shown, while in Figure 1.19, b, the unrolling of a semicylinder after its rotation around the Earth's axis in order to project several zones is shown. With such a projection, all maps are constructed on the scales: 1:500,000, 1:200,000, 1:100,000, 1:50,000, and 1:25,000. The latter are essentially charts. /31

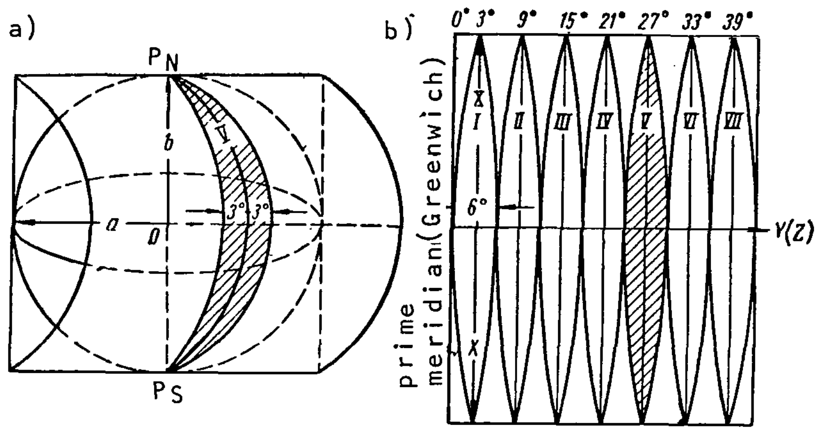

Fig. 1.19. Isogonal Transverse-Cylindrical Gaussian Projection.

Each zone on maps with a scale of 1:200,000 and larger has its own special *X* and *Y*(*Z*) rectangular coordinate system, which is called the Gaussian kilometer system. Meridians and parallels on maps of this projection are curved lines and do not coincide with the Gaussian system. The vertical lines of the rectangular Gaussian system are parallel to the central meridian of the zone and do not coincide with other meridians of the zone.

The angle between the vertical line *X* of the Gaussian system and the line to the object (point) is called the *directional angle*. In order to obtain the true or magnetic direction (angle), the angles of the convergence of the system with the true and magnetic meridians are indicated on the edge of the map. In addition, the vertical section of a map (frame) always runs in the direction of the true meridian.

By means of the Gaussian system and figures in the frames of the maps, it is possible to determine the distance from the equator and from the central meridian of the zone to the object (point). Distortions of lengths on these maps are insignificant and do not exceed 0.14% along the edges of the zone in the latitude which is equal to zero (140 m at 100 km).

Maps on an isogonal transverse-cylindrical Gaussian projection are used both in aviation for a detailed orientation and location

of targets, and in many branches of the national economy for linking projects, equipment, and radio engineering facilities in a location, /32
for determining geodesic reference points, and for accurate geodesic calculations of distances and directions, etc.

Conic Projections

Conic projections are constructed by projecting the surface of the Earth's spheroid (globe) on a tangent or intersecting cone, with its subsequent unrolling to form a plane surface (Fig. 1.20, a).

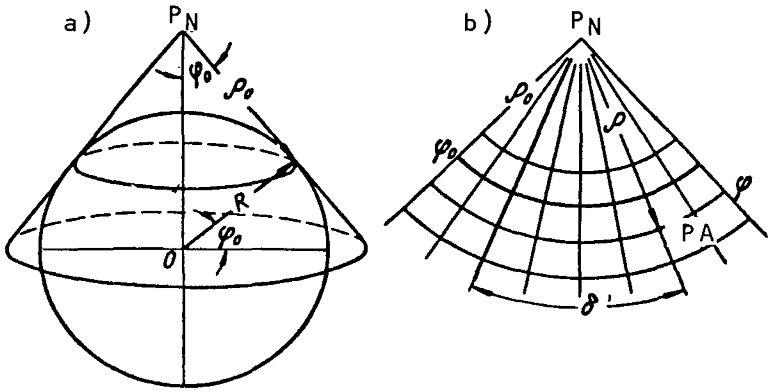

Fig. 1.20. Construction of Conic Projections: (a) Tangent (intersecting) cone; (b) Unrolling of the Cone to Form a Plane.

According to the positions of the axes of the globe and cone, conic projections can be normal, transverse, and oblique. However, in our publications normal projections are generally used when the axis of the cone coincides with the axis of the globe.

In a normal conic projection, meridians are represented by straight lines, while parallels are represented by arcs of concentric circles (Fig. 1.20, b).

From Figure 1.20, a, it is easy to see that the radius of a parallel of tangency (ρ_0) can be expressed by the Earth's radius:

$$\rho_0 = R \operatorname{ctg} \varphi_0,$$

where R is the radius of the Earth (globe) and ϕ_0 is the latitude of the parallel of tangency.

The equation of this projection is written in the following form:

$$\delta = \alpha\lambda; \qquad \rho' = \rho_0 + R(\varphi_0 + \varphi); \qquad (1.23)$$

Translator's note: ctg = cot.

where δ and ρ are the principal directions in the polar coordinate system along the parallel and meridian, respectively, and α is the coefficient for the angle of convergence of the meridians. /33

Simple normal conic projection

A simple normal conic projection is constructed with the consideration that the meridians on the whole map and the parallel of tangency be transferred from a globe without distortions to their natural value (i.e., m = 1), while for the parallels of tangency (ϕ_0) $m = n = 1$.

Such a projection forms the basis of the improved intersecting conic Kavrayskiy projection (Fig. 1.21, a). It is equally spaced, since m = 1, while on the intersecting parallels it is isogonal and equally large (Fig. 1.21, b).

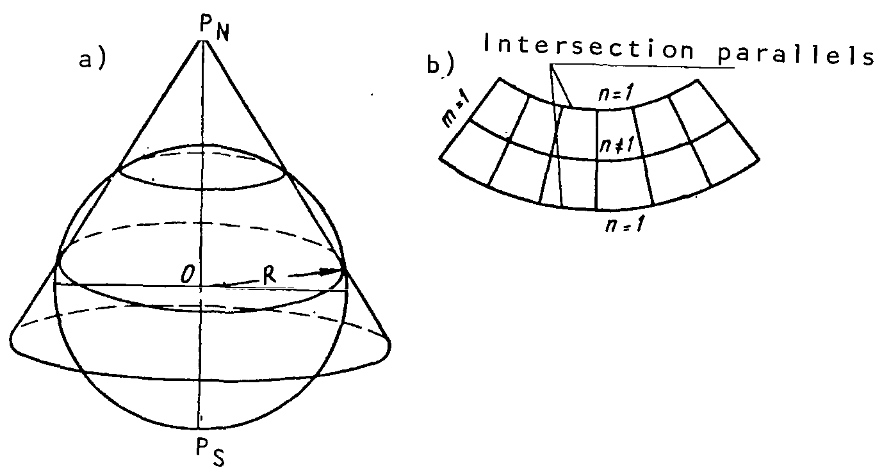

Fig. 1.21. Simple Normal Conic Projection. (a) Intersecting Cone; (b) Unfolding of the Cone to a Plane.

Many aircraft maps with scales of 1:2,500,000, 1:2,000,000, and even 1:1,500,000, which are used in aircraft navigation for general orientation and the approximate determination of the position of an aircraft by means of radio engineering facilities (aircraft radio compasses, ground radiogoniometers, etc.), have been published.

Their positive feature if the insignificant distortion of lengths in the strip ±5° from the intersecting parallels, which does not exceed 0.34% (340 m for 100 km). Their disadvantage is the distortion of directions, which increases with distance from the intersecting parallels.

Isogonal conic projection

By analogy with the construction of an isogonal cylindrical Mercator projection, destroying the equal spacing, a simple normal conic projection is transformed into an isogonal projection by reducing (equating) the scale along the meridians to the scale along the parallels ($m = n$). This is more valuable for use in aviation.

Aircraft maps with a scale of 1:2,000,000 and survey maps on scales of 1:3,000,000, 1:4,000,000, and 1:5,000,000 are published with a normal isogonal conic projection for aviation. /34

Maps with a scale of 1:2,000,000 in this projection, besides having the basic advantage of isogonality, also have distortions of length which are permissible in the practice of aircraft navigation. On an intersecting cone in a strip from 40° to 70° in latitude, the maximum length distortions do not exceed ±1.8 km for 100 km.

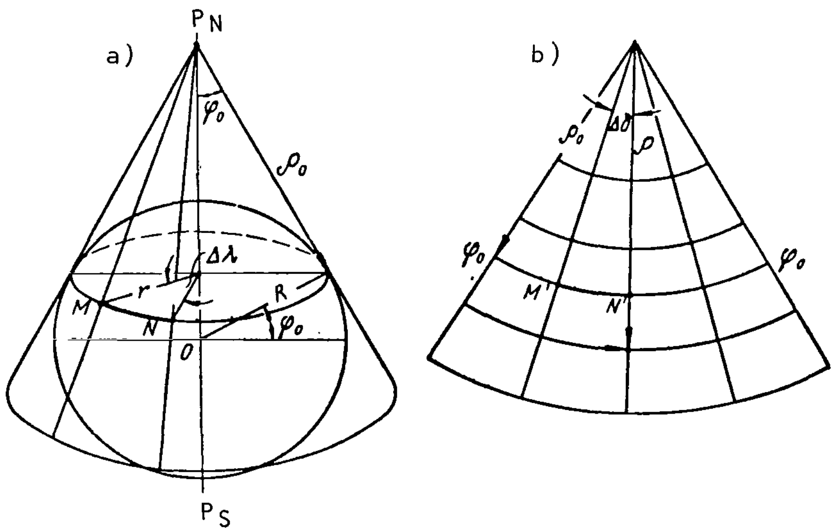

Fig. 1.22. Angle of Convergence of the Meridians of a Tangent Conic Projection: (a) Arc of a Parallel on a Globe; (b) Arc of a Parallel on a Map.

The **orthodrome** on maps of an isogonal conic projection for distances up to 1200 km appears as a practically straight line. This valuable quality is used during flights on civil aviation airlines of average length by using gyroscopic and astronomical compasses for following the orthodrome. At great distances, the orthodrome (as a result of a change in scale) is bent by a bulge tending toward a larger scale.

The **loxodrome** is represented by an arc of a logarithmic spiral.

This creates difficulties in aircraft navigation by means of magnetic compasses. In these instances, for distances up to 500-800 km in directions which intersect the meridians on a map, a straight line is constructed, while measurement of the flight angle is carried out along the central meridian of the route which is maintained in flight by means of a magnetic compass.

It is also possible to construct (continue) the loxodrome along an angle measured in the middle of the straight line joining the control (rotating) landmarks of the route. /35

The disadvantage of all maps with conic projections is the presence of an angle of convergence of the meridians from the parallels of tangency (parallels of intersection) to the pole. It is necessary to consider this angle when determining directions (flight angles) or the location of the aircraft by means of aircraft radio compasses. In addition, depending on the parallels of intersection or tangency, the angle of convergence of the meridians will be different.

Convergence angle of the meridians

The principal scale of conic projections is taken along the meridians and parallels of tangency or intersection (ϕ_0). Therefore, the arc MN is equal to the arc M_1N_1 (Fig. 1.22). It is known that on a globe (spheroid) (Fig. 1.22, a), the arc $MN = r\Delta\lambda$, where r is the radius of the parallel. On a map of a conic projection (Fig. 1.22, b) the arc $M_1N_1 = \rho_0\Delta\delta$; then

$$r\Delta\lambda = \rho_0\Delta\delta. \qquad (1.24)$$

But $r = R \cos \phi_0$ and $\rho_0 = R \cot \phi_0$, and from the equation of a conic projection (1.23), $\Delta\delta = \alpha\Delta\lambda$.

Substituting the values of r, ρ_0, and $\Delta\delta$ in (1.24) and carrying out the necessary reductions, we obtain:

$$\alpha = \sin \varphi_0. \qquad (1.25)$$

Obviously, on the equator the coefficient of convergence of the meridians $\alpha = 0$, since $\sin 0° = 0$; at the poles $\alpha = 1$, since $\sin 90° = 1$, and in the general case for central latitudes, $0<\alpha<1$.

Knowing the coefficient α, it is not difficult to determine any angle of convergence of the meridians δ along a parallel of tangency or intersection:

$$\delta = \Delta\lambda\alpha, \qquad (1.26)$$

where $\Delta\lambda$ is the difference in longitude between the given meridians.

At any other latitude, the coefficient α will be different from the coefficient α at a latitude of tangency (intersection). Therefore, for approximate calculations in the practice of aircraft navigation during the determination of flight angles or location of the aircraft, the mean latitude of the route, part of the route, or the distance between the aircraft and the radio station, is taken as

$$\delta=(\lambda_2-\lambda_1)\ \sin\phi_{mid} \text{ or } \delta=(\lambda_r-\lambda_a)\ \sin\phi_{mid}$$

where λ_2 and λ_1 are the longitudes of the final and initial points, λ_r and λ_a are the longitudes of the radio station and aircraft, respectively, and ϕ_{mid} is the middle latitude between the indicated points (places).

In some cases, for approximate determinations of the location of an aircraft or the flight angles, the coefficient α is assumed constant for a given map of a conic projection. Thus, for example, for a map with a scale of 1:2,000,000 and a normal isogonal conic /36
projection, it is possible to let $\alpha \approx 0.8$, which corresponds to the sine of the latitude of the middle parallel between the intersection parallels, where the map scale will be minimum.

Polyconic projections

Polyconic (multiconic) projections are the greatest perfection of conic projections for the purpose of decreasing distortions of lengths and angles in projecting the Earth's surface onto a plane.

The principle of construction of such projections is shown in Figure 1.23, a. The central meridian of the projections is a

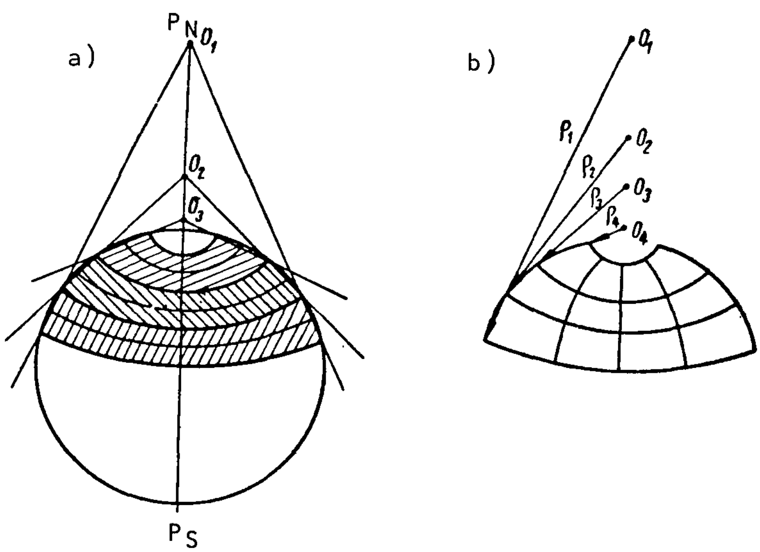

Fig. 1.23. Polyconic Projection: (a) Intersecting Cones on the Globe; (b) Unrolling of Cones on a Plane.

straight line, while meridians in the form of curved lines are situated to the west and east of it. The parallels are concentric circles with different centers, lying on the central meridian (Fig. 1.23, b). As a result of the increase in scale in proportion to the distance from the central meridian to the west and east, such projections are used only to represent the Earth's surface in countires extended along a meridian.

International projection

In terms of the method of construction, an international projection is related to a modified polyconic projection; in terms of the nature of the distortions, it is related to an arbitrary projection.

This projection was accepted at an international geophysical conference in London in 1909 and was called "A Projection of an International Map of the World, with a Scale of 1:1,000,000; Corrections by the Russian Geodesist Shchetkin". This projection, which is the most widely distributed projection at the present time in the Soviet Union, is used for the publication of flight maps with scales of 1:1,000,000, 1:2,000,000, and 1:4,000,000.

Every sheet of map with a scale 1:1,000,000, which encompasses /37
4° of latitude and 6° of longitude (in a range of latitudes from 0 to 64°), is constructed according to its own law, which is general for all the sheets of a given latitudinal strip. On each sheet, the principal scale is given along the outer parallels of the sheet as a result of the intersection of the globe by a cone along these parallels (where $n = 1$) and along the meridians, separated from the central meridian of the sheet by 2° to the west and east, where $m = 1$ (Fig. 1.24, a). In the range of latitudes from 64° to 80°, each sheet occupies 12° of longitude, and from 80 to 88°, 24°. The principal scale of these sheets is also given along the outer parallels of the sheet (through 4°) and along the meridians which are distant

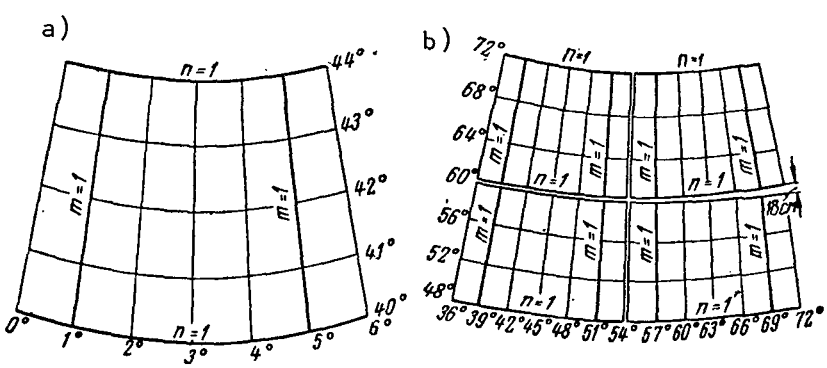

Fig. 1.24. International Projection: (a) Construction of the Sheet; (b) Breaks in the Splicing of Sheets.

from the central meridian of the sheet by 4 and 8°, respectively. The regions of the poles are projected onto separate sheets in a central (polar) projection.

The meridians in this projection are represented by straight lines which have an angle of convergence to the poles, similar to the conic projections, while the parallels are curved lines which are constructed according to a special mathematical law. The centers of the circle-parallels are situated on the central meridian of a given sample of sheets, while their radii are proportional to the cotangents of the intersection latitudes:

$$R_1 = \mathrm{ctg}\,\varphi_1;\quad R_2 = \mathrm{ctg}\,\varphi_2 \text{ etc.}$$

According to studies by Limnitskiy, distortions of lengths on maps with a scale of 1:1,000,000 with such a projection, in the middle latitudes does not exceed 0.076% (76 m in 100 km), while distortion of directions is 5'. The greatest distortions arise in the region of the equator: distortion of lengths up to 0.14%, angles up to 7'.

Insignificant distortions make it possible to consider the map as a practically isogonal, equally spaced, and with equally large projection.

According to this principle, sheets of maps with scales of 1:2,000,000 and 1:4,000,000 are constructed. In a range of latitudes from 0 to 64°, the sheet of a map with a scale of 1:2,000,000 occupies 12° of latitude and 18° of longitude (nine sheets of a millionth, 3 × 3), while a map with a scale of 1:4,000,000 occupies 24 and 36°, respectively. The principal scale of a 1:2,000,000 map is given along the outer parallels of the sheet and the meridians which are distant from the central median of the sheet by 6° to the west and east (Fig. 1.24, b), while on a map with a scale of 1:4,000,000 the parallels which are distant by 8°50' to the north and 8°10' to the south are given without distortions, and the meridians are 12° to the west and east from the central parallel and the central meridian, respectively. /38

The distortion of lengths in the middle latitudes on maps with a scale of 1:2,000,000 reaches 0.5%, and the distortion of the angles is 30'; on 1:4,000,000 maps, distortion of lengths reaches 1.5%, that of angles, 1°30'.

A disadvantage of maps in the international projection on all scales is the presence of discontinuities in the splicing of several sheets, as a result of the features of its construction. Sheets of maps of only one strip or one column are spliced without breaks. During splicing of nine sheets of maps on a scale of 1:1,000,000 (3 × 3), the discontinuities which arise are partially evened out by deformation of the paper, and the use of such a map does not result in perceptible distortions of lengths and angles. Splicing of a large number of sheets is not recommended.

It is even impossible to splice a map with a scale of 1:2,000,000 from four sheets (2 × 2) without a break. At a latitude of 60°, the discontinuity of the spliced sheets reaches 1.8 cm, i.e., 36 km (see Fig. 1.24, b). Therefore, it is possible to splice only one strip or one column of these maps.

The orthodrome with a length up to 1200 km on maps with a scale of 1:1,000,000 and 1:2,000,000 (within the limits of one sheet) appears in practice as a straight line, while the loxodrome is the arc of a logarithmic spiral. Usually, in directions which intersect the meridians, the loxodrome sections with a length up to 600 km are likewise constructed in the form of a straight line, while the flight angle is measured in the middle of a part of a route in order to lessen by a factor of 2 the error of the measured angle during flight with the use of a magnetic compass.

During the determination of the position of an aircraft by means of radio compasses, a correction is allowed for the convergence of the meridians just as in maps of conic projections, with an approximate formula

$$\delta = (\lambda_r - \lambda_a) \sin \varphi_{mid}$$

where λ_r is the longitude of the radio station; λ_a is the longitude of the aircraft; φ_{mid} is the mean latitude between the radio station and aircraft, or the mean latitude of the sheet (sheets) if the approximate position of the aircraft is unknown.

In civil aviation, maps with a scale of 1:1,000,000 and 1:2,000,000 on an international projection are used as flight maps, primarily on piston-engine aircraft and helicopters, and secondly on aircraft with gas-turbine engines. Maps with a scale of 1:4,000,000 are used as aircraft maps for general orientation and approximate determination of the location of an aircraft by means of radio-engineering facilities. /39

Azimuthal (Perspective) Projections

Azimuthal (perspective) projections are constructed according to the laws of a simple geometric perspective; therefore, they are often called perspective projections.

According to the position of the plane of the figure, azimuthal projections are divided into normal or polar (Fig. 1.25, a), transverse or equatorial (Fig. 1.25, b), and oblique or horizontal (Fig. 1.25, c); depending on the position of the center of the projection relative to the plane of the figure, they can be of the following types (Fig. 1.26):

a) *Central* or *gnomonic*, when the center of the projection coincides with the center of the Earth (globe): point *A*;

b) *Steriographic*, when the center of the projection is separated from the point of contact with the plane of the figure by a distance equal to the diameter of the Earth (globe): point *B*;

c) *Orthographic*, when the center of the projection is infinitely separated from the plane of the figure: point *C*;

d) *External*, when the center of the projection is located above the plane of the figure: point *D*.

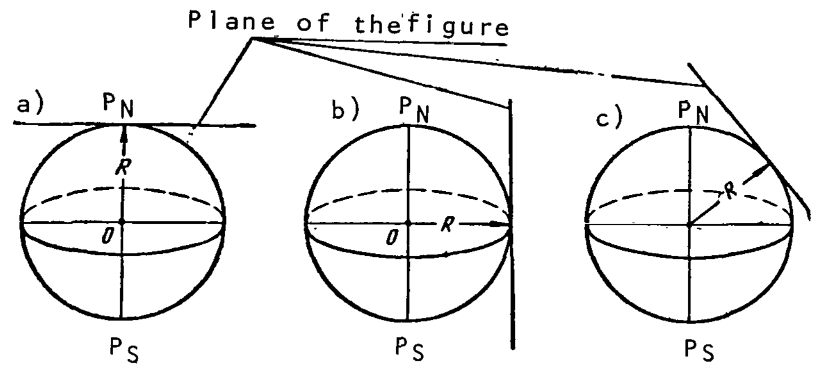

Fig. 1.25. Azimuthal Projections: (a) Normal; (b) Transverse; (c) Oblique.

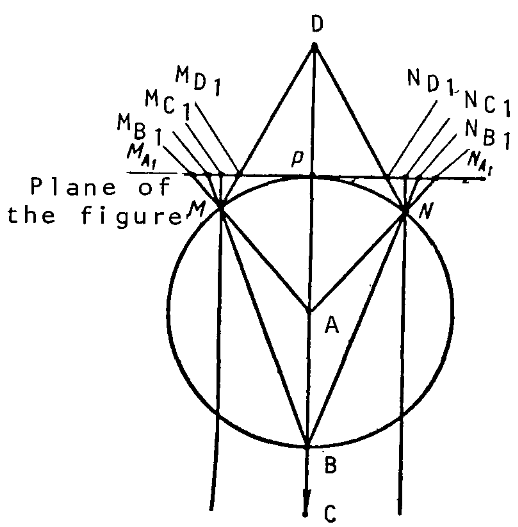

Fig. 1.26. Position of the Centers of Projection in Azimuthal Projections.

As is evident from Fig. 1.26, on such projections points *M* and *N* on the Earth's surface will be projected at a different distance from the point of tangency of the plane of the figure with the Earth's surface. /40

Meridians in azimuthal (polar) projections are represented by straight lines which converge to a pole at an angle equal to the difference in longitude: $\delta = \Delta\lambda$.

Parallels are represented by concentric circles, the radii of which depend on the center of the projection and the latitude of the position.

In aviation, central polar and stereographic polar projections are generally used.

Central polar (gnomonic projection)

The center of projection in this projection coincides with the center of the Earth (globe) at the point O (Fig. 1.27, a).

From Figure 1.27, it is possible to write the equation of this projection:

$$\delta = \lambda;$$
$$\rho = R \operatorname{ctg} \varphi.$$

In order to have a complete idea of the projection, let us find the special scales (m, n) for the principal directions (meridians and parallels):

$$m = \frac{dS_{\text{map}}}{dS_{\text{globe}}} = \frac{-d\rho}{Rd\varphi} = \frac{-d(R \operatorname{ctg} \varphi)}{Rd\varphi},$$

where $d\rho$ is the increase in the radius of the unrolling, i.e., a positive increase in latitude (ϕ) corresponds to a negative increase /41
in the radius (ρ). Integrating the latter, we obtain:

$$m = \frac{+Rd\varphi}{Rd\varphi \sec^2\varphi} = \frac{1}{\sec^2\varphi}$$

or

$$m \operatorname{cosec}^2 \varphi;$$

$$n = \frac{dS_{\text{map}}}{dS_{\text{globe}}} = \frac{\rho d\delta}{rd\lambda} = \frac{R \operatorname{ctg} \varphi d\lambda}{R \cos \varphi d\lambda} = \frac{\operatorname{ctg} \varphi}{\cos \varphi} = \frac{1}{\sin \varphi}; \quad (1.27)$$

Here, $r = R \cos \phi$ is the radius of the parallel, i.e.,

$$n = \operatorname{cosec} \varphi. \quad (1.28)$$

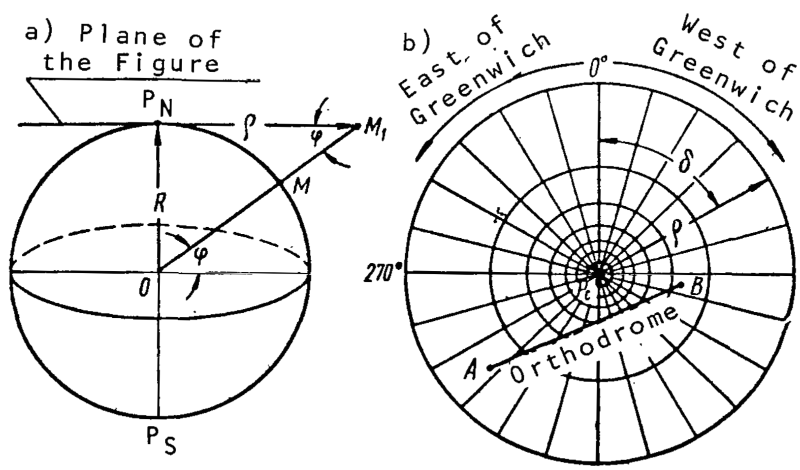

Fig. 1.27. Central Polar (Gnomonic) Projection: (a) Position of the Plane of the Figure; (b) Appearance of the Projection.

Translator's note: cosec = csc.

Therefore, the projection is not isogonal ($m \neq n$), not equally spaced ($m \neq 1$ and $n \neq 1$), and not equally large ($mn \neq 1$).

Although the projection is not isogonal, the orthodrome on it is represented by a straight line. This remarkable property is explained by the fact that the plane of the circumference of a great circle (plane of the orthodrome) always passes through the center of the Earth, which in this case appears as the center of the projection, while the intersection of the plane of a great circle with the plane of the figure is a straight line.

Since the projection is not isogonal, the moving azimuth of the orthodrome on it, if it is not equal to 0, 180, 90, or 270°, does not correspond to the azimuth on the Earth's surface.

Distortion of directions on the map will be equal: /42

$$\operatorname{tg}\beta = \frac{n}{m}\operatorname{tg}\alpha = \frac{\operatorname{cosec}\varphi}{\operatorname{cosec}^2\varphi}\operatorname{tg}\alpha = \sin\varphi\operatorname{tg}\alpha, \qquad (1.29)$$

while it is possible to calculate the actual direction of the orthodrome at the location analogously with the aid of the measured angle on the map:

$$\operatorname{tg}\alpha = \frac{m}{n}\operatorname{tg}\beta = \operatorname{cosec}\varphi\cdot\operatorname{tg}\beta, \qquad (1.30)$$

where β is the measured angle on the map of a given projection, α is the corresponding angle in a location, and ϕ is the latitude of the final point of the orthodrome.

The distortions of directions and distances on this projection are great. In this connection, it is impossible to use a protractor to measure the directions and a scale to measure the distances on the map without corresponding corrections.

A central polar projection is used for constructing gnomonic systems, while the regularity in the distortion of directions is used for calculating the nomograms of the orthodrome direction.

The gnomonic system and the nomogram of the orthodrome direction can be used for the graphic (approximate) calculation of the length of the orthodrome, the coordinates of its intermediate points, and the direction. The loxodrome and other lines of position of the given projection are represented by complex curves.

The property of orthodromicity of a central polar projection has been used for the publishing of oblique central projections which have been used at radiogoniometric points in civil aviation. The middle of the base (the middle of the orthodrome distance between two radiogoniometers) was taken as the point of tangency of the plane of the figure of such maps. In this case, the coordinates of the position of the aircraft are very easily defined as the intersection of two straight orthodrome bearings (lines) extended from the radiogoniometers.

Maps of the differential rangefinding (hyperbolic) system of long-range navigation (DSLN-1) are made on such a projection, since the spherical hyperbola on the projection is also expressed by a hyperbola.

Equally spaced azimuthal (central) projection

This projection is constructed by calculating and transforming conventional meridians (radii) to full size, equal to the principal scale transferred from the globe. The projection is used only for the publication of special small-scale maps (1:40,000,000), which are used as reference maps for measuring distances from a central point on the map.

Usually a large administrative or aviation center, from which it is necessary to know the shortest orthodrome distance in any direction to a given point on a map, is chosen as the point of tangency of the plane of the figure of the projection. In such a projection, for example, a map is constructed with the point of tangency at Vnukovo Airport, with circles plotted at equal distances from the airport. The geographic meridians and parallels are represented by complex curves. This does not allow the directions to be measured. /43

Stereographic polar projection

The center of projection in a stereographic polar projection

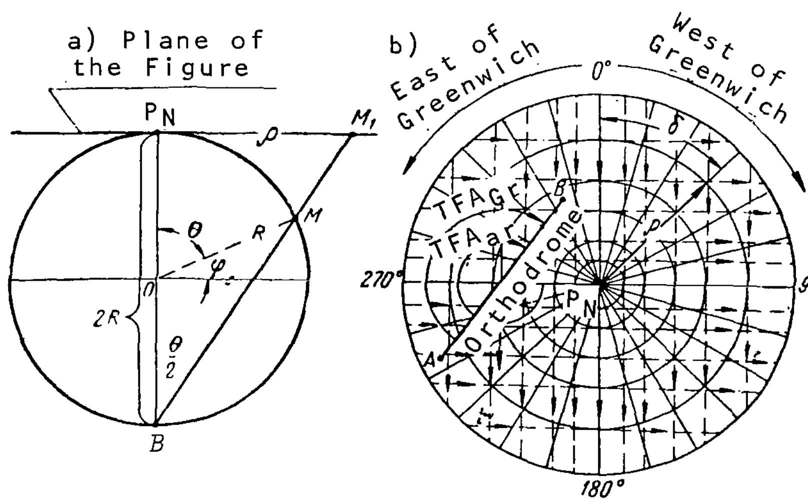

Fig. 1.28. Stereographic Polar Projection: (a) Position of the Point of Projection; (b) Appearance of the Projection.

is separated from the point of tangency of the plane of the figure by two radii of the globe at the point B (Fig. 1.28, a).

Here the angle $\theta = 90° - \phi$, while the angle

$$P_N BM = \frac{\theta}{2} = \frac{90 - \varphi}{2}.$$

An equation of the projection can be derived from equations of the elements shown in Figure 1.28.

$$\delta = \lambda;$$
$$\rho = 2R \operatorname{tg} \frac{\theta}{2}.$$

The meridians in the projection are straight lines which diverge radially from the pole (Fig. 1.28, b), and from the point of tangency of the plane of the paper at an angle equal to the difference in longitude: $\delta = \Delta\lambda$.

The parallels are concentric circles, whose radii are proportional to the tangent of the latitude. /44

The special scale along the parallel is determined by the equation

$$m = \frac{dS_{map}}{dS_{globe}} = \frac{d\rho}{-Rd\varphi} = \frac{2Rd \operatorname{tg} \frac{\theta}{2}}{Rd(90° - \theta)};$$

Here $\phi = 90° - \theta$, while after integration

$$m = \frac{1}{\cos^2 \frac{\theta}{2}} = \sec \frac{2\theta}{2}. \tag{1.30}$$

The special scale along the parallel is determined by the equation

$$n = \frac{dS_{map}}{dS_{globe}} = \frac{\rho d\lambda}{r d\lambda} = \frac{2R \operatorname{tg} \frac{\theta}{2}}{R \cos \varphi}$$

but $\cos \phi = \cos (90 - \theta) = \sin \theta$, so that

$$n = \frac{2 \operatorname{tg} \frac{\theta}{2}}{\sin \theta} = \sec^2 \frac{\theta}{2}, \tag{1.31}$$

i.e.,

$$m = n = \sec^2 \frac{\theta}{2} = \sec^2 \left(\frac{90 - \varphi}{2}\right).$$

Therefore, this projection is isogonal ($m = n$), but not equally spaced ($m \neq 1$ and $n \neq 1$) or equally large ($mn \neq 1$).

On maps of a stereographic projection, a circle drawn on the globe is represented by a circle on the plane (map); however, the center of this circle does not coincide with the projection of the center of the circle on the globe. This makes the projection ineffective for use in rangefinding systems, since lines of equal length will be represented by eccentric circles.

The basic advantage of the projection is its isogonality and insignificant distortion of lengths in the polar regions. Therefore, it has been used for publishing maps of the polar regions with scales of 1:2,000,000, 1:3,000,000, and 1:4,000,000, which are used for general orientation and approximate determination of the position of an aircraft by means of radio devices.

The maximum distortion of lengths at 70° latitude does not exceed 3% (3 km in 100 km), whereas if the plane of the figure is intersected (for example, at 70° latitude), the distortion of the lengths at the poles does not exceed 3% (and at 60° latitude, 4%).

The orthodrome on maps of a stereographic projection has an /45
insignificant bend toward the equator and is constructed in practice as a straight line.

The loxodrome is represented by a logarithmic spiral. It is possible to continue it (just as in conic projections) along the flight angle, which is measured in the middle of the part of the straight line joining the control (rotating) points of the flight path.

In determining the position of an aircraft by means of an aircraft radio compass, a correction for the angle of convergence of the meridians is allowed according to the formula

$$\delta = \lambda_r - \lambda_a$$

where λ_r and λ_a are the longitude of the radio station and the aircraft, respectively.

On maps of a stereographic projection, in order to facilitate determining directions in the polar regions according to a suggestion by V. I. Akkuratov, a supplementary system of "arbitrary" meridians (Fig. 1.28, b) parallel to the Greenwich meridian ($\lambda = 0°$) and perpendicular to it ($\lambda = 90°$) is plotted. Then the true Greenwich flight angle will equal:

$$TFA_{Gr} = TFA_{ar} \pm \lambda_w^e$$

where TFA_{ar} is an arbitrary flight angle measured from a direction perpendicular to the Greenwich meridian ($\lambda = 90°$); $\lambda^e = 90°$ is the location of the route (part of the path) to the east of the Greenwich meridian; and $\lambda_W = 270°$ is the location of the route (part of the path) to the west of the Greenwich meridian.

Nomenclature of Maps

At the present time, a map with a scale of 1:1,000,000 (1 cm = 1 km) which is executed in an international projection is considered the basic topographical map of the world. As described above, each sheet of this map encompasses a territory within the limits of 4° of latitude and 6° of longitude. This has made it possible to compile an international designation for the sheets of maps.

For the purpose of quickly choosing a given sheet of a map, each of them bears a designation of its rank in a definite system.

This designation is called *international map nomenclature.*

The sheets are situated in rows along parallels which run from the equator to a latitude of 84°. There are a total of 21 rows in each hemisphere. Each row is designated by a letter in the Latin alphabet: A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U (maps for latitudes greater than 84° are constructed in perspective projections).

Each sheet of a row has an ordinal number from 1 to 60. Count- /46
ing of the sheets begins from the 180th meridian and proceeds from west to east. The map sheets referring to the prime (Greenwich) meridian from the east have the ordinal number 31. Thus, columns of map numbers are obtained.

To choose the necessary map sheet, it is necessary to know the approximate coordinates of the point for whose region the sheet is selected.

For example: the point coordinates latitude 50° N, longitude 69° E.

Let us divide the latitude of the point by 4, and we will obtain the necessary row of map sheets: $50 \div 4 > 12$. Therefore, the map sheet is located in the thirteenth row, which is designated by the letter *M*.

Let us divide the longitude of the point by 6, and we will obtain: $69 \div 6 > 11$. The ordinal number of the sheet will then be $30 + 12 = 42$.

For convenience in selecting map sheets, composite tables have been constructed. These tables are executed on small-scale maps with a straight, equally spaced cylindrical projection by ruling the indicated map every 4 degrees in latitude and every 6 degrees in longitude, with corresponding designations showing the rows and columns of ordinal numbers of the maps (Supplement 1).

In addition, on the face of each map sheet is a diagram showing how the given sheet fits to the adjoining one (Fig. 1.29). The sheet on which this diagram is drawn fits in the middle and is shaded.

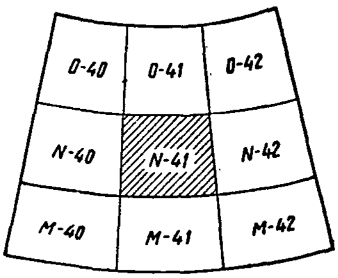

Fig. 1.29. Scheme for Splicing Map Sheets with an International Projection.

Sheets of maps with larger scales have standard schemes of arrangement within the limits of a sheet with a scale of 1:1,000,000.

For example, a map sheet with a scale of 1:1,000,000 contains 4 map sheets with a scale of 1:500,000, which are designated by letters of the Russian alphabet: A, B, C, and D.

By an analogous method, the division of a map sheet with a scale of 1:1,000,000 into sheets with larger scales is carried out. Roman and Arabic numerals are used for their designation. Here the map nomenclature retains the designation of the sheets in the initial division, beginning with a scale of 1:1,000,000 and up (Fig. 1.30).

The nomenclature of map sheets with small scales (1:2,000,000, 1:2,500,000, and 1:4,000,000) is not international and is established when they are printed in accordance with the regions for which they are published and in accordance with the dimensions of the map sheets.

Maps Used for Aircraft Navigation /47

Depending on the nature of the tasks to be fulfilled, it is possible to divide maps into several groups according to their scales.

1) Maps with detailed orientation, with scales of 1:500,000 and up, are used in civil aviation during flights for special purposes (geomagnetic mapping and photography, chemical treatment of areas, searching for small objects in the execution of special tasks, "joining" of radio engineering projects in airport regions, compilation of diagrams for piercing clouds, and for other purposes).

2) Flight maps with scales of 1:2,000,000, 1:1,000,000, and 1:500,000 are used in civil aviation as basic flight maps. Crews of light aircraft and helicopters at comparatively low speeds use maps with scales of 1:1,000,000 and 1:500,000, while crews of high-speed aircraft use maps with scales of 1:2,000,000 and 1:1,000,000. /48

3) Aircraft maps with scales of 1:4,000,000, 1:3,000,000, 1:2,500,000, and 1:2,000,000 are used in civil aviation for general orientation and plotting of position lines with the aid of radio-engineering and astronomical facilities. For these purposes, crews of light aircraft at low speeds and helicopters use maps with only the last two scales.

4) Special maps with scales of 1:40,000,000 and up (to 1:2,000,000), with special emphasis on different purposes of application: lines of equal distance from definite points, a hyperbolic system, azimuths from radio-engineering installations, etc. are used. These include maps with reference materials of smaller scales: maps with time zones, magnetic declinations, composite tables of map sheets, etc.

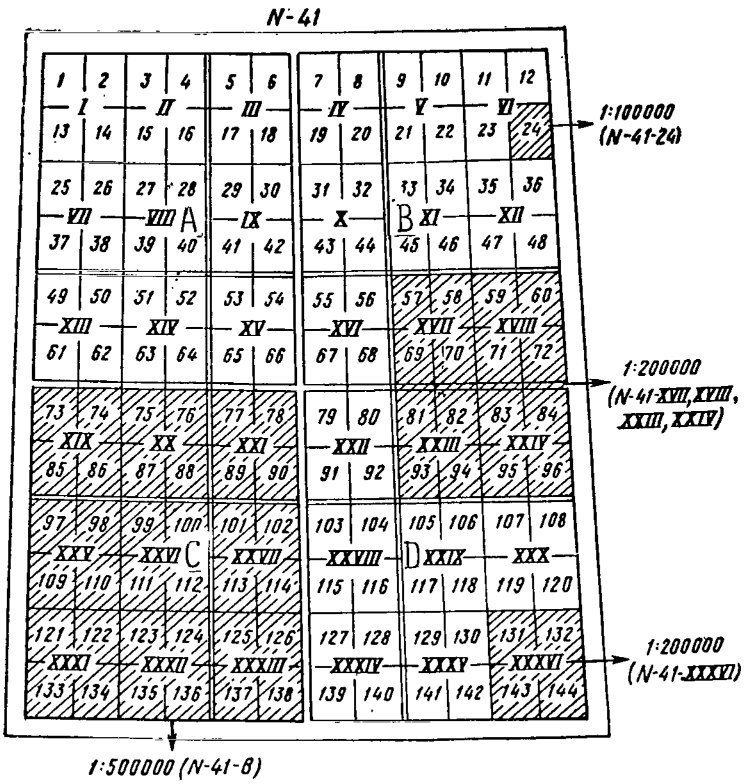

Fig. 1.30. Scheme for Dividing a Map Sheet with an International Projection.

Also, special flight maps with scales of 1:1,000,000 and 1:2,000,000 with plotted and marked flight routes are published for civil aviation. As a rule, they are compiled on oblique cylindrical or oblique conic projections, with the least distortions of angles and lengths along the route. The orthodrome on such maps is practically a straight line.

The contents of a map depend on its scale, the aerographic features of the regions for which it is compiled, and the purpose of the map.

On maps of all scales, the following are drawn in some kind of detail:

(a) relief;
(b) hydrography (seas, rivers, lakes);
(c) populated points;
(d) network of railroads, highways, and country roads;
(e) vegetation or ground cover (large forests, meadow, swamp, sand, desert, etc.);
(f) isolines of magnetic declinations and magnetic anomalies.

The legends of the indicated elements are usually executed on the maps at the lower edge of the sheet.

On maps, a relief is expressed by three methods:

1) It is expressed by isolines of equal height on the surface of the relief (horizontals), i.e., lines formed at the intersection of a relief with horizontal planes which are situated one above the other, with height intervals depending on the scale of the map; the height of the horizontals above sea level is designated by numbers.

2) It is expressed by layered coloring; a special color designated on a special (hypsometric) scale on the lower edge of the map is assigned to each interval of relief height.

3) It is expressed by brown shading, i.e., by special coloring with thickening of brown in the highest areas of the relief and the steepest slopes. This use of color gives a natural, volumetric idea /49
of the nature of the relief.

In addition to the above methods of representing relief on maps, marks of command heights (which exceed neighboring heights), with an indication of the height of these points above sea level, are shown.

Hydrography is shown on maps by a blue color. Its detail depends on the scale and purpose of the map.

Populated points, depending on the scale of the map and the areal dimensions of the points, are represented by contours or conventional symbols in accordance with the point's dimensions or its population.

In lightly populated areas, all populated points are designated. On small-scale maps of densely populated areas, some of the points are omitted. The number of points drawn depends on the scale of the map and the population density of the area.

The detail of the highway network depends on its density, the vegetative or ground cover, and the scale of the map and its purpose.

Besides the above general contents of maps, specially prepared flight maps represent a navigational situation, i.e., the arrangement of radio-engineering facilities for aircraft navigation, position lines of aircraft, and special markings for navigational measurements and calculations are shown.

On some forms of specially prepared maps (map-diagrams), some of the elements of the general contents are omitted or simplified for the purpose of a more detailed and graphic representation of the navigational situation.

6. Measuring Directions and Distances on the Earth's Surface

Orthodrome on the Earth's Surface

In the practice of aircraft navigation at the present time, an orthodrome direction is the main and most widespread direction.

In order to explain all the problems connected with measuring moving angles, distances, and coordinates in flight along an orthodrome, let us examine an orthodrome on the Earth's surface (Fig. 1.31).

An orthodrome, in general, lies at an angle to the Earth's equator and intersects it at two points, the distance between which (along the arc of the equator) is equal to 180°. Only the equator, which likewise appears as an orthodrome, is an exception.

In Figure 1.31, a and b, line $\lambda_0 M_1$ is the arc of the equator, line $\lambda_0 M$ is the orthodrome examined by us; points λ_0 and λ_0 + 180 are the points of intersection of the orthodrome with the equator; /50
$P_N \lambda_0 P_S$ is the meridian of the point of intersection of the orthodrome with the equator; $P_N M M_1 P_S$ is the meridian of the point M on

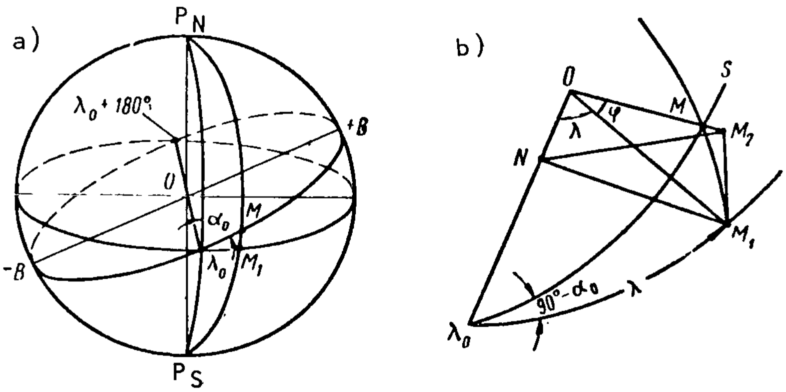

Fig. 1.31. Orthodrome on the Earth's Surface: (a) Position of the Orthodrome on a Sphere; (b) Relationship between Longitudes and Latitudes of Points on the Orthodrome.

the orthodrome; 90° - α_0 is the angle between the plane of the equator and the plane of the orthodrome; λ is the londigude of the point M; ϕ is the latitude of the point M; $-B$, $+B$ are points on the orthodrome of maximum latitude, which are called vertex points.

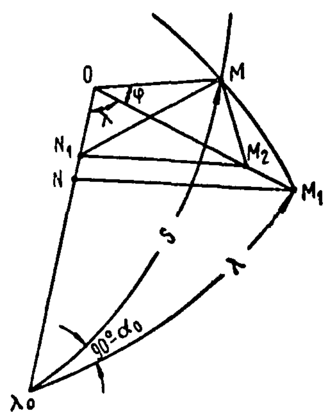

Fig. 1.32. Determining Distance on an Orthodrome.

Let us erect a normal to the plane of the equator at point M_1 (see Fig. 1.31, b) and extend it to an intersection with the vertical of point M on the orthodrome (point M_2). It is obvious that the triangle $O\ M_1M_2$ will be a right triangle. Here M_1M_2 will be the tangent line of the angle ϕ.

Let us drop from points M_1 and M_2, perpendiculars to the aperture axis of the orthodrome with the equator $\lambda_0 O$. One of them will lie in the plane of the equator, the second in the plane of the orthodrome; both will converge at one point on the aperture axis (point N).

It is obvious that line M_1N will be a line of the sine of angle λ, while angle M_1NM_2 will be the aperture angle of the plane of the equator with the plane of the orthodrome (90° - α_0). Here the triangle NM_1M_2 will also be a right triangle.

Thus, for point M and for any point on the orthodrome, the following equation will be valid: /51

$$\operatorname{tg}(90^\circ - \alpha_0) = \frac{\operatorname{tg}\varphi}{\sin\lambda}$$

or

$$\operatorname{tg}\alpha_0 = \frac{\sin\lambda}{\operatorname{tg}\varphi}. \qquad (1.32)$$

Formula (1.32) is valid only for cases when the point λ_0 is the point of origin of the longitude. When the longitude of the point is not equal to zero, the longitude of the point λ_0 must be subtracted from the longitude of the point $M(\lambda_M)$, i.e., the reference system of longitudes must be reduced to this point. Then

$$\operatorname{tg}\alpha_0 = \frac{\sin(\lambda_M - \lambda_0)}{\operatorname{tg}\varphi_M}. \qquad (1.32a)$$

In the future, for the sake of simplicity, we will consider the longitude of λ_0 equal to zero.

It is possible to determine the moving azimuth according to the formula

$$\operatorname{tg}\alpha = \operatorname{tg}\alpha_0 \sec\lambda \sec\varphi, \qquad (1.33)$$

Considering that tg $\alpha_0 = \frac{\sin \lambda}{\text{tg } \phi}$, it is possible to reduce (1.33) to the form:

$$\left.\begin{aligned}\text{tg}\,\alpha &= \text{tg}\,\lambda\,\text{cosec}\,\varphi \\ \text{or} \quad \text{ctg}\,\alpha &= \text{ctg}\,\lambda\,\sin\varphi.\end{aligned}\right\} \tag{1.33a}$$

Formulas (1.33) and (1.33a) are obtained by differentiation of (1.32).

Since the ratio $\frac{\sin \lambda}{\text{tg } \phi}$ = tg α_0 = const remains valid for every length of an orthodrome, it is obvious that the elementary difference quotient sin λ and tg ϕ will also be constant for every length of an orthodrome and will equal:

$$\frac{d \sin \lambda}{d\, \text{tg}\, \varphi} = \text{tg}\, \alpha_0 = \text{const.}$$

Therefore, it is possible to write (1.32) in the form:

$$\frac{\frac{d \sin \lambda}{d\lambda}}{\frac{d\, \text{tg}\, \varphi}{d\varphi}} = \frac{d\lambda}{d\varphi} = \text{tg}\, \alpha_0,$$

whence

$$\frac{\cos \lambda}{\sec^2 \varphi} \cdot \frac{d\lambda}{d\varphi} = \text{tg}\, \alpha_0$$

or

$$\frac{d\lambda}{d\varphi} = \frac{\text{tg}\, \alpha_0}{\cos \lambda \cos^2 \varphi}$$

On the Earth's surface, the linear scale of longitude is equal to the linear scale of latitude multiplied by the cosine of latitude. Therefore, the tangent of the moving azimuth of the orthodrome will be expressed by the derivative $\frac{d\lambda}{d\phi}$, divided by the cosine of the latitude: /52

$$\text{tg}\, \alpha = \frac{\frac{d\lambda}{d\varphi}}{\cos \varphi} = \frac{\text{tg}\, \alpha_0}{\cos \lambda \cos^2 \varphi}$$

or, considering that $\text{tg}\, \alpha_0 = \frac{\sin \lambda}{\text{tg}\, \varphi}$,

we arrive at (1.33a): $\text{tg}\, \alpha = \text{tg}\, \lambda\, \text{cosec}\, \varphi.$

In the practice of aircraft navigation, it is usually necessary to deal with two points on the Earth's surface. With the exception of special cases, neither of them is on the equator.

Formulas (1.32) and (1.33) can be used only in those cases when the point of intersection of an orthodrome with the equator (i.e., the longitude of a point on the orthodrome, the width of which is equal to zero) is known.

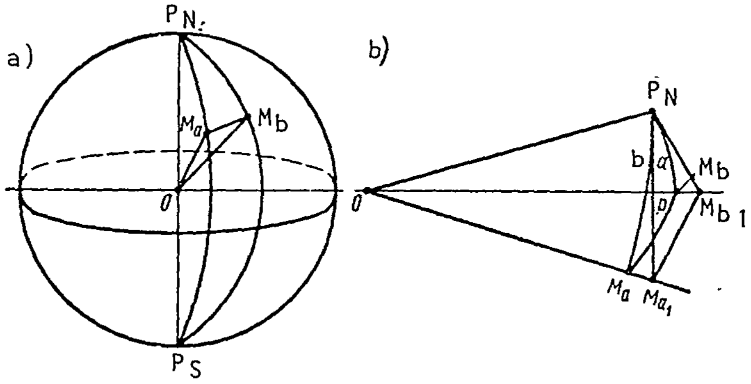

Fig. 1.33. Elements of a Spherical Triangle. (a) Triangle on a Sphere; (b) Relationship of Angles and Sides of a Spherical Triangle.

Let us derive an equation which makes it possible to determine the coordinate λ of a given point on an orthodrome on the basis of the coordinates of two known points on it.

Let us assume that we have two arbitrary points on the Earth's surface with coordinates $\phi_1\lambda_1$ and $\phi_2\lambda_2$. We will take the difference of the longitudes of these points as $\Delta\lambda(\Delta\lambda = \lambda_2 - \lambda_1)$. Then, according to (1.32a),

$$\frac{\sin(\lambda_1 - \lambda_0)}{\operatorname{tg}\varphi_1} = \frac{\sin[(\lambda_1 - \lambda_0) + \Delta\lambda]}{\operatorname{tg}\varphi_2}.$$

Transforming the right-hand side of this equation, we obtain: /53

$$\frac{\sin(\lambda_1 - \lambda_0)}{\operatorname{tg}\varphi_1} = \frac{\sin(\lambda_1 - \lambda_0)\cos\Delta\lambda + \cos(\lambda_1 - \lambda_0)\sin\Delta\lambda}{\operatorname{tg}\varphi_2}$$

Dividing both sides of the equation by $\sin(\lambda_1 - \lambda_0)$ and multiplying by $\operatorname{tg}\phi_2$, we will have:

$$\frac{\operatorname{tg}\varphi_2}{\operatorname{tg}\varphi_1} = \frac{\sin(\lambda_1 - \lambda_0)\cos\Delta\lambda + \cos(\lambda_1 - \lambda_0)\sin\Delta\lambda}{\sin(\lambda_1 - \lambda_0)} =$$
$$= \cos\Delta\lambda + \operatorname{ctg}(\lambda_1 - \lambda_0)\sin\Delta\lambda,$$

from which

$$\operatorname{ctg}(\lambda_1 - \lambda_0) = \operatorname{tg}\varphi_2 \operatorname{ctg}\varphi_1 \operatorname{cosec}\Delta\lambda - \operatorname{ctg}\Delta\lambda. \qquad (1.34)$$

Equation (1.34), which makes it possible to determine the longitude of the point of intersection of an orthodrome with the equator (λ_0), is very important. Knowledge of this coordinate makes it possible to calculate easily all the remaining elements of the orthodrome.

Having substituted the value λ in (1.34) for the value (λ_1 - λ_0), as before, and substituting into (1.33a) the value ctg λ from (1.34), we obtain the following equation for a point with the coordinates $\phi_1\lambda_1$:

$$\operatorname{ctg}\alpha = \operatorname{tg}\varphi_2 \cos\varphi_1 \operatorname{cosec}\Delta\lambda - \operatorname{ctg}\Delta\lambda \sin\varphi_1. \tag{1.35}$$

Formula (1.35) is usually used for calculating the azimuth of an orthodrome at the initial point of the straight-line segment of the path when there is no necessity for determining the remaining elements of the orthodrome.

In general, it is better to solve (1.34) independently, and then find the solution by substituting λ into (1.32) and (1.33).

Simple transformations of (1.32) reduce to formulas which make it possible to determine the coordinates of intermediate points on an orthodrome:

$$\operatorname{tg}\varphi = \sin\lambda \operatorname{ctg}\alpha_0, \tag{1.36}$$

$$\sin\lambda = \operatorname{tg}\varphi \operatorname{tg}\alpha_1. \tag{1.36a}$$

Given the arbitrary value of a point coordinate on the orthodrome ϕ or λ, it is possible to obtain the value of the second coordinate of this point on the basis of these formulas.

The formulas from (1.32) through (1.36), given by us, make it possible to determine the initial and moving azimuths of the orthodrome, and also the coordinates of its intermediate points.

In order to determine the length of the orthodrome or distances along it (S) let us derive equations which connect the coordinates of the points of the orthodrome with its length.

In Figure 1.32, the triangles ONM_1 and ON_1M_2 are similar. The straight line ON is a line of the cosine of the arc λ, while ON' is a line of the cosine of arc S.

The hypotenuse of triangle ONM_1 is equal to the radius of the Earth, while the hypotenuse of triangle ON_1M_2 is the line of the cosines of arc ϕ. /54

Therefore,

$$\cos S = \cos\lambda \cos\varphi. \tag{1.37}$$

Equation (1.37) makes it possible to determine the distance from the starting point of the orthodrome to any of its points with known coordinates.

If the initial point of the orthodrome and the coordinates of any two points along it are known, the distance (S) between the latter is determined as the difference between the distances to the initial point:

$$S_{1,2} = S_2 - S_1.$$

If the coordinates of the starting point are not known, and the necessity for determining the other elements of the orthodrome (besides the distance between the two points) is lacking, then the indicated distance can be determined by the formula

$$\cos S = \sin \varphi_1 \sin \varphi_2 + \cos \varphi_1 \cos \varphi_2 \cos \Delta\lambda. \quad (1.38)$$

Formula (1.38) is not derived from simple geometric ratios. For its derivation, it is necessary to use the spherical triangle ($P_N M_a M_b$) (Fig. 1.33a).

Let us join points $P_N M_a$ and M_b by verticals with the center of the Earth O. Let us draw tangents to the arcs $P_N M_a$ and $P_N M_b$ at the point P_N up to the intersection with the indicated verticals at points M_{a1} and M_{b1} (Fig. 1.33, b). We will obtain two plane triangles $P_N M_{a1} M_{b1}$ and $OM_{a1} M_{b1}$ with the common side $M_{a1} M_{b1}$. Obviously,

$$M_{a1} M_{b1} = (P_N M_{a1})^2 + (P_N M_{b1})^2 - 2 P_N M_{a1} P_N M_{b1} \cos M_{a1} P_N M_{b1}$$

At the same time,

$$M_{a1} M_{b1} = (OM_{a1})^2 + (OM_{b1}) - 2 OM_{a1} OM_{b1} \cos M_{a1} OM_{b1} \quad (1.39)$$

Since M_{a1} M_{b1} is the common side of the triangle, the left-hand side of the first equation is equal to the right-hand side of the second.

Taking the radius of the Earth as equal to 1, from the right triangles $OP_N M_{a1}$ and $OP_N M_{b1}$ we find:

$$P_N M_{a_1} = \mathrm{tg}\, b; \; P_N M_{b1} = \mathrm{tg}\, a; \; OM_{a_1} = \sec b;$$
$$OM_{b1} = \sec a; \; \angle M_{a_1} P_N M_{b1} = P; \; \angle M_{a_1} OM_{b1} = p.$$

Substituting the indicated values into (1.39), we obtain:

$$\mathrm{tg}^2 b + \mathrm{tg}^2 a - 2 \mathrm{tg}\, b\, \mathrm{tg}\, a \cos P = \sec^2 a + \sec^2 b - 2 \sec a \sec b \cos p;$$
$$\sec^2 a = 1 + \mathrm{tg}^2 a; \quad \sec^2 b = 1 + \mathrm{tg}^2 b$$

Therefore,

$$2\, \mathrm{tg}\, a\, \mathrm{tg}\, b \cos P = 2 - 2 \sec a \sec b \cos p. \quad (1.40)$$

Multiplying both sides of (1.40) by $\frac{\cos a \cos b}{2}$ we obtain:

$$\sin a \sin b \cos P = \cos a \cos b - \cos p$$

or

$$\cos p = \cos a \cos b + \sin a \sin b \cos P. \quad (1.41)$$ /5!

Formula (1.41) is the first basic formula of spherical trigonometry and is widely used in aircraft navigation with the use of astronomical facilities (the remaining formulas of spherical trigonometry are given in Supplement 2).

In our case,

$$\angle P = \Delta\lambda; \quad \angle p = Sab; \quad \angle b = 90° - \varphi_2; \quad \angle a = 90° - \varphi_1,$$

i.e., (1.41) has the form:

$$\cos S = \sin \varphi_1 \sin \varphi_2 + \cos \varphi_1 \cos \varphi_2 \cos \Delta\lambda.$$

When determining point coordinates of the orthodrome, there is the same necessity to solve the inverse problems according to the known orthocromic distance (S).

For this let us return to Figure 1.32, in which it is obvious that the line MN_1 is the line of the sine of the arc S, while line MM_2 is equal to $MN_1 \cos \alpha_0$. At the same time, MM is the line of the sines for arc ϕ. Therefore,

$$\sin \varphi = \sin S \cos \alpha_0 \tag{1.42}$$

Formula (1.42) makes it possible to determine the coordinate ϕ along the traversed orthodrome distance from the initial point. The coordinate λ in this case is determined according to (1.36a).

$$\sin \lambda = \operatorname{tg} \varphi \operatorname{tg} \alpha_0.$$

Thus, we have an analytical form of all the necessary transformations for determining the elements of the orthodrome on the Earth's surface. However, in the practice of aircraft navigation it is sometimes more convenient to apply other formulas which determine separate elements of the orthodrome.

For example, if the coordinates of two points on the Earth's surface and the orthodrome distance between them S are known, the azimuth of the orthodrome (α) at the starting point can be determined by the formula

$$\sin \alpha = \frac{\cos \varphi_2 \sin \Delta\lambda}{\sin S}. \tag{1.43}$$

Formula (1.43) can be transformed to determine the distance between points at a known azimuth:

$$\sin S = \frac{\cos \varphi_2 \sin \Delta\lambda}{\sin \alpha}. \tag{1.43a}$$

It is obvious that both formulas are obtained from the equation

$$\sin S \sin \alpha = \cos \varphi_2 \sin \Delta\lambda,$$

which in turn is derived by means of Figure 1.34, where line BB_1 is a perpendicular dropped from point B to the plane of the equator, and is a line of the sine of the latitude of this point, while /56 line B_1A_2 is a perpendicular dropped from point B_1 to the plane of the meridian which passes through point A. Obviously,

$$A_2B_1 = \cos\varphi_2 \sin\Delta\lambda.$$

Let us erect another perpendicular to the plane of the equator at point A_2; we will then obtain plane $A_1A_2B_1B$ perpendicular to the plane of the equator and the plane of the meridian of point A.

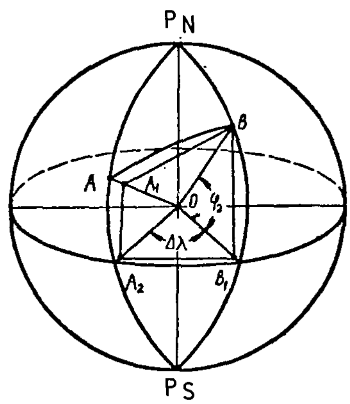

Fig. 1.34. Determining Special Elements of an Orthodrome.

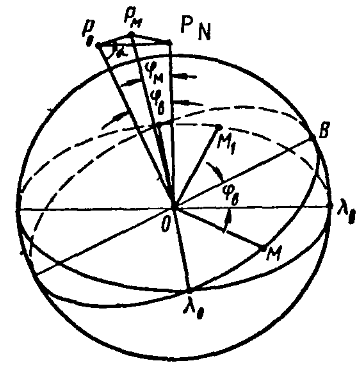

Fig. 1.35. Determining the Initial Azimuth or the Vertex of an Orthodrome.

If we rotate the indicated plane around the line BA_1 in such a way that it remains perpendicular to the plane of the meridian of point A up to the moment when line A_2A_1 becomes perpendicular to the vertical of point A, the distance BA_1 will not change. In this case, straight line BA_1 will be the line of the sine of the arc AB, while its length will be determined by the formula

$$A_1B = \frac{A_2B_1}{\sin\alpha},$$

from which it follows that

$$\sin S \sin\alpha = \cos\varphi_2 \sin\Delta\lambda.$$

By a similar method, the initial angle of the orthodrome or the latitude of the vertex point is determined if the azimuth of the orthodrome at any point on the Earth's surface is known.

In Figure 1.35, arc $\lambda_0\lambda_B$ is the equator; arc λ_0MB is the orthodrome; line OP_N is the axis of the Earth; OP_0 is the axis of the orthodrome; M is a point on the Earth's surface; and B is the vertex point.

Let us erect a perpendicular $P_M O$ to the vertical of point M from the center of the Earth so that it is located in the plane of the meridian of point M. Let us also draw a plane parallel to the plane of the equator through the poles P_N, P_O, and P_M. The angles $P_O P_N O$ and $P_M P_N O$ will be right angles, since lines $P_O P_N$ and $P_M P_N$ lie in a plane perpendicular to $P_N O$. /57

Angle $P_O P_M O$ is also a right angle, since the plane of triangle $P_O P_M O$ has a slope to the axis perpendicular to $P_M O$ and parallel to $P_O P_M$. It is obvious that angle $P_O O P_N$ is equal to the latitude of the vertex point, while angle $P_M O P_N$ is equal to the latitude of point M. Therefore,

$$OP_N = OP_0 \cos\varphi_B = OP_M \cos\varphi_M;$$
$$OP_M = OP_0 \sin\alpha,$$

whence

$$\cos\varphi_B = \sin\alpha_0 = \cos\varphi_M \sin\alpha. \quad (1.44)$$

Formula (1.44) is used to find the latitude of the vertex point, which also appears as a complement of the initial angle of the orthodrome up to 90°. The longitude of point M relative to the starting point of the orthodrome in this case can be determined by (1.36a).

With a known azimuth of the orthodrome at any point on the Earth's surface, the coordinates of its starting point can be obtained directly according to (1.33a), from which it follows that

$$\operatorname{tg}\lambda = \sin\varphi \operatorname{tg}\alpha, \quad (1.45)$$

Then it is not difficult to determine the initial angle of the orthodrome.

In some cases, in order to calculate the elements of the orthodrome, coordinates of the vertex point rather than of the starting point are used. In these cases, the functions of the α_0 angle are replaced by inverse functions of the latitude of the vertex point which are equal to them, just as functions of longitude are, since these angles differ by 90°.

For example, (1.36a) has the form:

$$\cos\lambda_B = \operatorname{tg}\varphi \operatorname{ctg}\varphi_B,$$

while (1.45) has the form:

$$\operatorname{ctg}\lambda_B = \sin\varphi \operatorname{tg}\alpha.$$

To explain the procedure for determining all the elements of an orthodrome, let us examine (as an example) an orthodrome which passes through two points on the Earth's surface with these coordinates: M_1 = latitude 60° N, longitude 30° E; M_2 = latitude 80° N, longitude 40° E.

First we shall carry out the general solution of the problem of finding the elements of an orthodrome. For this we shall use (1.34). Substituting into this equation the functions of the coordinates of the points M_1 and M_2, we obtain:

$$\operatorname{ctg}(\lambda_1-\lambda_0)=\operatorname{tg}80^\circ\operatorname{ctg}60^\circ\operatorname{cosec}10^\circ-\operatorname{ctg}10^\circ;$$

$$\operatorname{ctg}(\lambda_1-\lambda_0)=\frac{5{,}671\cdot 0{,}5774}{0{,}1736}-5{,}671=13{,}228;$$

$$\lambda=(\lambda_1-\lambda_0)=4^\circ18'; \quad \lambda_0=25^\circ42'.$$

The initial azimuth of an orthodrome, according to (1.32), will be:

$$\operatorname{tg}\alpha_0=\sin\lambda\operatorname{ctg}\varphi.$$

Let us determine it on the basis of the coordinates of point M_1: /58

$$\operatorname{tg}\alpha_0=\sin 4^\circ18'\cdot\operatorname{ctg}60^\circ=0{,}574\cdot 0{,}075=0{,}0433;$$

$$\alpha_0=2^\circ29'.$$

According to (1.33a), the moving azimuth of the orthodrome (α) for point M_1 is equal to

$$\operatorname{ctg}\alpha=\operatorname{ctg}4^\circ18'\cdot\sin 60^\circ=13{,}228\cdot 0{,}866=14{,}455;$$

$$\alpha=5^\circ.$$

The distance from the starting point of the orthodrome to point M_1, according to (1.37), equals

$$\cos S_1=\cos 4^\circ18'\cdot\cos 60^\circ=0{,}9972\cdot 0{,}5=0{,}4986;$$

$$S_1=60^\circ4',$$

while the distance to point M_2, according to the same formula, is

$$\cos S_2=\cos 14^\circ18'\cdot\cos 80^\circ=0{,}969\cdot 0{,}1736=0{,}1682;$$

$$S_2=80^\circ17'.$$

The distance between M_1 and M_2 is then defined as the difference between the distances to the starting point:

$$S=S_2-S_1=80^\circ17'-60^\circ04'=20^\circ13'.$$

Coordinates of any intermediate point can be determined according to (1.36) or (1.36a).

For example, the longitude of point M_2 according to its latitude is

$$\sin\lambda_2=\operatorname{tg}80^\circ\operatorname{tg}2^\circ29'=5{,}671\cdot 0{,}0433=0{,}246;$$

$$\lambda=(\lambda_2-\lambda_0)=14^\circ15'; \quad \lambda_2=39^\circ57'.$$

Thus, all the necessary elements of an orthodrome are easily determined.

Let us now assume that we had to determine only the azimuth of the orthodrome at point M_1. For this we will use (1.35):

$$\operatorname{ctg} \alpha = \frac{0{,}5 \cdot 5{,}671}{0{,}1736} - 0{,}866 \cdot 5{,}671 = 11{,}4225;$$
$$\alpha = 5°.$$

Knowing the azimuth of the orthodrome at one of its points makes it possible to determine the distance to any point by using (1.43a), or in our example:

$$\sin S = \frac{0{,}1736 \cdot 0{,}1736}{0{,}0872} = 0{,}3456;$$
$$S = 20°13'.$$

Using the azimuth of the orthodrome at one point, it is possible to determine the latitude of the vertex point or the initial azimuth of the orthodrome according to (1.44).

For our orthodrome, using point M_1, we obtain:

$$\sin \alpha_0 = \cos 60° \sin 5° = 0{,}5 \cdot 0{,}0872 = 0{,}0436;$$
$$\alpha_0 = 2°30'.$$

After this, the intermediate points of an orthodrome are easily determined. /59

Thus, it is possible to determine the elements of an orthodrome beginning with the distance between two points according to (1.38), changing to the moving azimuth according to (1.43), and then to the latitude of the vertex point according to (1.44).

Orthodrome on Topographical Maps of Different Projections

Let us examine an orthodrome on maps of a simple equally spaced cylindrical projection, which essentially represents a geographical coordinate system on a scale of angles.

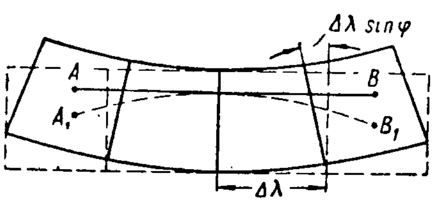

Fig. 1.36. Elementary Segment of an Orthodrome on a Map of a Cylindrical Projection.

To explain this, however, let us draw on the Earth's surface an elementary normal cone at some latitude; we shall examine it on the above projection (Fig. 1.36).

As is already known, the radius of this cone when unrolled equals:

$$\rho_0 = R \operatorname{ctg} \varphi_0$$

or, taking the radius of the Earth as 1,

$$\rho_0 = \text{ctg}\,\varphi_0.$$

According to (1.20) the scale of the projection along the parallel is equal to:

$$n = \frac{1}{\cos\varphi}.$$

In order to draw our unrolled cone on a cylindrical surface, it is necessary to straighten the cone first and then extend it. Obviously, the segments of the meridians remain straight lines during straightening of the cone, but they must be unrolled together with the surface elements to an angle equal to $\Delta\lambda \sin\phi$.

Let us now draw a straight line *AB* in the east-west direction on an elementary cone.

During straightening of the cone, the indicated straight line will acquire a curvature, the radius of which will be equal to the radius of the unrolled cone (*r*), but curved in the opposite direction. Therefore,

$$\rho = r_{A_1B_1} = \text{ctg}\,\varphi.$$

During extension of our cone along a parallel to a scale *nz*, each of its elements (including elements of our straight line) will undergo an extension equal to $\frac{1}{\cos\phi}$. Therefore, the radius of the straight-line element will increase and will equal: /60

$$r_{A_1B_1} = \frac{\text{ctg}\,\varphi}{\cos\varphi} = \text{cosec}\,\varphi.$$

As is evident, the straight-line element, situated along the parallel (in general, in a direction perpendicular to the axis of the cylinder) acquires a curvature. The straight-line element situated in the direction of the axis of the cylinder does not acquire a curvature. Therefore, if the straight-line element is situated at an angle to the axis of the cylinder, its radius of curvature will equal:

$$r_{A_1B_1} = \frac{\text{ctg}\,\varphi}{\cos\varphi \sin\alpha} = \text{cosec}\,\varphi\,\text{cosec}\,\alpha. \quad (1.46)$$

In geometry, the curvature of a curve is considered to be a value inverse to the radius of the curvature. Therefore, the curvature of our element will equal:

$$\frac{1}{r_{A_1B_1}} = \sin\varphi \sin\alpha. \quad (1.47)$$

An orthodrome on the Earth's surface does not have its own curvature of each element of an orthodrome on a map in a normal, equally spaced cylindrical projection will be expressed by (1.47), from which it is obvious that the maximum curvature of the orthodrome will be observed at its vertex points, while the starting points, i.e., the points of intersection of the orthodrome with the equator (Fig. 1.37), will appear as points of inflection.

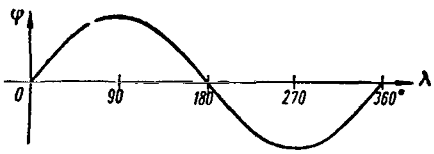

Fig. 1.37. Graph of an Orthodrome in a Cylindrical Projection.

Thus, the orthodrome on a map of an equally spaced cylindrical projection has a form reminiscent of a sine curve. This curve is the graph of the ratio of the coordinates of the orthodrome with a known initial azimuth (α).

As a result of the nonisogonality of an equally spaced projection, the slope of a tangent to the curve of the orthodrome does not reflect its directly moving azimuth, with the exception of the azimuth at starting points.

The moving azimuth of an orthodrome along a curve can be determined if we consider the relationship between the scales m and n at the investigated points. With equal scales, the dip angle of the tangent to the curve is determined by the formula

$$\operatorname{tg} \alpha = \frac{d\lambda}{d\varphi};$$

In our case, the scale $n_\lambda = n_0 \sec \phi$ or $n_\lambda = n_\phi \sec \phi$. Therefore, the actually moving azimuth of the orthodrome in an equally spaced cylindrical projection is determined by the formula /61

$$\operatorname{tg} \alpha = \frac{d\lambda}{d\varphi} \cos \varphi.$$

It is obvious that in an isogonal normal cylindrical projection, the orthodrome will also have a shape reminiscent of a sine curve. However, as a result of the extension of the scale along the latitude ($n = n_0 \sec \phi$), the amplitude of this curve will be increased. The more it is increased, the smaller the initial azimuth of the orthodrome will be, and the greater the latitude of the vertex points.

In contrast to an equally spaced projection, in this projection the dip angle of the tangent to the curve will correspond to the moving azimuth of the orthodrome at any point, since the scales along the longitude and latitude are equal to:

$$m = n = \sec \varphi.$$

Thus, the orthodrome in a cylindrical projection has the form of a curve which is convex in the direction of the increase in the scale of the projection. This feature of the orthodrome is common to all projections which have an increase in one direction.

Let us cite a brief analysis of the bend of the orthodrome with a varying map scale, in accordance with the general case.

Let us assume that we have a spherical trapezoid which is represented on a map in the form of a rectangle (Fig. 1.38). The length of any parallel (L_λ) on the trapezoid is equal to its length on a rectangle divided by the scale of its representation. The scale of representation of the meridians in any part of the rectangle is equal to one.

$$L_\lambda = \frac{1}{n}; \quad L_\varphi = 1.$$

During extension of the trapezoid into a rectangle, each straight-line element on its surface acquires a curvature.

$$\frac{1}{r_\lambda} = \frac{\partial \frac{1}{n}}{d\varphi}; \quad \frac{1}{r_\varphi} = 0.$$

Therefore, for an equally spaced cylindrical projection,

$$\frac{1}{r_\lambda} = \frac{\partial \cos\varphi}{d\varphi} = -\sin\varphi.$$

Since $\frac{1}{r_\lambda} = 0$, for a straight line passing at an angle to the meridian we will have

$$\frac{1}{r} = -\sin\varphi \sin\alpha.$$

A minus sign shows that bending occurs in the direction of a decrease in the value of $\frac{1}{n}$ or in the direction of an increase in the scale.

/62

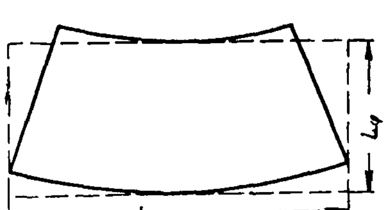

Fig. 1.38. Conical Trapezoid Represented in the Form of a Rectangle.

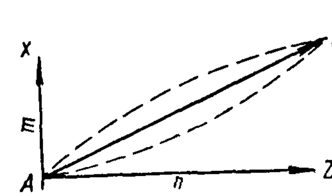

Fig. 1.39. Bending of an Orthodrome in the Directions of Scale Increases.

Let us now assume that we have a projection, a change in the scale of which occurs in two principal directions [for example, simultaneously in the north and east (Fig. 1.39)].

It is obvious that a straight line AB, passing at an angle to the meridian with a change in the scales in two principal directions, will simultaneously undergo bending in opposite directions, i.e., its component curvatures will be subtracted:

$$\frac{1}{r} = \frac{\partial \frac{1}{n_\lambda}}{d\varphi} \sin\alpha - \frac{\partial \frac{1}{m_\varphi}}{d\lambda} \cos\alpha$$

For the general case,

$$\frac{1}{r} = \frac{\partial \frac{1}{n}}{dz} \sin\alpha - \frac{\partial \frac{1}{mz}}{dx} \cos\alpha.$$

Therefore, the orthodrome on maps constructed with tangential cylindrical projections will have convexity:

a) at latitudes greater than the latitude of a parallel which is tangent to a geographic pole;

b) at latitudes lower than the latitude of a parallel which is tangent to the equator (Fig. 1.40).

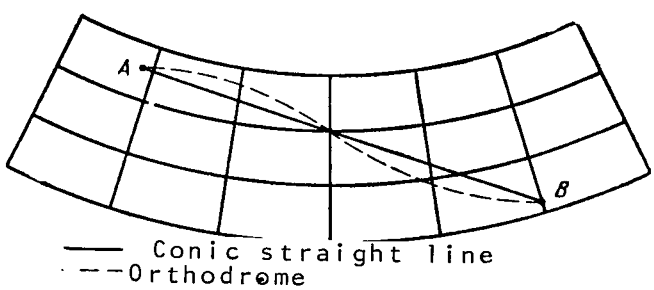

Fig. 1.40. Orthodrome on a Map of a Tangential Conic Projection.

In intersecting conic projections, the orthodrome has the same form as in tangential projections. Here, its point of inflection is situated on the middle parallel between the parallels of intersection. /63

Of special interest is the orthodrome on maps in an international projection. As is known, the scale along the latitude in these maps remains practically constant, while the scale along the longitude has small changes in the limits of each map sheet. Therefore, an orthodrome drawn at an angle to the meridian within the limits of one sheet on this map will be a wavy line around a straight principal direction. However, on maps with a scale of 1:1,000,000, deviations from the principal direction will be so insignificant that they are practically unnoticeable, while in the places where separate map sheets are spliced, both parallels and the orthodrome will have breaks.

As we have already shown, the orthodrome in a central polar projection is expressed by a straight line. However, its moving azimuth, with the exception of the directions 0, 90, 180, and 270° cannot be determined by simple measurements on a map, but demand the introduction of corrections according to (1.29) and (1.30).

In a polar stereographic projection, the orthodrome is also a nearly straight line. However, to determine its azimuth, it is necessary to use general equations of an orthodrome on the Earth's surface.

Loxodrome on the Earth's Surface

The loxodrome direction at the present time is used only to determine the mean path angle of flight on short segments of a path by the use of magnetic compasses. With the use of magnetic compasses, not a geographic but a magnetic loxodrome direction is used. This leads to a bending of the flight path which does not lend itself to precise analytical descriptions.

The use of maps in an international projection, on which an orthodrome with a length up to 1200 km is practically represented by a straight line, as well as the use of radio-engineering facilities for aircraft navigation by orthodrome bearings, led to the position that the orthodrome line of the path of an aircraft is practically maintained without being dependent on a system of measuring directions on the Earth's surface. In this case, a strictly loxodromic line is not taken as the loxodrome, but rather an orthodrome section with an indication of the mean path angle.

Application of precise gyroscopic and astronomical navigational devices generally eliminates the necessity of using the loxodrome direction of flight. Therefore, in studying the properties of a loxodrome on the Earth's surface, we are limited only by the small amount of available geometric information.

As we already know, a *loxodrome* is a line on the Earth's surface which joins two points and intersects the meridians at a constant angle. /64

In general, a loxodrome is a spiral line which goes to the Earth's poles. As a result of this, it has curvature not only in a vertical plane, but in a horizontal plane as well. Meridians, the equator, and parallels which are also loxodrome lines, expressed in the first two cases by a great circle and in the last case by a small circle on the Earth's surface, are the exception.

The curvature of a loxodrome in a horizontal plane increases sharply with an approach to the Earth's poles. As a result, it is not used at all for flights in polar latitudes.

Let us determine the curvature of a loxodrome, its extension, and its deflection as compared to the orthodrome direction at a given latitude ϕ.

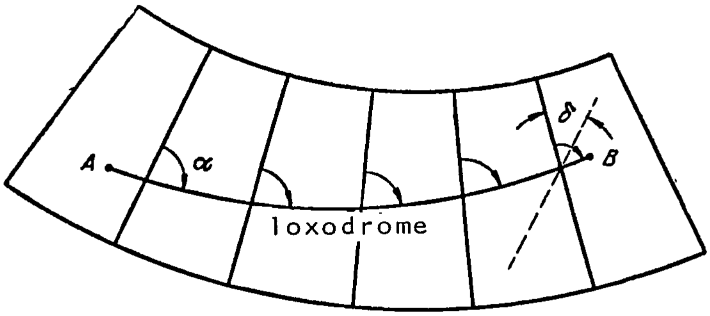

Fig. 1.41. Loxodrome on a Map of a Conic Projection.

The maximum curvature of a loxodrome at a given flight altitude will occur when the flight is in an easterly or westerly direction, and it will vanish in a flight to the north or south.

Let us assume that a flight at altitude ϕ occurs in an easterly direction. In this case, the angle of turn of the loxodrome from point A to point B will be equal to the angle of convergence of the meridians (δ) between these points (Fig. 1.41).

$$\delta = -(\lambda_B - \lambda_A)\sin\varphi,$$

Its length (S) from point A to B will be

$$S = (\lambda_B - \lambda_A)\cos\varphi,$$

where λ_A and λ_B are the longitudes of points A and B and ϕ is the mean latitude between points A and B.

The radius of curvature of the loxodrome (r_λ) can be determined as the ratio of the length of part (S) to the angle of turn (δ). /65
If we take the radius of the Earth as 1, then

$$r_\lambda = \frac{S}{\delta} = \operatorname{ctg}\varphi. \qquad (1.48)$$

The part of the loxodrome which runs along the meridian does not have a horizontal curvature. Therefore, if the loxodrome passes at an angle to the meridian, the radius of its curvature at any point will equal:

$$r = r_\lambda \operatorname{cosec}\alpha = \operatorname{ctg}\varphi \operatorname{cosec}\alpha. \qquad (1.49)$$

Example: Determine the radius of curvature of a loxodrome passing at an angle of 30° to the meridian at a latitude of 45°.

Solution:

$$r = R_3 \operatorname{ctg} 45° \operatorname{cosec} 30° = 2R_3 = 12742 \text{ km}$$

where R_3 is the radius of the Earth.

The curvature of the loxodrome in a horizontal plane creates some lengthening of the straight-line parts of the path. The lateral deviations from the line of the orthodrome direction may turn out to be very significant here.

In Figure 1.42, the straight line *AB* is the orthodrome; arc *AB* is the loxodrome; δ is the angle of turn of the loxodrome from point *A* to point *B*. The length of the straight line is

$$AB = 2R \sin \frac{\delta}{2},$$

while the length of the arc is

$$AB = R\delta.$$

Lengthening of the path along the loxodrome (ΔS) is determined by the formula

$$\Delta S = R\delta - 2R \sin \frac{\delta}{2}. \tag{1.50}$$

Example: Determine the lengthening of the path along the loxodrome passing through points *A* and *B* on the Earth's surface, with the following coordinates: A: latitude 55° N, longitude 38° E; B: latitude 55° N, longitude 68° E.

Since the latitude of the starting and end points is the same, the direction of the loxodrome coincides with the Earth's parallel at a latitude of 55°.

The radius of curvature of the loxodrome will be:

$$r = r_\lambda = R_з \operatorname{ctg} 55° = 6371 \cdot 0{,}7002 = 4461 \text{ km}$$

The angle of turn of the loxodrome is determined by the formula

$$-\delta = (\lambda_2 - \lambda_1) \sin \varphi; \quad \delta = -30° \sin 55° = -30° \cdot 0{,}8192 = 24{,}576°,$$

Then

$$\sin \frac{\delta}{2} = \sin 12°17' = 0{,}2127.$$

Substituting the value of the radius of curvature and the angle of turn of the loxodrome into (1.50), we obtain: /66

$$\Delta S = \frac{24{,}576 \cdot 4461}{57{,}3} - 2 \cdot 4461 \cdot 0{,}2127 \approx 15{,}6 \text{ km}$$

From this example, it is obvious that at middle latitudes, with flight paths up to 2,000-3,000 km long, the curvature of the loxodrome creates relatively small lengthenings of the path (in our example, less than 1%); however, in approaching the polar latitudes, lengthening of the path will increase, together with a decrease in the radius of curvature of the loxodrome.

Significant lengthenings of the path along the loxodrome occur at middle latitudes with very long distances between points on the Earth's surface. For example, at a latitude of 40°, with a distance of 11,000 km between points, lengthening of the path along the loxodrome can exceed 4,000 km, i.e., more than 30%.

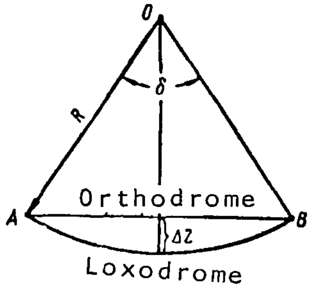

Fig. 1.42. Radius of Curvature of a Loxodrome.

In Figure 1.42, it is obvious that with a constant radius of curvature of the loxodrome, its greatest discrepancy with respect to the orthodrome (deflection) will be observed at half the path between points A and B.

Here,

$$\Delta Z = R - R\cos\frac{\delta}{2}$$

or

$$\Delta Z = R\left(1 - \cos\frac{\delta}{2}\right). \quad (1.51)$$

In the example analyzed by us,

$$\Delta Z = 4461\,(1 - 0{,}9771) = 102{,}6 \text{ km}$$

Thus, the discrepancy between the loxodrome line of the path and the orthodrome, even at comparatively small distances between points on the Earth's surface, will be very substantial. This is the basic cause of the limitation of the length of the loxodrome segments of the path.

In the practice of aircraft navigation, since the loxodrome direction of flight is used only in limited path segments, the azimuth of the orthodrome (α), measured on the central meridian between the starting and end points of the segment is taken as the loxodrome direction of the flight.

This angle can also be determined on the basis of the approximate formula /67

$$\operatorname{tg}\alpha = \frac{\lambda_2 - \lambda_1}{\varphi_2 - \varphi_1}\cos\varphi_{av} \quad (1.52)$$

The length of the loxodrome segment of the path (S) is determined by the formula

$$S = \frac{\varphi_2 - \varphi_1}{\cos\alpha} \quad (1.53)$$

or

$$S = \frac{\lambda_2 - \lambda_1}{\sin\alpha}\cos\varphi_{av} \quad (1.54)$$

Formulas (1.54) and (1.52) are approximate and have a simple geometrical interpretation.

Formula (1.53) is derived analytically.

Considering that the loxodrome intersects the meridians at a constant angle, the ratio remains constant:

$$\frac{dS}{d\varphi}=\frac{1}{\cos\alpha},$$

from which

$$S=\frac{1}{\cos\alpha}\int_{\varphi_1}^{\varphi_2} d\varphi=\frac{\varphi_2-\varphi_2}{\cos\alpha}.$$

In the majority of cases, in calculating the distance along the loxodrome, it is more advantageous to apply (1.53). However, with loxodrome directions close to 90 or 270°, the values $\phi_2 - \phi_1$ and cos α simultaneously approach zero. This leads to large arithmetic errors in calculation and ultimately to an ambiguity in the solution. In these cases, it is more advantageous to use (1.54), the errors in which will be negligibly small, since a small difference in the latitudes between the points means that the mean cosine of the latitude becomes practically equal to the cosine of the mean latitude.

Example: Determine the loxodrome direction and the distance between points *A* and *B* on the Earth's surface, the coordinates of which are: *A*: latitude 56° N, longitude 38° E; *B*: latitude 68° N, longitude 47° E.

Solution: According to (1.38), let us find the direction of the loxodrome:

$$\operatorname{tg}\alpha=\frac{47-38}{68-56}\cos 62^\circ=0{,}3521;\quad \alpha=19^\circ 24'.$$

Let us determine the loxodrome distance according to (1.53):

$$S=111{,}1\,\frac{68-56}{\cos 19^\circ 24'}=1413\text{ km}$$

Loxodrome on Maps of Different Projections /68

A loxodrome has the appearance of a straight line only on maps of a normal isogonal cylindrical projection.

On maps of normal isogonal conic and azimuthal projections, the loxodrome is a curved line intersecting the meridians at a constant angle α. Therefore, knowing the direction of the loxodrome in order to draw it on a map it is sufficient at the starting point to plot this direction up to the intersection with the next meridian, where the indicated direction must be extended to the next meridian in line. Continuing our plotting to the final point, we will obtain a broken line very close to the loxodrome.

On maps with nonisogonal projections, the loxodrome will have a variable angle to the meridians, which depends on the ratio of the scales

$$\mathrm{tg}\,\alpha_m = \mathrm{tg}\,\alpha \frac{n}{m}, \tag{1.55}$$

where α is the angle of intersection of the loxodrome with the meridian at a location; α_M is the angle of intersection of the loxodrome with the meridian on a map; n and m are the scales of a map at a given point along the principal directions east-west and north-south, respectively.

For example, on maps with an equally spaced normal cylindrical projection, where $\frac{n}{m} = \sec\phi$,

$$\mathrm{tg}\,\alpha_m = \mathrm{tg}\,\alpha \sec\varphi,$$

i.e., the loxodrome will have a curvature in the direction of a pole, whereas it has a natural curvature in the direction of the equator.

General Recommendations for Measuring Directions and Distances

Orthodrome directions and distances for straight-line segments of a path of more than 1200-1500 km in all cases must be determined by analytical means, independently of the scales and map projections used. With a length of the path segments of more than 2000 km, the intermediate points of the orthodrome must also be determined in such a way that the distance between them does not exceed 800-1000 km.

On short path segments (up to 1200-1500 km), the methods of determining directions and distances depend on the scale and projection of the maps, as well as on the means and methods of aircraft navigation used. For example, in using precise automatic navigational devices, it is always advantageous to use analytical /69
forms to solve these problems.

It is possible to carry out direct measurement of distances and directions on maps by means of a scale and protractor, with the length of the path segments being not more than 1500 km if these maps are executed on an international polyconic projection and have a scale of 1:1,000,000 or 1:2,000,000 (the latter within the limits of one (or, in extreme cases, two) adjoining sheets).

We must note that good results in measuring directions and distances can be obtained on route maps constructed on oblique cylindrical or oblique conic projections when the flight direction coincides with or is located close to the axis of the route map. However, in directions at an angle to the axis of the route map, the results of measurements are significantly worse than on maps with an international polyconic projection.

In using maps constructed with all other projections, only the analytical form of determining distances and directions, with calculation of intermediate points along the orthodrome after every 200-300 km of the path, must be applied.

The loxodromic flight direction can be measured directly only on maps with an isogonal normal cylindrical projection. Here, segments of distances up to 300-400 km on this projection can be measured by means of a varying scale located on the edge of the map.

On maps in other projections, generally speaking, there is no need to measure and plot the loxodrome line of the path in parts of more than 300-400 km.

Since the loxodromic flight direction in short path segments is used as the mean orthodrome direction, it is considered equal to the orthodrome as indicated by the mean meridian between the starting and end points of the path segment.

In view of the fact that in short segments of the path the loxodrome line does not show significant deviation from the orthodrome as a rule, it is not plotted on maps but is considered coincident with the direction of the orthodrome.

7. Special Coordinate Systems on the Earth's Surface

In the practice of aircraft navigation, rectangular and geographic coordinate systems are insufficient, and it is necessary to use at least three or four coordinate systems simultaneously.

Actually, elements of aircraft movement are examined in a moving rectangular coordinate system. The center of a rectangular system moves in one of the surface coordinate systems which is connected with the given flight path, which in turn is determined in a geographic coordinate system. /70

The indicated order of the connection of the coordinate systems is minimal. For some purposes, it is advantageous to examine aircraft movement relative to the airspace, i.e., a supplementary coordinate system whose center shifts in the moving rectangular system.

With the use of gyroscopic devices as well as astronomical ones, it is necessary to use a universal (stellar) coordinate system. The use of radio-engineering navigational facilities is connected with the use of a whole series of special types of surface coordinates by which the position of the aircraft on the Earth's surface is determined.

Let us examine the most important surface coordinate systems used in aircraft navigation.

Orthodromic Coordinate System

The orthodromic coordinate system for calculating the path of an aircraft is the one most widely used at the present time.

In this system, the direction of the straight-line path segment (Fig. 1.43) is taken as the main axis X. The line perpendicular to the X-axis and also situated in the plane of the horizon is the second axis, Z.

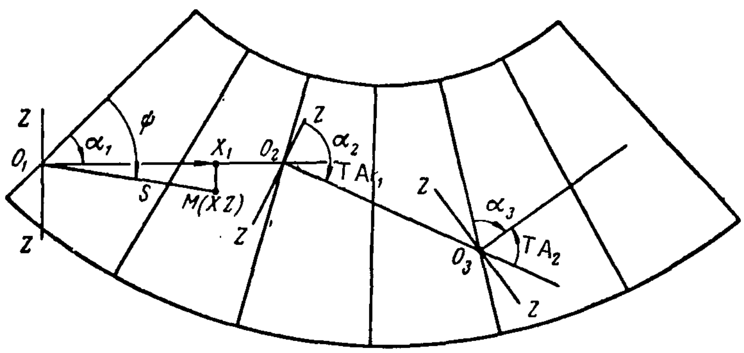

Fig. 1.43. Orthodromic Coordinate System.

In Figure 1.43, angles α_1 and α_2 are the directions of the first and second straight-line segments of the path, measured from the meridians of their starting points. Points O_1 and O_2 are the starting points of the segments, the coordinates of which are determined in the geographic coordinate system. The orthodrome dis- /71
tances O_1O_2 and O_2O_3 are the lengths of the straight-line segments; the angle TA_1 is the angle of turn of the orthodromic coordinate system at point O_2.

Since an aircraft moving above the Earth's surface in a given direction has only small random deviations from the given flight path (as a rule, not more than 20-30 km), it is possible to take the spherical surface of the Earth within the area of the possible deviations of the aircraft from the X-axis of the orthodrome system as a cylindrical surface. Then the unrolling of the cylinder gives us a rectangular system XZ on a plane.

Let us assume that an aircraft moves from point O_1 at a small angle to the O_1X_1 axis equal to $\psi - \alpha_1$, and covers a distance S.

The coordinates of the aircraft at point M_{xz} are determined by the equations:

$$\left.\begin{aligned} X_a &= S\cos(\psi - \alpha_1); \\ Z_a &= S\sin(\psi - \alpha_1). \end{aligned}\right\} \quad (1.56)$$

Measuring the X_a coordinate constitutes checking of the path of the aircraft according to distance, while measuring the Z_a coordinate constitutes checking of the path according to direction.

Periodic measurement of the X_a and Z_a coordinates make it possible to determine all the basic elements of aircraft movement; for example:

a) Direction of aircraft movement (ψ):

$$\psi = \text{arctg}\,\frac{Z_{a_2} - Z_{a_1}}{X_{a_2} - X_{a_1}} + \alpha, \quad (1.57)$$

where X_{a1}, Z_{a1} are coordinates of the aircraft at the first point. X_{a2}, Z_{a2} are coordinates of the aircraft at the second point;

b) Speed of aircraft movement along a given flight path (W)

$$W = \frac{X_{a_2} - X_{a_1}}{t} \quad (1.58)$$

where t is the flying time of the aircraft between points X_{a_1} and X_{a_2};

c) Remaining flying time to point O_2

$$t_{rem} = \frac{X_{rem}}{W}, \quad (1.59)$$

where $X_{rem} = O_1O_2 - X_a$.

d) Necessary flight direction for arrival at point O_2:

$$\psi = \alpha_1 - \text{arctg}\,\frac{Z}{X_{rem}}. \quad (1.60)$$

Formulas (1.55) to (1.60) are entirely obvious and do not require special derivations or proofs.

To refine the coordinates of the aircraft in the orthodrome system, we can use correction points (CP), visual or radar landmarks on the Earth's surface, locations of ground radio facilities, etc. (Fig. 1.44). /72

Translator's note: arctg = $\cot^{-1}$.

If the correction point is observed from an aircraft at an angle θ to the given route, at a distance from the aircraft equal to R, the coordinates of the aircraft will be determined by the formulas:

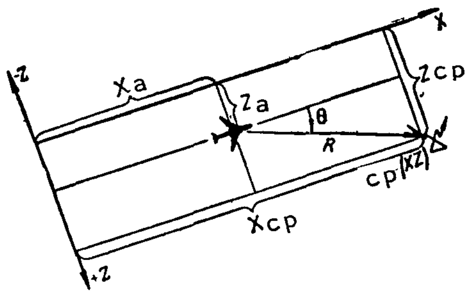

Fig. 1.44. Determining the Orthodromic Coordinates of an Aircraft from a Correction Point.

$$\left.\begin{aligned} X_a &= X_{cp} - R\cos\theta; \\ Z_a &= X_{cp} - R\sin\theta. \end{aligned}\right\} \qquad (1.61)$$

During flight over the correction point, i.e., when this point is observed at an angle equal to 90° to the flight path, (1.61) is simplified and takes the form:

$$X_a = X_{cp}$$
$$Z_a = Z_{cp} - R.$$

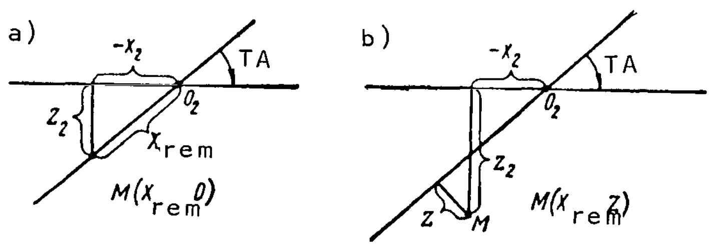

Fig. 1.45. Transfer of the Next Stage in a Course to an Orthodromic Coordinate System: (a) with the Aircraft Position on the Path of the Given Course; (b) with Deviation of the Aircraft from the Path of a Given Course.

The simplicity of the geometrical transformations and the natural perception of the coordinates of the aircraft in an orthodrome system, both of the path covered by the aircraft and of the deviation allowed from the given path, make it the most acceptable coordinate system for a given flight path.

In high-speed aircraft (as a result of a large turning radius), in order to emerge without deviation at the next stage of the orthodrome path, it is necessary to consider the linear advance to the angle of turn (*TA*). This transfer is connected with transform- /73
ations of the coordinates of the aircraft from the orthodrome system of the preceding stage to the system of the following stage.

In Figure 1.45, a, point *M* located on the flight path of the preceding stage of flight is the point of the beginning of turn for arrival at the flight path of the following stage. Obviously, the coordinates of this point in the system of the following stage will be equal to

$$\left.\begin{aligned} -X_2 &= X \,\text{rem} \cos \text{TA}; \\ Z_2 &= X \,\text{rem} \sin \text{TA}. \end{aligned}\right\} \quad (1.62)$$

In general, when the coordinate Z at the beginning of the turn is not equal to zero, i.e., if the aircraft is not located strictly on the given flight path when beginning the turn, the transformation of the coordinates must be carried out according to the following formula (Fig. 1.45, b):

$$\left.\begin{aligned} X_2 &= Z \sin \text{УП} - X \,\text{rem} \cos \text{TA}; \\ Z_2 &= Z \cos \text{УП} + X \,\text{rem.} \sin \text{TA}. \end{aligned}\right\} \quad (1.63)$$

In the process of turning, the coordinates of the aircraft are measured in the system of the following part of the flight in which their calculation after turning is carried out.

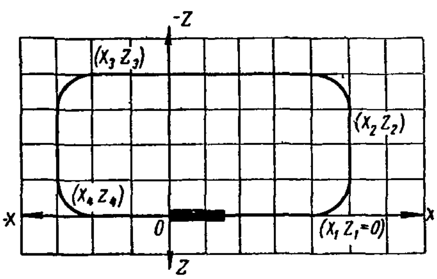

Fig. 1.46. Rectangular Coordinate System for Flight over an Area.

The orthodrome system examined by us is sometimes called the *stage orthodromic coordinate system.* In some instances, a rectangular coordinate system is used for flight over an area, e.g., for maneuvering of an aircraft in the region of an airport and for special-purpose flights, etc. In these cases, the direction of the meridian at the point of origin of the coordinates or some other direction [for example, the direction of the take-off-landing zone at an airport (Fig. 1.46)] is taken as the X-axis, and a rectangular coordinate system is constructed from this.

The flight is carried out along the given coordinates of the points of the route [for example, along the coordinates of the beginning of each of the four turns in the rectangular maneuver of making an approach to land at an airport $(X_1 Z_1)$, $(X_2 Z_2)$, $(X_3 Z_3)$, $(X_4 Z_4)$].

The limits of applicability of an areal rectangular coordinate system are limited by the effect of the sphericity of the Earth on the precision of measurements. In practice, without noticeable distortions, such a system can be used within a radius of 300-400 km /74
from the point of origin of the coordinates.

It is also applied with the use of navigational indicators in flight, when the orthodrome direction of part of the course is taken as the X axis.

Arbitrary (Oblique and Transverse) Spherical and Polar Coordinate Systems

In the solution of navigational problems with a geographical coordinate system in polar regions, very significant errors arise.

A special chapter is devoted to problems of accuracy in aircraft navigation. In the present section, for the purpose of illustration, only (1.36a) is examined.

It is obvious that with the approach of the aircraft to a latitude equal to 90°, the tangent ϕ will approach infinity. Therefore, small errors in measuring the latitude of the location of the aircraft will cause the errors in calculating the longitude to grow indefinitely.

To avoid a loss of accuracy in solving navigational problems, especially by automatic navigational devices, random spherical coordinate systems are employed.

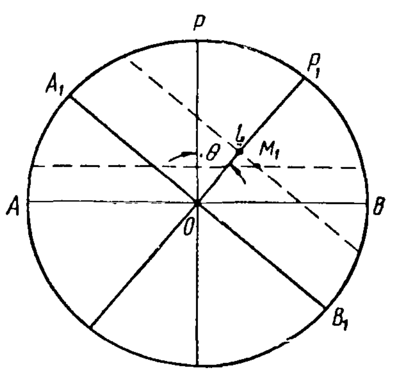

Fig. 1.47. Transformation of Spherical Coordinates on the Earth's Surface.

Arbitrary spherical systems differ from a geographical system by the fact that the poles of these systems do not coincide with the geographic poles. Therefore, in these systems all the analytical transformations of distances and directions which are carried out for a geographic coordinate system are justified.

For transferring from a geographical coordinate system to an arbitrary spherical system, or vice versa, it is necessary to derive special equations: let us examine Figure 1.47.

In Figure 1.47, a cross section of the Earth's sphere is shown. Here the plane of the cross section is chosen in such a way as to pass through the center of the Earth and the poles of the geographic system and the arbitrary coordinate systems, i.e., so as to appear as the plane of the meridian in both the geographic and arbitrary systems simultaneously. It is obvious that such a plane exists with /75
any distribution of the poles of an arbitrary spherical system.

Let us agree that a reading of the longitude both in the geographical and arbitrary systems will run from the indicated plane of intersection. Lines AB and A_1B_1 in Figure 1.47, and the lines parallel to them, appear as lines of intersection with the planes of the equator and the parallels in the geographical and arbitrary systems. Point P is the pole of the geographic coordinate system; P_1 is the pole of the arbitrary system; angle θ is a combination of the axes of the geographical and random systems.

Let us choose point M ($\phi_1 \lambda_1$) on the Earth's surface and project it onto the plane of the cross section (point M_1). It is obvious that OL will appear as the line of the sines of the latitude of point M in an arbitrary system, while LM_1 will appear as the line of the cosines of the longitude of point M of this system in the plane of its parallel, i.e.,

$$LM_1 = \cos\lambda_1 \cos\varphi_1.$$

The latitude of point M in the geographical system will equal:

$$\sin\varphi = OL \cos\theta - LM_1 \sin\theta$$

or

$$\sin\varphi = \sin\varphi_1 \cos\theta - \cos\lambda_1 \cos\varphi_1 \sin\theta. \tag{1.64}$$

It is obvious that a perpendicular dropped from point M to the plane of intersection (point M_1) will be the line of the sine of the longitude in the arbitrary system in the plane of the parallel of this point; at the same time, the line of the sine of the longitude in the plane of the parallel of the point in the geographic coordinate system will be

$$\sin\lambda = \sin\lambda_1 \cos\varphi_1 \sec\varphi.$$

from which it follows that

$$MM_1 = \sin\lambda_1 \cos\varphi_1 = \sin\lambda \cos\varphi, \tag{1.65}$$

Formulas (1.64) and (1.65) make it possible to determine the coordinates of a point in a geographical coordinate system according to its coordinates, known in the arbitrary system under the condition that the plane coinciding with the axes of both systems is taken as the initial meridian. After solving the problems according to (1.64) and (1.65), it is necessary to introduce a correction into the λ coordinate equal to the longitude of the P_1 pole in the geographic coordinate system.

Since the principles of construction of spherical and geographic coordinate systems are identical, for the solution of the reverse task (transferring from the geographic system to the arbitrary one), it is sufficient to drop the subscripts in the functions of the coordinates of (1.64) and (1.65) whereever they occur and to add them where they are absent:

$$\sin\varphi_1 = \sin\varphi \cos\theta - \cos\lambda \cos\varphi \sin\theta;$$

$$\sin\lambda_1 = \sin\lambda \cos\varphi \sec\varphi_1.$$

Formulas (1.64) and (1.65) were given with a consideration of the flattening of the Earth at the poles, i.e., the Earth was taken as a sphere with a mean radius. /76

Position Lines of an Aircraft on the Earth's Surface

Thus far, we have examined coordinate systems on the Earth's surface as systems which connect the position of an aircraft with the Earth's surface during its movement in a given direction.

In aircraft navigation, it is often necessary to determine the elements of aircraft movement according to consecutive coordinates. It is obvious that means and methods for measuring the coordinates of an aircraft are necessary for this purpose.

Usually the two-dimensional surface coordinates of an aircraft are determined separately according to two lines of the aircraft's position measured at different times or according to two lines measured simultaneously. In some cases, it is sufficient to determine one line of the aircraft's position.

The geometric locus of points of the probable location of an aircraft on the Earth's surface is called the *position line of an aircraft*. Similar groups of aircraft position lines are called a *family of position lines*.

For example, if the latitude of the location of an aircraft is determined by astronomic means based on the elevation of Polaris, the parallel on which the aircraft is located will be a position line of the second family.

Let us assume that the longitude of an aircraft was determined simultaneously on the basis of the altitude of a star, the azimuth of which is equal to 90 or 270°. The longitude obtained by such a method is a position line of the second family.

Direct measurement of the geographic coordinates of an aircraft is possible only by astronomic means, and not in all cases.

In determining the location of an aircraft by optical or radiometric means, the families of position lines generally do not coincide either with the grid of geographic coordinates or with the given flight direction.

At the present time, there are several types of coordinate systems which are used as families of aircraft position lines in the application of radio-engineering and astronomic facilities of aircraft navigation. They include the following:

1) *A two-pole azimuthal system*, in which the radial lines (*bearings*) diverging from two points on the Earth's surface with known coordinates are families of position lines.

2) *Polar or azimuthal range-finding system*, in which the bearings from a point on the Earth's surface with known coordinates are /77
the first family of position lines of this system, and concentric

circles at equal distances from the indicated point are the second family.

3) *Lines of equal azimuths* (LEA), which are position lines relative to known points on the Earth's surface, at each of which the azimuth of a known point retains a constant value.

4) *Difference-rangefinding (hyperbolic system)*, in which each family of position lines is bipolar; a constant difference of distances to the poles of the system is preserved on each position line.

5) *Over-all rangefinding (elliptical) system*, in which the family of position lines is bipolar; a constant sum of the distances to the poles of the system is preserved on the position lines.

6) *confocal hyperbolic-elliptical system*, in which the families of position lines are ellipses and hyperbolas confocal with them.

From the above list of coordinate systems, it is evident that each has arisen from the nature of the navigational values measured by the devices used. The indicated values are called *navigational parameters*.

For example, for a hyperbolic system the difference in distances serves as a navigational parameter, and in an azimuthal system (or for lines of equal azimuths) the azimuth serves as a navigational parameter, etc.

In evaluations of the accuracy of navigational measurements, considering that the intersecting segments of the position lines of any system can be assumed to be straight-line segments in the region of the location of the aircraft, the concept of a *unified coordinate system* is sometimes introduced for the purpose of studying the general properties of all the above systems, including the geographic and orthodrome systems.

In studying these coordinate systems, it is necessary to connect each of them with the geographic system for locating the intermediate points of the position lines, in order to plot them on a map. In addition, it is necessary to know the analytical form for determining the coordinates of an aircraft in a geographic or orthodrome system on the basis of known parameters of navigational systems without plotting position lines on the map, as is done in automatic navigational devices.

Bipolar Azimuthal Coordinate System

Bearings for an aircraft, i.e., orthodrome lines diverging from two points on the Earth's surface with known coordinates, are position lines in the azimuthal coordinate system.

Let us assume that we have two points O_1 and O_2 on the Earth's surface (Fig. 1.48).

If the map being used has been executed on a projection having the properties of isogonality and orthodromicity, e.g., on an inter- /78 national projection, the indicated position lines on the map can be taken as straight lines originating at points O_1 and O_2.

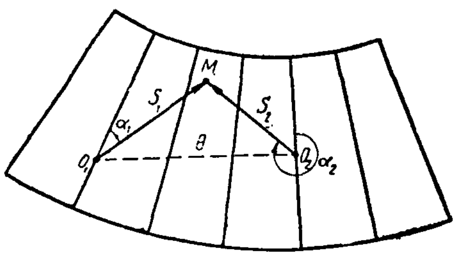

Fig. 1.48. Bipolar Azimuthal Coordinate System.

However, satisfactory accuracy in determining the coordinates of an aircraft at the intersection of the bearings as straight lines on a map is preserved at comparatively small distances and only on maps with an international projection.

In general, for the precise plotting of position lines on a map, let us consider the points O_1 and O_2 as poles of an arbitrary spherical coordinate system. Let us consider the distances S from these points to any point on the Earth's surface M as complements of the latitude of point M in these coordinate systems, up to 90°:

$$S_1 = O_1M = 90° - \varphi_1; \quad S_2 = O_2M = 90° - \varphi_2.$$

In this case, the coordinates of point M in the geographical system are determined according to (1.64) and (1.65).

Taking the meridian of point O_1 as the prime meridian of the geographic system, O_1 as the azimuth α for the longitude in the spherical system, and a value of 90° for S_1 as the latitude in this system, let us obtain (in the geographical coordinate system)

$$\sin\varphi = \cos S_1 \cos\theta - \sin\alpha_1 \sin S_1 \sin\theta,$$

where

$$\theta = 90° - \varphi_{0_1};$$

$$\sin\lambda = \sin\alpha_1 \sin S_1 \sec\varphi.$$

Given the definite value S_1 and substituting different values for α_1, e.g., greater than 1°, formulas let us use the given formulas to find the coordinates of the points of intersection of the azimuthal lines with the circle of equal distance S_1 in the geographic system.

Given another value for S_1 and having carried out the same operations with α_1, we will obtain the coordinates of the points of intersection of the azimuthal lines with a circle of equal distance having this radius.

Continuing to increase S_1 to a full radius of operation of a navigational device, let us obtain the coordinates of the intermediate points of the azimuthal position lines, running from pole O_1 in the geographical coordinate system, with the longitude changed to the value λ_{O1}. Introducing a correction in the values of the longitudes of the intermediate points for the indicated value λ_o, let us obtain the longitudes of these points from the prime meridian of the geographic system. On a map of any projection, by joining the points obtained by lines running from point O_1, we will obtain position lines of the first family. /79

In this way, it is possible to obtain the family of position lines from point O_2, taking it as the pole of the second arbitrary spherical coordinate system.

Let us now determine the coordinates of point M in the geographical system, based on known azimuths measured at points O_1 and O_2, without recourse to the plotting of position lines. Let us first solve this problem in the spherical system of one of the poles of a navigational device, e.g., O_2 (see Fig. 1.48), taking the azimuth of point O_1 as the prime meridian.

According to (1.64) and (1.65), the coordinates of point M in this system are determined by the equations:

$$\sin\varphi_1 = \sin\varphi_2\cos\theta - \cos\lambda_2\cos\varphi_2\sin\theta,$$

where Θ is the angular distance of O_1O_2;

$$\sin\lambda_1 = \sin\lambda_2\cos\varphi_2\sec\varphi_1,$$

where $\lambda_1\lambda_2$ are respectively α_1, α_2.

From (1.65) it is evident that

$$\frac{\sin\lambda_1}{\sin\lambda_2} = \frac{\cos\varphi_2}{\cos\varphi_1} \quad \text{or} \quad \frac{\sin\lambda_2}{\sin\lambda_1} = \frac{\cos\varphi_1}{\cos\varphi_2}. \qquad (1.66)$$

Substituting into (1.52), instead of cos ϕ_1 its value according to (1.64), we obtain:

$$\frac{\sin\lambda_2}{\sin\lambda_1} = \operatorname{tg}\varphi_2\cos\theta - \cos\lambda_2\sin\theta$$

or

$$\operatorname{tg}\varphi_2 = \sin\lambda_2\operatorname{cosec}\lambda_1\sec\theta + \cos\lambda_2\operatorname{tg}\theta. \qquad (1.67)$$

Since the azimuth of point M in the O_2 system is considered known, we obtained both coordinates of point M in this system.

For transferring to the geographical coordinate system, it is again possible to use (1.64) and (1.65), considering as angle Θ the value ϕ_{O2}, and as the prime meridian the longitude of the point O_2.

It is obvious that here it is necessary to introduce a correction into the λ_2 coordinate for the value of the azimuth of point O_1 from point O_2, i.e., the corrected value of λ_2 will equal:

$$\lambda_{2C} = \lambda_2 + \alpha_{O_1}. \qquad (1.68)$$

It is also obvious that after transforming the coordinates into geographical ones, it is necessary to introduce a correction into coordinate λ_2 for the longitude of point O_2: /80

$$\lambda_C = \lambda + \lambda_{O_2}. \qquad (1.69)$$

Formulas (1.64) and (1.65) also make it possible to implement a transfer from a spherical system with pole O_2 to the orthodrome system. This is necessary for determining the position of an aircraft relative to a given flight path. Actually, it is possible to consider the orthodrome system as a spherical system if we measure the X- and Z-coordinates not as linear but as angular measures, i.e., we take the X-coordinate as λ and the Z as ϕ.

In this instance, it is advantageous to take the X-coordinate of point O_2 as the prime meridian and the value 90° - Z of this point as the angle Θ. The coordinates of point M in the orthodrome system will then equal:

$$\sin X = \sin\varphi_2 \cos\theta - \cos\lambda_2 \cos\varphi_2 \sin\theta;$$
$$\sin Z = \sin\lambda_2 \cos\varphi_2 \sec X.$$

If the X_0-coordinate, not equal to X_{O2}, is taken as the prime meridian, then after transforming the coordinates according to (1.64) and (1.65) a correction equal to X_{O_2} is introduced in the X_M coordinate.

Goniometric Range-Finding Coordinate System

The goniometric range-finding system is the most convenient system for conversion to the geographic or orthodrome system.

Since direction and distance are measured simultaneously in this system, for conversion to the geographical system it is sufficient to use (1.64) and (1.65), taking the value 90° - ϕ_{O1} for angle Θ and λ_{O1} for the prime meridian.

In this instance the value 90° - S is considered the latitude of the point M in the coordinate system with pole O_1, while the azimuth of point M is considered as the longitude. In the geographical system, the coordinates of the point M will equal:

$$\sin\varphi = \sin\varphi_1 \cos\theta - \cos\lambda_1 \cos\varphi_1 \sin\theta;$$
$$\sin\lambda = \sin\lambda_1 \cos\varphi_1 \sec\varphi.$$

After transformation, it is necessary to introduce a correction to the coordinate λ for the longitude of point O_1:

$$\lambda_C = \lambda + \lambda_{O_1}. \qquad (1.70)$$

The conversion to the orthodromic coordinate system is implemented in the same manner as was done in the bipolar azimuthal system after solving (1.67).

If the radius of action of the boniometer range-finding coordinate system is small (on the order of 300-400 km), it is possible to disregard the sphericity of the Earth in converting to the orthodrome system and the problem of transfer is considerably simplified (Fig. 1.49). /81

In Figure 1.49, it is evident that with known values of R and α in the goniometer range-finding system, the coordinates of point M in the orthodrome system can be determined according to the following formulas:

$$X = X_{O_1} + R\cos(\alpha - \psi); \qquad (1.71)$$

$$Z = Z_{O_1} + R\sin(\alpha - \psi), \qquad (1.71a)$$

where ψ is the direction of the orthodrome segment of the path relative to point O_1.

Bipolar Range-Finding (Circular) Coordinate System

In a bipolar range-finding system (Fig. 1.50), the distance to two points on the Earth's surface with known coordinates is a measured navigational parameter.

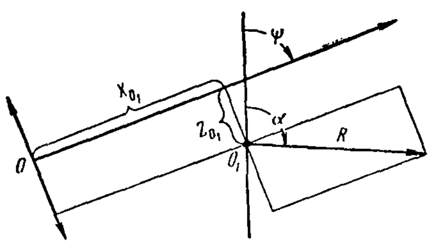

Fig. 1.49. Conversion of Polar (Goniometer-Range-Finding) Coordinates to Orthodromic Coordinates.

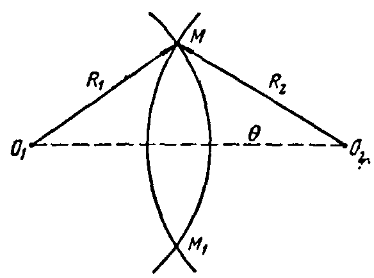

Fig. 1.50. Bipolar Range-Finding Coordinate System.

The indicated distance is usually determined according to the time of passage of radio signals from the aircraft to the ground radio-relay equipment and back to the aircraft.

In Figure 1.50, it is evident that the task of determining the coordinates of an aircraft in a circular system is double-valued. The point of intersection of the circles of equal distance to the poles O_1 and O_2 is considered the location of the aircraft. Since there are two such points for any pair of circles, additional signs are used for choosing the actual point, e.g.:

a) Provisional aircraft position at the moment of measurement.

b) Tendency toward a change in distance during flight in a definite direction.

In Figure 1.50, it is obvious that in flying from north to south, the distances R_1 and R_2 will decrease at point M and increase at point M_1. /82

On maps with different projections, circular position lines will have a different appearance. Usually they are plotted on maps on an oblique central and international projection on which the approximate form of the circles is preserved.

For plotting the indicated lines on a map with any projection, it is necessary to determine the coordinates of their intermediate points. This problem is solved in the same way as for bipolar azimuthal systems, with the sole difference being that after determining the coordinates of intermediate points, the latter are not joined by radial position lines, but by circular lines.

In converting from a circular to a geographic or orthodrome system, it is necessary first to determine the coordinates of point M in the spherical system relative to one of the poles of the circular system.

Considering the line O_1O_2 as the initial meridian of this system, the latitude of point M in the system O_1 according to (1.64) will be

$$\sin\varphi_1 = \sin\varphi_2 \cos\theta - \cos\lambda_2 \cos\varphi_2 \sin\theta,$$

where θ is the angular distance between points O_1 and O_2; ϕ_1; ϕ_2 are $90° - R_1$ and $90° - R_2$, respectively.

Carrying out simple transformations, we obtain:

$$\cos\lambda_2 = \frac{\sin\varphi_2 \cos\theta - \sin\varphi_1}{\cos\varphi_2 \sin\theta}. \qquad (1.72)$$

Formula (1.72) makes it possible to determine the λ-coordinate in the O_2 system. Since the ϕ-coordinate in this system is determined directly as $90° - R_2$, it is possible to consider the problem solved.

The conversion to the geographic or orthodrome system is implemented by the same means as in the azimuthal and goniometer range-finding systems.

Lines of Equal Azimuths

Lines of equal azimuths (LEA) are a family of aircraft position lines which converge at one point on the Earth's surface, on each of which the azimuth of the known point retains a constant value (Fig. 1.51).

For finding the location of an aircraft, it is used along one line of equal azimuths of two families, as is done along two bearings in an azimuthal bipolar system.

Lines of equal azimuths were widely used in the period when the radiocompass (aircraft radiogoniometer), measuring the distance from the aircraft to the ground radio station, was the most refined navigational facility.

Along with lines of equal azimuths, a method of determining the coordinates of an aircraft by plotting bearings from a radio station to an aircraft (taking account of the convergence of the meridians between them) has become widespread. /83

An advantage of the lines of equal azimuths, in comparison with bearings for an aircraft, is the fact that the solution of the problem of determining an aircraft's coordinates is independent of its location, whereas in order to plot bearings it is necessary to know the approximate coordinates of the aircraft for calculating the convergence of the meridians.

In examining lines of equal azimuths, there is no sense in deriving an analytical form of transformations for converting to the

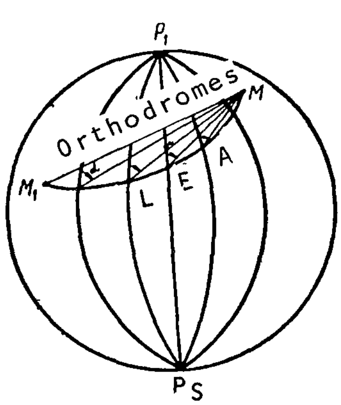

Fig. 1.51. Line of Equal Azimuths (LEA).

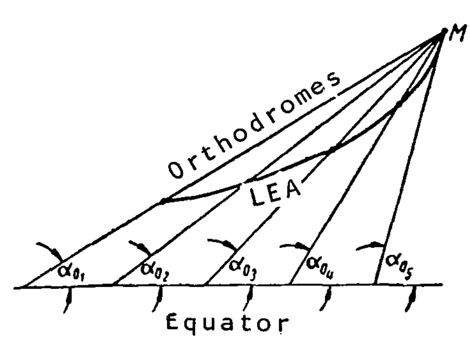

Fig. 1.52. Determining the Coordinates of Intermediate Points of an LEA.

geographic or orthodromic coordinate system. Let us limit ourselves to an examination of the means of calculating intermediate points for plotting them on a map in order to make it possible to determine the coordinates of an aircraft according to the intersection of the lines of equal azimuths of two families on a map with any projection.

In Figure 1.52, one of the lines of equal azimuths of a family converging at point *M* is shown. At this point, the orthodromes intersecting the equator at different angles α_{0_1}, α_{0_2}, etc. converge.

According to (1.32), it is possible to find the longitude λ_0 of the points of intersection of a family of orthodromes with the equator, given the values of the initial angles α_0.

According to (1.44),

$$\cos\varphi_B = \cos\varphi \sin\alpha.$$

Or since $\cos\phi_B = \sin\alpha_0$,

$$\cos\varphi = \frac{\sin\alpha_0}{\sin\alpha}. \qquad (1.73)$$

Formula (1.73) makes it possible to determine the latitude of a point on any line of the family of orthodromes which converge at point *M*, where the azimuth of point *M* is equal to the given value of α. /84

The longitude of the indicated point can be determined according to (1.36a) by substituting into it the given initial angle of the orthodrome and the latitude obtained from (1.73). It is obvious that the longitude obtained will be measured from the starting points of the family of orthodromes. Therefore, to reduce it to the geographic system, it is necessary to introduce a correction for the longitude of the indicated initial points.

Having solved this problem for every value of α_0 with given values of α, let us obtain the intermediate points of the family of lines of equal azimuths.

The problem of determining the coordinates of intermediate points on lines of equal azimuths of the second family, whose plotting on a map yields a grid of intersecting aircraft position lines, is solved analogously.

Difference-Range-Finding (Hyperbolic) Coordinate System

The circular range-finding system of aircraft position lines examined earlier is used with comparatively small distances from the ground radio-engineering equipment to the aircraft, since the sending and radio-relaying of radio signals to the aircraft over great distances involves technical difficulties.

The technical solution of the problem is greatly simplified if, instead of relaying aircraft radio signals, we send simultaneous radio signals from two ground radio-engineering installations, with their subsequent reception by the aircraft.

However, in this instance it is advantageous to measure not the absolute distances from the ground installations to the aircraft, but only the difference in distances to them.

The *system of position lines* for the difference in the distances to two points on the Earth's surface is called the *difference-range-finding or hyperbolic system.*

The geometric locus of points, the difference in whose distances to two given points (foci) is a constant value equal to $2a$ (Fig. 1.53), is called a *hyperbola.*

The distance along the focal axis from the point of intersection of the focal and conjugate axes to the peak of the hyperbola is the value "a". It is possible to designate hyperbolic aircraft position lines on a map by doubling the value of "a" as an ordinal /85
number. The distance along the focal axis from the focus to the intersection with the conjugate axis is designated by the value "c".

To determine the position of an aircraft, two families of hyperbolic position lines constructed from three points forming pairs of focal axes are usually used. At each of these points, ground radio-engineering installations for synchronous transmission of radio signals are established.

In order to plot hyperbolic position lines on a map with any projection, the intermediate points of hyperbolas in the spherical system of one of its foci, e.g. F_1, are determined first. Here the direction F_1F_2 is taken as the initial meridian of this system.

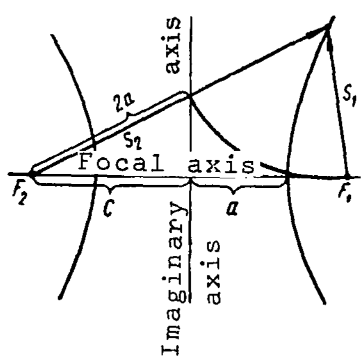

Fig. 1.53. Difference-Range-Finding (Hyperbolic) Coordinate System.

Bearing in mind the fact that the latitude of any point in the F_1 system equals $90° - S_1$ and $90° - S_2$ in the F_2 system, the value $S_2 = S_1 + 2a$, and taking the distance F_1F_2 as the angle θ, it is possible to write (1.64) in the following form:

$$\cos(S_1 + 2a) = \cos S_1 \cos 2c - \cos\lambda_1 \sin S_1 \sin 2c,$$

Hence,

$$\cos\lambda_1 = \frac{\cos S_1 \cos 2c - \cos(S_1 + 2a)}{\sin S_1 \sin 2c}. \qquad (1.74)$$

Given the definite values of S as circles of equal radius and changing the values of $2a$, it is possible to determine the value of λ of all the hyperbolas of the family at points of intersection with the indicated circles.

For conversion to the geographical coordinate system, the intermediate points are recalculated according to (1.64) and (1.65), after which they are plotted on a map and joined by smooth lines.

Hyperbolic coordinate systems are usually used in the application of radionavigational devices with a large effective radius. Therefore, the automatic conversion of the hyperbolic coordinates to geographical or orthodromic coordinates is advantageous.

The problem indicated is solved comparatively easily when all three foci of the hyperbolic system are situated on one orthodrome line (Fig. 1.54).

According to (1.74),

$$\cos\lambda_1 = \frac{\cos S_1 \cos 2c_1 - \cos(S_1 + 2a_1)}{\sin S_1 \sin 2c_1} = \frac{\cos S_1 \cos 2c_2 - \cos(S_1 + 2a_2)}{\sin S_1 \sin 2c_2}$$

Expanding the value of the cosines of the sum of the angles and carrying out a reduction, we obtain:

$$\frac{\cos 2c_1 - \cos 2a_1 + \operatorname{tg} S_1 \sin 2a_1}{\sin 2c_1} = \frac{\cos 2c_2 - \cos 2a_2 + \operatorname{tg} S_1 \sin 2a_2}{\sin 2c_2}.$$

Multiplying both sides of the equation by $\sin 2c_1 \cdot \sin 2c_2$ and rearranging the terms, we obtain: /86

$$\cos 2c_1 \sin 2c_2 - \cos 2c_2 \sin 2c_1 - \sin 2c_2 \cos 2a_1 + \sin 2c_1 \cos 2a_2 = \operatorname{tg} S_1 (\sin 2c_1 \sin 2a_2 - \sin 2c_2 \sin 2a_1)$$

or

$$\operatorname{tg} S_1 = \frac{\sin 2c_2 (\cos 2c_1 - \cos 2a_1) - \sin 2c_1 (\cos 2c_2 - \cos 2a_2)}{\sin 2c_1 \sin 2a_2 - \sin 2c_2 \sin 2a_1}. \qquad (1.75)$$

The task is simplified even more if the distances FF_1 and FF_2 which are chosen are identical, i.e. $2c_1 = 2c_2$. In this case

$$\operatorname{tg} S_1 = \frac{\cos 2a_2 - \cos 2a_1}{\sin 2a_2 - \sin 2a_1}. \qquad (1.75a)$$

Formulas (1.75) are used for determining the coordinates of an aircraft in a spherical system with the pole at point F, bearing in mind that $\phi = 90° - S_1$.

The λ-coordinate with a known value of S_1 is easily determined on the basis of (1.74). For conversion to the geographic or orthodrome system, the same formulas (1.64) and (1.65) are used.

The problem of conversion to the spherical (and consequently, to the geographic coordinate system) if the foci of the hyperbolic system are not located on one orthodrome line (Fig. 1.55), is much more complicated to solve.

It is obvious that (1.74) can be reduced to the form:

$$\cos\lambda_1 = \frac{\cos S_1 \cos 2c_1 - \cos S_1 \cos 2a_1 + \sin S_1 \sin 2a_1}{\sin S_1 \sin 2c_1}.$$

Carrying out simple transformations, we obtain:

$$\cos\lambda_1 = \operatorname{ctg} S_1 \operatorname{ctg} 2c_1 - \operatorname{ctg} S_1 \cos 2a_1 \operatorname{cosec} 2c_1 + \sin 2a_1 \operatorname{cosec} 2c_1$$

or

$$\operatorname{ctg} S_1 = \frac{\cos\lambda_1 - \sin 2a_1 \operatorname{cosec} 2c_1}{\operatorname{ctg} 2c_1 - \cos 2a_1 \operatorname{cosec} 2c_1}. \quad (1.76)$$

In Fig. 1.55 it is evident that in the FF_2 system $\lambda_M = \lambda_1 + \beta$; therefore, the following equation is valid: /87

$$\operatorname{ctg} S_1 = \frac{\cos(\lambda_1 + \beta) - \sin 2a_2 \operatorname{cosec} 2c_2}{\operatorname{ctg} 2c_2 - \cos 2a_2 \operatorname{cosec} 2c_2} \quad (1.76a)$$

Designating the second terms of the numerators of (1.76a) by X and the denominators by Y and reducing to a common denominator, we obtain:

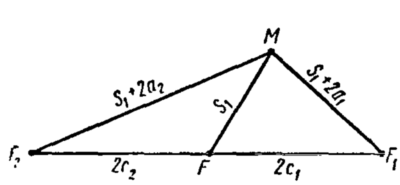

Fig. 1.54. Conversion of Hyperbolic Coordinates to Spherical Coordinates (Special Case)

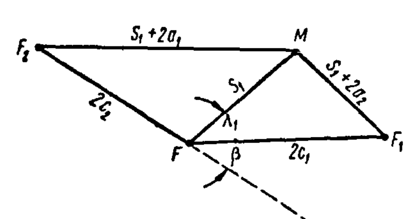

Fig. 1.55. Conversion of Hyperbolic Coordinates to Spherical Coordinates (General Case).

or

$$(\cos\lambda_1 - X_1)\,Y_2 = [\cos(\lambda_1 + \beta) - X_2]\,Y_1$$

$$Y_2\cos\lambda_1 - Y_2X_1 = Y_1\cos\lambda_1\cos\beta - Y_1\sin\lambda_1\sin\beta - Y_1X_2.$$

Rearranging the terms, replacing sin λ, by $\sqrt{1 - \cos^2\lambda}$, and squaring both sides of the equation we obtain:

$$\cos\lambda_1(Y_2 - \cos\beta Y_1) - X_1Y_2 + X_2Y_1 = -\sqrt{1-\cos^2\lambda_1}\,\sin\beta Y_1$$

or

$$\cos^2X_1(Y_2^2 - 2Y_1Y_2\cos\beta + Y_2^2) + 2\cos\lambda_1(X_2Y_1 + X_1Y_2)\times$$
$$\times(Y_2 - Y_1\cos\beta) + (X_2Y_1 - X_1Y_2)^2 - \sin^2\beta Y_1^2$$

Thus, the coordinate λ_1 is determined by the solution of a quadratic equation

$$\cos\lambda_1 = -\frac{(X_2Y_1 - X_1Y_2)(Y_2 - \cos\beta Y_1)}{Y_1^2 - 2Y_1Y_2\cos\beta + Y_2^2} \pm$$
$$\pm\frac{\sqrt{\begin{array}{c}(X_2Y_1 - X_1Y_2)^2(Y_2 - \cos\beta Y_1)^2 - \\ -(Y_1^2 - 2Y_1Y_2\cos\beta + Y_2^2)[(X_2Y_1 - X_1Y_2)^2 - \sin^2\beta Y_1^2]\end{array}}}{Y_1^2 - 2Y_1Y_2\cos\beta + Y_2^2} \qquad (1.77)$$

where

$$X_1 = \sin 2a_1 \operatorname{cosec} 2c_1;$$
$$X_2 = \sin 2a_2 \operatorname{cosec} 2c_2;$$
$$Y_1 = \operatorname{ctg} 2c_1 - \cos 2a_1 \operatorname{cosec} 2c_1;$$
$$Y_2 = \operatorname{ctg} 2c_2 - \cos 2a_2 \operatorname{cosec} 2c_2.$$

There is no sense in substituting the indicated values X_1, X_2, Y_1, and Y_2 into (1.77), since this hampers its solution greatly. In practice, it is easier to determine the numerical values of these magnitudes first, on the basis of the known values of $2a_1$, $2a_2$, $2c_1$, and $2c_2$, and to substitute them into (1.77).

Knowing the coordinate λ makes it possible to determine easily the coordinate ϕ_1, e.g., according to (1.76), keeping in mind the fact that $\phi_1 = 90° - S_1$, and then to convert to the geographic or orthodrome system using (1.64) and (1.65).

Overall-Range-Finding (Elliptical) Coordinate System /88

Hyperbolic navigational systems are the most easily implemented technologically of all the range-finding systems. However, from the point of view of use in flight, they are geometrically disadvantageous.

Both families of position lines are divergent, and at distances exceeding $2c$ from the center of the system are practically directed along the radii of this center. This leads to an increase in error in determining aircraft coordinates with an increase in the distance from the center.

In addition, with an increase in distance, the angle of intersection of the hyperbolic lines of the two families decreases. This also lowers the accuracy of determining the aircraft coordinates.

Combination of hyperbolic position lines with elliptical lines turns out to be more advantageous (Fig. 1.56).

It is known that the geometrical place of points, the sum of whose distances to two given points (foci) is a constant magnitude equal to $2a$, is called an *ellipse*. The distance along the major axis from its intersection with the minor axis to the top of the ellipse, i.e., its semi-axis, is considered to be the value "a", in this case. The distance from the intersection of the axes to the foci is considered to be the "c" value.

If, in addition to the difference in distances to the foci, the distance to one of them is measured, it is easy to implement the hyperbolic-elliptical system of position lines.

Actually, if one distance S_1 is known and the difference in the distances $\Delta S = 2a_h$, obviously the distance to the second focus is

$$2a_e = 2S_1 + 2a_h$$

where a_e is the major semi-axis of the ellipse and a_h is the parameter a of the hyperbola.

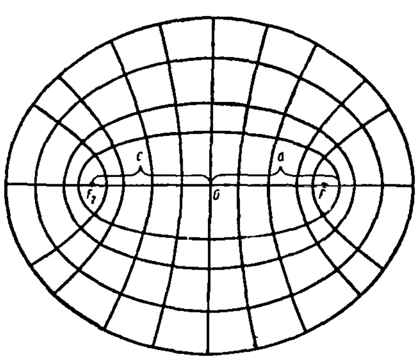

Fig. 1.56. Hyperbolic-Elliptical Coordinate System.

Obvious advantages of the hyperbolic-elliptical system include the following:

(a) There is an absence of divergence in the second family of position lines. The elliptical position lines are closed, so that the accuracy of determining the aircraft's position along them does not decrease with an increase in distance.

(b) Orthogonality of the /89

position lines appears at any point of the system. In a confocal hyperbolic-elliptical system, the position lines intersect only at a right angle.

(c) Two foci, instead of the three for a hyperbolic system, are sufficient for the construction of a confocal hyperbolic-elliptical system. This leads to a simplification of the transformations during conversion to a geographic or orthodrome system.

However, in spite of the advantages of a hyperbolic-elliptical system indicated above, a wide distribution was not obtained. This was connected with great technical difficulties in measuring distance to points on the Earth's surface at distances exceeding straight-line geometric visibility of the object from flight altitude.

The above problem is solved by keeping on board the aircraft a reference frequency (quartz-crystal clock) which permits synchronization of the transmission of radio signals from the ground with reference signals on board the plane. It is therefore possible to determine the travel time of the signals.

Hence, strict stabilization of the reference frequency on board the aircraft is the main task for the technical implementation of hyperbolic-elliptical systems.

To plot elliptical lines on a map with any projection, the intermediate points are determined according to the same formulas as the family of hyperbolas. For example, in (1.74), considering the second distance in it (S_2) not as the sum $S_1 + 2a_h$, but as the difference $2a_e - S_1$,

$$\cos\lambda_1 = \frac{\cos S_1 \cos 2c - \cos(2a - S_1)}{\sin S_1 \sin 2c}, \tag{1.78}$$

where λ_1 is an angle with the vertex at point F_1, measured from the major axis of the ellipse.

Given the different values of S_1 and determining the values of λ_1 for each of them with a constant value of $2a$, we shall obtain intermediate points of an ellipse in a spherical system F_1.

Changing the value of $2a$ and performing these operations with S_1, we obtain intermediate points of the next elliptical position line, etc. Recalculation of intermediate points is implemented in the geographic system according to (1.64) and (1.65) as in other cases analyzed by us.

In the hyperbolic-elliptical system, the conversion to the geographic or orthodromic coordinate system is very simple.

In fact, the value of S_1 and the parameter $2a$ in this system are measured. Therefore, (1.78) is useful for the problem of calculating the spherical coordinates of an aircraft along measured parameters and for a subsequent transfer to the geographic or orthodrome system.

8. Elements of Aircraft Navigation /90

Aircraft flights are carried out in airspace. The physical composition of airspace, as well as the speed and direction of its shift relative to the Earth's surface, exert a substantial influence on the trajectory of aircraft movement in a geographic or orthodromic coordinate system.

Until recently, the direct measurement of the speed and direction of aircraft movement relative to the Earth's surface was a problem. At the present time, this problem has been solved. However, it is not advisable to install the complex and expensive equipment which measures the indicated parameters on all aircraft.

In solving navigational problems, parameters of aircraft movement relative to the airspace are usually measured to the greatest extent possible, and then additional parameters of the movement of the aircraft which are connected with movement in airspace are found.

Summarizing the measured parameters of aircraft movement, the value and direction of the speed vector of the aircraft relative to the Earth's surface are found.

The parameters of aircraft movement with which we must concern ourselves in carrying out aircraft navigation are called *elements of aircraft navigation*.

Elements of aircraft navigation are divided into three groups which determine the direction, speed, and altitude of flight.

Elements which determine Flight Direction

The basic element which determines the direction of aircraft movement in airspace is called the aircraft course.

The *aircraft course* (generally designated by γ) is the angle between the direction of a meridian on the Earth's surface and the direction of the longitudinal axis of the aircraft in a horizontal plane.

Usually it is considered that the airspeed vector of an aircraft in the plane of the horizon coincides with the direction of the longitudinal axis of the aircraft, although this is actually not entirely true. Therefore, an understanding of the course often coincides with an understanding of the direction of the flight airspeed vector.

Depending on the reference system chosen, the following special varieties of aircraft courses can be distinguished:

(a) A *true course* (TC) is measured from the northern end of a geographic meridian which passes through the point of intersection of the Earth's surface with the vertical of the aircraft. The latter is usually called the position point of the aircraft (PA):

(b) The *orthodrome course* (OC) is measured from the northern end of a geographic meridian of the starting point of a rectilinear /91 (orthodrome) segment of the path or from another conditionally chosen (reference) meridian along which the zero point of the course-reading scale is established.

(c) The *magnetic course* (MC) is read from the northern end of the magnetic meridian which passes through point PA.

In addition to these varieties of aircraft courses, there is another concept, the *compass course* (CC), i.e., a course based on the responses of a compass. In textbooks on aircraft navigation, the concept of compass course has included only magnetic compasses, but we have broadened this concept to include all methods of measuring an aircraft course with an instrument.

Aircraft courses are measured by three different methods, i.e., stabilization of the zero reading of the compass along the meridians:

Magnetic course, by means of magnetic systems.

True course, by means of astronomical systems.

Reference course, by means of gyroscopic devices.

All of these methods have instrumental errors or a *deviation* designated by Δ_c. Individual components of errors in the course devices are components of the deviation.

Any of the three types of aircraft courses can be obtained from responses of a course device, allowing for its deviation, e.g.,

$$\left.\begin{aligned} OC &= CC_{orth} + \Delta_c \\ TC &= CC_{astr} + \Delta_c \\ MC &= CC_{mag} + \Delta_c \end{aligned}\right\} \qquad (1.79)$$

In the general case,

$$\gamma = CC + \Delta_c$$

As a correction for any measurement, the value Δ_c is considered positive when the compass underestimates the value of the measured magnitude, and negative when the compass readings are too high.

In the future, when we study the relationship among the three types of aircraft courses, we will consider that the value of each has been corrected for the deviation of the device.

The interrelationship between magnetic and true flight courses is established with the least difficulty, since these courses are measured from the meridians which pass through point PA (Fig. 1.57a).

In Fig. 1.57a the northern geographic meridian is designated by P_n, the direction of the magnetic meridian by P_M.

Since the magnetic meridian is shifted to the left relative to the geographic meridian, the magnetic declination at the given point is negative. With a positive deviation, the magnetic meridian is shifted to the right relative to the geographic meridian. /92

From the figure, it is evident that

$$\left.\begin{array}{l} MC = TC - \Delta_M; \\ TC = MC + \Delta_M. \end{array}\right\} \quad (1.80)$$

In the case of Δ_M, the value is negative; therefore, the absolute value of the true course turns out to be less than the magnetic course.

Converting from the true or magnetic course to the orthodrome course (or vice versa) is more complex (Fig. 1.57, b).

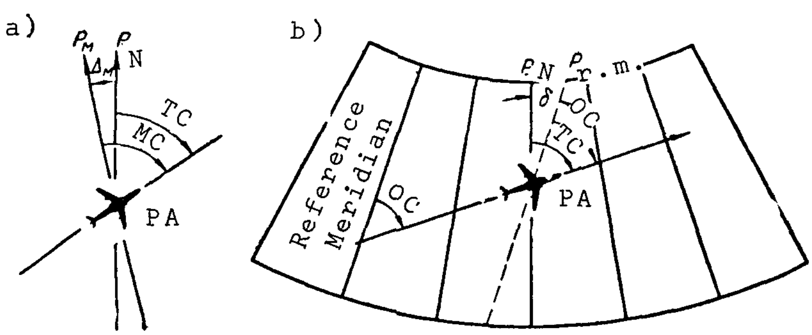

Fig. 1.57. Interrelationship of Aircraft Courses; a: Magnetic and True; b: True and Orthodrome.

The direction of the geographic meridian passing through point PA is designated by P_n; the direction of the reference meridian $P_{r.m.}$ is shown by a dotted line which intersects the geographic meridian at angle δ. Therefore

$$\left.\begin{array}{l} TC = OC - \delta; \\ OC = TC + \delta. \end{array}\right\} \quad (1.81)$$

The value δ, the angle of convergence of the meridians, is considered positive when the direction of the geographic meridian at point PA is proportional to the reference meridian extended clockwise (to the right) and negative when the geographic meridian is shifted to the left.

On small path segments (500-600 km), the angle of convergence of the meridians is approximately equal to:

$$\delta = (\lambda_{r.m.} - \lambda_{PA}) \sin \varphi_{av} \qquad (1.82)$$

In general, for converting from the orthodrome course to a true course or vice versa, it is necessary to determine the longitude of the starting point of the orthodrome of each rectilinear path segment, e.g., on the basis of (1.34), or, if the azimuth of the orthodrome is known at the starting point of the path segment, on the basis of (1.33a). /93

The angle of convergence of the meridians between the starting point of the path section M_1 and any moving point M on a section, according to (1.33a), will be equal to

$$\delta = \alpha - \alpha_1 = \operatorname{arctg}(\operatorname{tg}\lambda \operatorname{cosec}\varphi) - \operatorname{arctg}(\operatorname{tg}\lambda_1 \operatorname{cosec}\varphi_1), \qquad (1.83)$$

where λ and λ_1 are measured from the starting point of the orthodrome.

If the longitude of any other point on the Earth's surface, e.g., the point of take-off of the aircraft, is taken as the reference meridian, the angle of convergence of the meridians will be determined as the sum:

$$\delta = \delta_1 + \delta_2 + \ldots \delta_c$$

where δ_1; δ_2 ... are the angles of convergence of the meridians between the starting and end points of the preceding path segments determined on the basis of (1.83); δ_c is the angle of convergence of the meridians from the starting point to the moving point of the last path segment (on the basis of the same formula).

The angle of convergence of the meridians calculated in this way allows conversion from a true course to an orthodrome course and vice versa at any flight distance with any number of breaks in the path.

For conversion from an orthodrome course to a magnetic course and vice versa, (1.80) and (1.81) are used, from which it follows that

$$\left.\begin{aligned} OC &= MC + \Delta_M + \delta; \\ MC &= OC - \Delta_M - \delta. \end{aligned}\right\} \quad (1.84)$$

The sum of the magnitudes $\Delta_M + \delta$ is taken as the overall correction for conversion from a magnetic to an orthodrome course and vice versa, and is designated by Δ.... Then (1.84) assumes the form:

$$OC = MC + \Delta;$$
$$MC = OC - \Delta.$$

The *drift angle* is the second element determining the direction of aircraft movement.

In an aircraft, the angle between the airspeed vector and the groundspeed vector in a horizontal plane is called the *drift angle* (Fig. 1.58). In general, the drift angle is designated by the Latin letter *a*. In those instances when special designations for courses are used in the solutions of navigational problems the drift angle is designated by the Russian letters for DA.

In Fig. 1.58. OP_N is the direction of the meridian at point PA; $\overline{OV}$ is the direction of the airspeed vector and the longitudinal axis of the aircraft; $\overline{OW}$ is the direction of the groundspeed vector relative to the Earth's surface; $\bar{u}$ is the wind speed vector. /94

The drift angle of an aircraft is considered positive when the groundspeed vector (vector of aircraft movement relative to the Earth's surface) is further to the right of the longitudinal axis of the aircraft and negative if it is further to the left.

The angle between the northern end of the meridian and the groundspeed vector or the vector of the speed of the aircraft relative to the Earth's surface is called the *flight angle* (FA). The general designation for the flight angle is ψ.

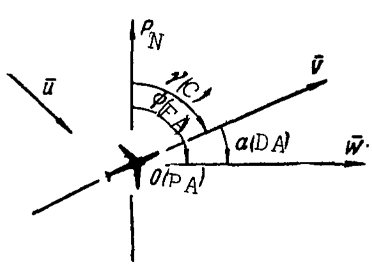

Fig. 1.58. The Path Angle of Flight.

The flight angle, like the course of the aircraft, can be measured from the reference meridian, the geographic meridian, and a magnetic meridian passing through point PA.

Special values of the flight angles have the following designations:

(a) The orthodrome flight angle is OFA, with obligatory indication by a subscript of the longitude of the reference meridian. For example, $OFA_{40} = 96°$.

(b) The true flight angle is TFA.

(c) The magnetic flight angle is MFA.

In Figure 1.58, it is evident that the flight angle is generally

$$\psi = \gamma + a$$

or in special cases:

$$\left.\begin{aligned} OFA &= OC + DA; \\ TFA &= TC + DA; \\ MFA &= MC + DA; \end{aligned}\right\} \qquad (1.85)$$

The interrelationship between the special values of the flight angles and the method of conversion from one special value to another corresponds completely to the interrelationship between special values of aircraft courses:

$$\begin{aligned} OFA &= TFA + \delta = MFA + \Delta; \\ TFA &= OFA - \delta = MFA + \Delta_m; \\ MFA &= OFA - \Delta = TFP - \Delta_m. \end{aligned}$$

In determining the direction of aircraft movement relative to the Earth's surface, it is sufficient to know the course of the aircraft as the angle between the direction of the meridian and the lateral axis of the aircraft, and the drift angle as the angle between the lateral axis of the aircraft and the direction of its movement. These elements, together with elements of flight speed, make it possible to determine approximately the speed and direction of the wind at flight altitude. /95

For precisely determining the wind at flight altitude, it is necessary to separate the part of the drift angle of an aircraft caused by the wind. It is obvious that to do this it is necessary

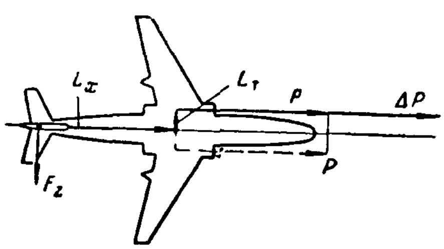

Fig. 1.59. Moment and Control Force with Assymetry of Engine Thrust.

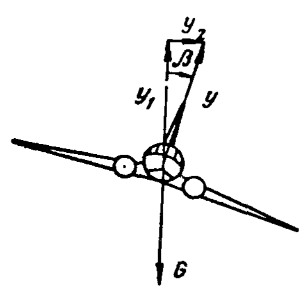

Fig. 1.60. Lateral Glide with Transverse Roll.

to determine the direction of the airspeed vector of the aircraft as the course and glide of dynamic origin, arising in flight.

There are several causes of lateral glide in aircraft during flight. The basic causes are the following.

1. Assymetry of the Engine thrust or Aircraft Drag (Fig. 1.59)

Let us assume that with symmetrical drag, one of the engines has a somewhat greater thrust than the other. The difference in thrust ΔP will produce torque in the aircraft relative to the vertical axis, i.e., the course of the aircraft will change.

For stabilizing the flight direction, a moment must be applied to the empennage of the aircraft which is equal in magnitude and opposite in direction to the moment of thrust, i.e.

$$F_z L_x = -\Delta P L_T,$$

where ΔP is the assymetry of the thrust; F_z is the control force; L_T is the arm of thrust assymetry (from the axis of the engine to the axis of the aircraft); and L_x is the arm of control (from the center of the empennage area to the center of gravity of the aircraft) /96

The lateral force which causes gliding of the aircraft will be:

$$F_z = \frac{\Delta P L_T}{L_x}. \qquad (1.86)$$

In an analogous manner, the force which causes gliding of an aircraft with assymetry of drag arises.

In this instance, the moment of rotation of an aircraft causes excess drag on one wing of the aircraft. The distance from the lateral axis of the aircraft to the center of its application is the arm of this force.

2. Allowable Lateral Banking of an Aircraft in Horizontal Flight.

With allowable lateral banking (Fig. 1.60), the horizontal component of the lift will appear:

$$Y_z = G \operatorname{tg} \beta,$$

where G is the weight of the aircraft and β is the angle of lateral banking.

For example, with a flying weight of the aircraft of 75 t, the allowable banking in horizontal flight, equal to 1°, causes a lateral component of lift ≈ 1.3 t.

3. Coriolis Force

During flight in the Earth's atmosphere, as a result of the diurnal rotation of the Earth's surface, a lateral Coriolis force acts on the aircraft:

$$F_C = 2\omega_e Wm \sin\varphi, \qquad (1.87)$$

where ω_e is the angular velocity of the Earth's rotation; W is the speed of the aircraft relative to the Earth's surface; m is the mass of the aircraft; and ϕ is the latitude of the point PA.

4. Two-dimensional Fluctuations in the Aircraft Course

During two-dimensional rolls (without banking), an aircraft (as a result of inertia) tries to maintain the initial direction of movement. This causes lateral gliding of the aircraft V_z, equal to:

$$V_z = V \sin\Delta\gamma, \qquad (1.88)$$

where V is the speed of the aircraft relative to the airspace, and $\Delta\gamma$ is the magnitude of the change in the aircraft's course.

The indicated lateral gliding of an aircraft gradually dies down as a result of acceleration caused by the lateral airflow over its surface.

5. Gliding During Changes in the Lateral Wind Speed Component at Flight Altitude /97

This type of gliding arises as a result of the inertia of the aircraft. First, lateral airflow over the aircraft or something similar (gliding in airspace) will appear, followed by a change in the direction of aircraft movement.

From the above five examples of lateral aircraft gliding, constant lateral forces are the causes of the gliding in the first three cases, while abrupt beginning and gradual diminution of gliding are the causes in the last two cases.

The magnitude of stable gliding with a constantly acting lateral force can be determined according to the formula

$$Z = c_z S \frac{\rho V^2}{2} \qquad (1.89)$$

or

$$V_z = \sqrt{\frac{2Z}{c_z S\rho}}$$

where Z is the operative lateral force; c_z is the coefficient of lateral drag of the aircraft; S is the area of the longitudinal section of an aircraft with a vertical plane; and ρ is the mass density of the air flight altitude.

To calculate gliding in flight, it must be integrated and converted to angular glide (α_{gl}):

$$\mathrm{tg}\, \alpha_{gl} = \frac{V_z}{V_x}, \tag{1.90}$$

where V_z is the lateral component of the airspeed and V_x is the longitudinal component of longitudinal speed.

The direction of the airspeed vector is determined by the formula

$$\gamma_V = \gamma + \alpha_{gl} \tag{1.91}$$

The drift angle of an aircraft, whose cause is the action of the wind at flight altitude, will be:

$$\alpha = \psi - \gamma_V = \psi - \gamma - \alpha_{gl} \tag{1.92}$$

As we have already said, determining the gliding of an aircraft in airspace is necessary only for precise measurements of wind speed and direction at flight altitude. For the purposes of aircraft navigation, there is no need to separate out the causes of lateral aircraft movement.

Elements Which Characterize the Flight Speed of an Aircraft

The flight speed of an aircraft is measured both relative to the airspace surrounding the aircraft and relative to the Earth's surface.

Measuring the speed of aircraft movement relative to the air- /98
space is significant both from the point of view of flight aerodynamics (stability and control of the aircraft) and from the point ov view of aircraft navigation.

It is known that the lift of a wing, the drag of an aircraft, and the stability and controllability of an aircraft depend on the square of the airspeed.

For example, at flight speeds which are significantly less than the speed of sound, the drag of an aircraft is determined by the formula

$$Q = c_x S \frac{\rho V^2}{2},$$

where Q is the lateral drag of an aircraft, c_x is the drag coefficient, S is the maximum area of the lateral cross section of an aircraft, and ρ is the mass air density at flight altitude.

The value $\frac{\rho V^2}{2}$ characterizes the aerodynamic pressure of the atmosphere on the surface of an aircraft.

All the aerodynamic characteristics of an aircraft are determined relative to this value.

In determining the aerodynamic characteristics of an aircraft, the aerodynamic pressure (and therefore the speed of flight) reduce to conditions in a standard atmosphere, i.e., to flight conditions near the Earth's surface, with an atmospheric pressure of 760 mm Hg and an ambient air temperature of 15° C. Therefore, speed indicators which measure airspeed on the basis of aerodynamic pressure are calibrated according to the parameters of a standard atmosphere.

With an increase in flight altitude, air density decreases. To preserve aerodynamic pressure at flight altitude, it is necessary to increase flight airspeed, although responses of the airspeed indicator which measure airspeed on the basis of aerodynamic pressure remain constant.

Flight airspeed which is measured on the basis of aerodynamic pressure and which influences the aerodynamics of the flight of the aircraft is called *aerodynamic speed* (V_{aer}).

It is necessary to consider, however, that with an increase in flight speed, especially in approaching the speed of sound, aerodynamic speed does not completely correspond to the aerodynamic characteristics of an aircraft which are determined under the conditions of a standard atmosphere and which are inherent in flight speed. This is becuase the factor of air compressibility begins to exert an influence. To ensure safe pilotage of the aircraft in these instances, a corresponding correction is introduced into the indications of aerodynamic speed.

For the purposes of aircraft navigation, it is necessary to know the actual speed of an aircraft in space. /99

The actual airspeed, which we shall call simply *airspeed* (V), can be obtained from aerodynamic speed by introducing corrections for the change in air density with flight altitude and temperature:

$$V = V_{aer} + \Delta V_H + \Delta V_t + \Delta V_{acmp}.$$

where V_{aer} is the aerodynamic speed; ΔV_H is the correction for speed as a result of flight altitudes; ΔV_t is the correction for speed as a result of air temperature; and $\Delta V_{a.cmp.}$ is the correction for speed as a result of air compressibility

Correction for flight altitude is of basic importance. Correction for air temperature is significantly smaller and is introduced only in those cases when the air temperature at flight altitude is significantly different from the temperature calculated for this altitude.

At the present time, there are devices which indicate flight airspeed directly, taking altitude into account. Corrections must be introduced in the responses of these devices only for instrumental errors of the devices and (in individual cases) for a discrepancy between the actual air temperature and the calculated temperature at a given altitude.

In published textbooks on aircraft navigation, airspeed has been classified as "indicated" (measured on the basis of aerodynamic pressure) but true, and as "indicated , corrected for methodological and instrument errors."

Since there are now devices which measure both these speeds, each of them is "indicated". In addition, increasing airspeeds have required the introduction of corrections in aerodynamic flight speed. This has caused a new classification of air speeds.

The speed of aircraft movement relative to the Earth's surface is called *flight groundspeed* (W).

Flight groundspeed can be measured directly by means of Doppler or inertial systems, determined by sighting along a series of landmarks on the Earth's surface, and also calculated on the basis of flying time between two landmarks on the Earth's surface. In addition, groundspeed can be determined by adding the airspeed and wind vectors, if the wind speed and direction at flight altitude are known.

Navigational Speed Triangle

The interrelationship of the elements of flight direction and speed in the chosen frame of reference of aircraft courses is clearly illustrated by a navigational speed triangle.

In Fig. 1.61 a navigational speed triangle is shown for a general case, i.e., independently of the meridian which is used as the basis for measuring an aircraft course. /1

Straight lines OP_N and O_1P_N in the figure show the direction of the meridian at point PA; V is the airspeed vector; W is the groundspeed vector; γ is the course of the aircraft (C), α is the drift angle (DA); ψ is the flight angle (FA), δ is the direction of the wind vector relative to the meridian for reading the aircraft course; δ_ψ is the flight angle of the wind (WA) read from the given line of the path; and $\delta_\gamma = \alpha + \delta_\psi$ is the course wind angle (CWA), read from the longitudinal axis of the aircraft.

A speed triangle can be solved graphically by construction of vectors on paper or by a mechanical apparatus, using a special device (a wind-speed indicator which is a combination of rules, dials, and hinges with movable and immovable joints).

A speed triangle is solved analytically on the basis of a known sine theorem. From Figure 1.63, it is clear that in the given case the sine theorem will have the form:

$$\frac{\sin\alpha}{\bar{U}} = \frac{\sin\delta_\psi}{\bar{V}} = \frac{\sin\delta_\gamma}{\bar{W}}. \tag{1.93}$$

From (1.93) the value of the drift angle and the flight groundspeed are easily determined on the basis of known values of the aircraft course, airspeed, and the speed and direction of the wind.

Now, let us define the path angle of the wind:

$$\delta_\psi = \delta - \psi. \tag{1.94}$$

The drift angle of an aircraft according to (1.93) is determined from the formula

$$\sin\alpha = \frac{u}{V}\sin\delta_\psi. \tag{1.95}$$

The value of the flight groundspeed is then easily determined:

$$W = \frac{V\sin\delta_\gamma}{\sin\delta_\psi}. \tag{1.96}$$

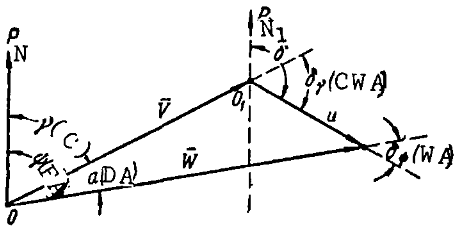

Fig. 1.61. Navigational Speed Triangle

These problems are especially simple to solve with slide rules having a combination of sine logarithms with a logarithm scale of linear values.

In this case, combining the logarithm of the sine of the wind angle with the logarithm of the airspeed, we obtain directly: /101

$$\lg \sin a - \lg u = \lg \sin \delta_{\psi} - \lg V = \lg \sin \delta_{\gamma} - \lg W$$

or on scales of navigational rulers with the designations used with special values for aircraft courses.

$$\frac{\mathrm{DA}}{u}, \frac{\mathrm{WA}}{V}, \frac{\mathrm{DA}+\mathrm{WA}}{W} \quad \boxed{\lg \sin} \quad \boxed{\lg s p} \tag{1.97}$$

To determine wind speeds and directions at flight altitude on the basis of known values of airspeed, groundspeed, and drift angle, let us use Figure 1.62.

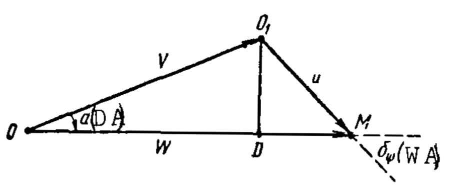

Fig. 1.62. Determining the Angle and Speed of the Wind with Known Values of the Groundspeed and Drift Angle of the Aircraft.

From the figure, it follows that

$$OD = V \cos \mathrm{DA};$$

$$O_1D = V \sin \mathrm{DA};$$

$$DM = OM - OD = W - V \cos \mathrm{DA}.$$

Therefore,

$$\operatorname{tg} \mathrm{WA} = \frac{O_1D}{DM} = \frac{V \sin \mathrm{DA}}{W - V \cos \mathrm{DA}}. \tag{1.98}$$

The flight angle of the wind determined in this way permits the further solution of problems on the basis of the sine theorem [Equation (1.93)].

With small drift angles (practically up to 10°), cos DA ≈ 1, i.e., it is possible to consider in approximation that

$$\operatorname{tg} \mathrm{WA} = \frac{V \sin \mathrm{DA}}{W - V} \tag{1.99}$$

For solution on a slide rule, (1.99) is reduced to the form:

$$\frac{\operatorname{tg} \mathrm{WA}}{V} = \frac{\sin \mathrm{DA}}{W - V},$$

i.e.,

$$\lg \operatorname{tg} \mathrm{WA} - \lg V = \lg \sin \mathrm{DA} - \lg (W - V)$$

Translator's note: lg = log.

or on a slide rule,

DA. WA	lg sin lg tg
$W - V$ V	lg sp

After finding the flight angle of the wind, the value of the wind speed is determined on the basis of the sine theorem. /102

Elements Which Determine Flight Altitude

The flight altitude of an aircraft (H) is measured from a special initial level of the Earth's surface. The initial level for measuring flight altitude is chosen depending on the purposes for which it is measured.

For example, in order to distribute the counter and incidental movements of aircraft in airspace (flight echelons), the initial level for measuring the altitude on each aircraft must be general. To ensure the safety of flights of individual aircraft at low altitudes, it is desirable that the flight altitude be measured from the surface of the relief over which the aircraft is flying. In making an approach to land at an airport, flight altitude is measured from the level of the landing point.

Usually 2 or 3 kinds of altitudes are measured at the same time. Therefore, it is necessary to classify them and to establish a relationship between them.

At the present time, the following kinds of altitudes are distinguished (Fig. 1.63):

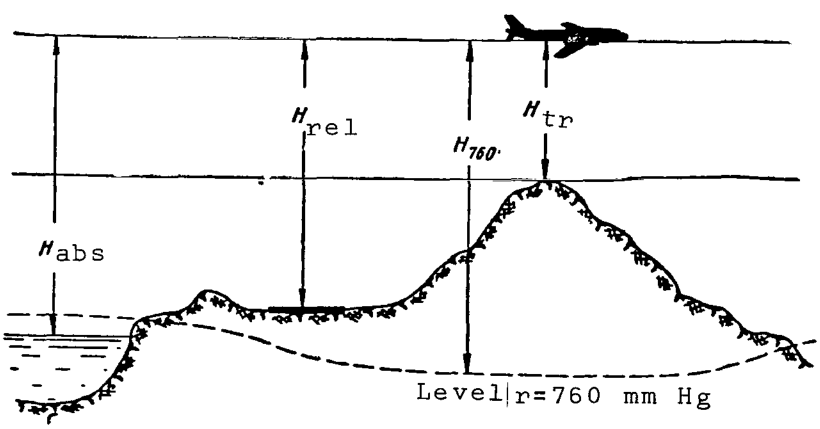

Fig. 1.63. Interrelationship of Different Systems for Measuring Flight Altitude

(a) *Absolute flight altitude* (H_{abs}) is measured from the mean level of the Baltic Sea in the same way as the height of a relief on the Earth's surface.

(b) *Relative flight altitude* (H_{rel}) is measured from the level of the take-off or landing airport. /103

(c) *True flight altitude* "H_{tr}" is measured from the surface of the relief over which the aircraft is flying.

(d) *Conventional barometric altitude* "H_{760}" is measured from the conventional barometric level on the Earth's surface, where the atmospheric pressure is equal to 760 mm Hg.

Absolute, relative, and true flight altitudes are determined by barometric altimeters with correction of their readings for instrumental and methodological errors. The latter can also be measured by radio altimeters and aircraft radar equipment or determined by aircraft sighting devices. There is a relationship between the three indicated altitudes which makes it possible to switch from one kind of altitude to another.

Conventional barometric altitude is measured by barometric altimeters without considering methodological errors. Therefore, it has no direct connection with the first three kinds of altitudes, and at a high flight altitude it can be distinguished from the absolute altitude closest to it by 900-1000 m.

The main advantage of a conventional barometric altitude is the convenience of using it for echeloning flights according to altitudes when the important thing is not the precise measuring of altitude but only the preservation of safe altitude intervals between neighboring echelons. The latter condition is satisfied, since if we permit two aircraft to meet in one region and at one altitude, the methodological corrections in these aircraft will be identical. Therefore, such a meeting cannot occur if aircraft maintain different altitudes based on instruments.

From Figure 1.63 it is evident that true flight altitude is distinguished from absolute flight altitude by the height of the relief over which the aircraft is flying, and from relative altitude by the height of the relief above the airport level from which relative altitude is measured:

$$\left.\begin{aligned} H_{tr} &= H_{abs} - H_r; \\ H_{tr} &= H_{rel} - \Delta H_r, \end{aligned}\right\} \qquad (1.100)$$

where H_r is the altitude of the relief above sea level; ΔH_r is the height of the relief above the level of the airport.

Relative altitude is distinguished from true altitude by the height of the relief, while it is distinguished from absolute altitude by the height of the airport above sea level:

$$\left.\begin{aligned} H_{rel} &= H_{tr} + \Delta H_r; \\ H_{rel} &= H_{abs} - \Delta H_{air}. \end{aligned}\right\} \qquad (1.101)$$

Finally, absolute flight altitude can be determined on the basis of the values of true or relative flight altitude: /104

$$\left.\begin{aligned} H_{abs} &= H_{tr} + H_r; \\ H_{abs} &= H_{tr} + H_{air}. \end{aligned}\right\} \qquad (1.102)$$

Calculating Flight Altitude in Determining Distances on the Earth's Surface

In measuring directions on the Earth's surface, flight altitude does not exert a direct influence on the value of the measured angles or on the accuracy of the measurements.

Actually, by direction on the Earth's surface we mean direction of the line of intersection of the horizon plane with the plane of a great circle (orthodrome) which joins two points on the Earth's surface.

Since the vertical at any of these points on the indicated line lies in the plane of a great circle, flight altitude does not exert an influence on the direction of the orthodrome and therefore on direction on the Earth's surface.

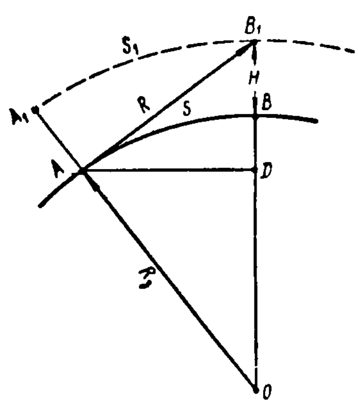

Fig. 1.64. Calculating Flight Altitude in Determining Distances.

In measuring distances on the Earth's surface, flight altitude can play an important role and can lead to large measurement errors if we do not allow for errors in flight altitude (Fig. 1.64).

In the figure, straight lines OA_1 and OB_1 are verticals of the position of an aircraft at points A and B.

Obviously the distance S between points A_1 and B_1 at flight altitude is greater than distance S between points A and B on the Earth's surface:

$$\frac{S_1}{S} = \frac{R_e + H}{R_e} \quad (1.103)$$

whence

$$S_1 = S\left(1 + \frac{H}{R_e}\right) \text{ or } \Delta S = S\frac{H}{R_e} \quad (1.104)$$

where R_e is the radius of the Earth (equal to 6371 km) and H is the flight altitude.

Each kilometer of flight altitude lengthens the path between points on the Earth's surface by a value expressed in percent: /105

$$\frac{1 \cdot 100}{6371} \approx 0.016\%.$$

For example, at a flight altitude of 10 km, a distance on the Earth's surface equal to 3000 km lengthens to the value

$$\frac{3000 \cdot 10 \cdot 0.016}{6371} = 4.8 \text{ км},$$

The indicated lengthening of the path of the aircraft does not exert a substantial influence on the time of the aircraft flight along the path. The influence of flight altitude on determination of the position of the aircraft by rangefinding and, especially, hyperbolic devices turns out to be more substantial.

Let us assume that a rangefinding device is located at point A on the Earth's surface, while the aircraft is located at point B_1, at flight altitude.

As is evident from Figure 1.66, the distance from the ground radio-engineering apparatus to the aircraft R along a straight line will equal AB_1, while the distance along the Earth's surface S is equal to AB.

Let us drop a perpendicular from point A on the Earth's surface to point D on the vertical OB_1. Obviously,

$$AB_1^2 = AD^2 + DB_1^2,$$

since

$$AB_1 = R;$$

$$AD = R_e \sin S;$$

$$DB_1 = R_e - R_e \cos S + H,$$

Then

$$R=\sqrt{R_e^2\sin^2 S+(R_e-R_e\cos S+H)^2}. \quad (1.105)$$

With small angular distances S (up to 6° along the arc of the orthodrome), when $R \sin S \approx S$, while $\cos S \approx 1$, (1.105) takes the form:

$$R=\sqrt{S^2+H^2}. \quad (1.106)$$

Figure 1.66 can likewise be used for determining the maximum distance of geometrical visibility of objects on the ground from on board the aircraft, or of an aircraft from the Earth's surface.

It is obvious that with maximum visibility, line AB_1 must be tangent to the Earth's surface, i.e., it is located in the plane of the horizon. In this case, angle OAB_1 will be a right angle.

Therefore, /106

$$OA^2+AB_1^2=OB_1^2$$

or

$$R_e^2+(R_e \operatorname{tg} S)^2=(R_e+H)^2.$$

Expanding the right-hand side of the equation, we obtain:

$$R_e^2+R_e^2 \operatorname{tg} S^2=R_e^2+2R_eH+H^2.$$

Considering that at distances up to 600-700 km, $R_e\operatorname{tg}S \approx S$, and disregarding the value H^2 as negligibly small in comparison with $2R_eH$, we obtain the approximate formula

$$S=\sqrt{2R_eH}. \quad (1.107)$$

Substituting in (1.107) the value of the radius of the Earth (6371 km) we obtain:

$$S=\sqrt{12742H}=113\sqrt{H}.$$

Bearing in mind that as a result of the refraction of light or radio waves in a vertical plane, the distance of geometrical visibility increases approximately by 8%, the practical result will be:

$$S_{vis} = 122\sqrt{H}. \quad (1.108)$$

Formula (1.108) determines the limits of applicability of (1.104) or (1.106). Since we have agreed to consider cos S = 1 and R sin S = S up to S = 6°, which on the Earth's surface corresponds to 666 km, it is obvious that at flight altitudes up to 25 km it is always possible to use (1.106).

It is necessary to use the precise formula (1.104) at distances of more than 700 km. This is possible at flight altitudes exceeding 25 km.

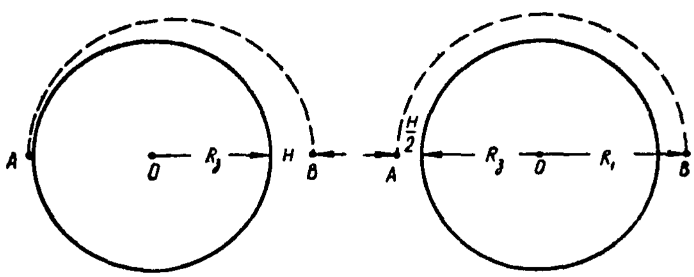

Fig. 1.65. Calculating Flight Altitude in Determining the Path Length of Electromagnetic Wave Propagation

Let us pause now to discuss the influence of flight altitude on the accuracy of measuring distances at very small ranges, i.e., in cases when long radio waves capable of traveling around the Earth's surface are used (Fig. 1.65). /107

In the figure, ground radio engineering equipment is located at point A on the Earth's surface; the aircraft is at point B at altitude H. Line AB is the curve of propagation of a radio wave front.

If we conditionally move the Earth's surface to the right by a value equal to $H/2$, then the line of radio wave propagation becomes concentric with the Earth's surface and will have a radius of curvature $R_1 = R_e + H/2$.

Therefore, the increase in distance from point A to point B can be considered as a lengthening of the orthodrome at a flight altitude equal to $H/2$, i.e.,

$$\Delta S = S_1 - S = S \cdot \frac{H}{2R_e}.$$

Elements of Aircraft Roll

It is known that the radius of aircraft roll in airspace at a given banking β equals:

$$R = \frac{V^2}{g \, \mathrm{tg} \, \beta}.$$

If a flight is carried out with a counter or incidental wind, rolling of an aircraft through an angle of 90° involves an increase or decrease in the mean radius of roll of an aircraft relative to the Earth's surface (Fig. 1.66).

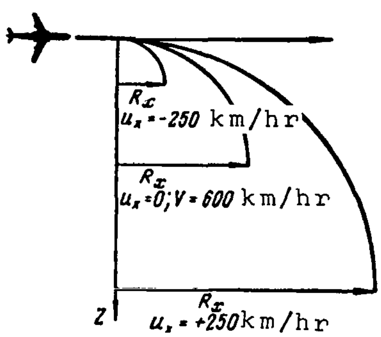

Fig. 1.66. Deformation of the Roll Trajectory in the Presence of Wind.

In fact, for a change in the direction of the groundspeed vector of an aircraft by 90°, with a shift from the plane of incident wind to a lateral plane, it is necessary to execute a roll of an aircraft to the right or left through an angle of 90° + DA, and in changing from the plane of incident wind to a lateral wind to a lateral plane through an angle of 90° - DA.

During rolling of an aircraft in airspace, in the first instance there will be a deviation of the aircraft from the original flight direction; in the second case, there /108
will be a deviation opposite to the original.

Example. Let us examine the roll of an aircraft through 90°, with a flight airspeed of 600 km/h and with a counter and incident wind speed of 250 km/h (70m/sec).

According to (1.6), the radius of the aircraft in airspace with banking of 15° will be

$$R = \frac{167^2}{9{,}81 \, \mathrm{tg} \, 15^\circ} = 10\,500 \text{ m}$$

The drift angle at the end of rolling through 90° will have the following value:

$$\mathrm{DA} = \mathrm{arctg} \frac{250}{600} = 23^\circ.$$

Therefore, in the first case it will be necessary to turn the aircraft through 113°, and in the second case through 67°.

The angular velocity of roll at V = 600 km/h (167 m/sec) and R = 10,500 m will be

$$W=\frac{V\cdot 57,3}{R}=\frac{167\cdot 57,3}{10\,500}=0,9\,\text{deg/sec}$$

Let us determine the additional shift of the aircraft as a result of wind during rolling in the first instance:

$$\Delta R_x=\frac{70\ \text{m/sec}\cdot 113^\circ}{0,9\,\text{deg/sec}}\approx 9\ \text{km}$$

and in the second instance:

$$\Delta R_x=\frac{70\cdot 67}{0,9}\approx 5\ \text{km}$$

Obviously, during roll (in the first case through 113° and in the second case through 67°) the movement of the aircraft in direction X will not be identical, since

$$B_x=R\sin \text{УР}.$$

Then the general path of an aircraft in direction X will equal: In the first case,

$$R_x=10,5\sin 113^\circ+9=18,5\ \text{km}$$

In the second case,

$$R_x=10,5\sin 67^\circ-5=4,5\ \text{km}$$

Let us now determine the lateral shift of the aircraft R_z during roll: In the first case,

$$R_z=R+R\sin 23^\circ=14,5\ \text{km}$$

and in the second case

$$R_z=R-R\sin 23^\circ=6\ \text{km}.$$

For comparison, let us examine the roll of an aircraft through 90°, with a radius calculated not on the basis of airspeed, but on the basis of groundspeed:

$$W=V\pm u_x.$$

In the first case, the radius of rolling is /109

$$R = \frac{237^2}{9{,}81 \operatorname{tg} 15^\circ} = 21 \text{ km}$$

and in the second case

$$R = \frac{97^2}{9{,}81 \operatorname{tg} 15^\circ} = 3{,}5 \text{ km}$$

Let us compile a table with the results obtained:

Roll parameters	$u_x=0$	$u_x=+250$ km/h	$u_x=-250$ km/h
R_v	10,5	10,5	10,5
R_w	10,5	21	3,5
X	10,5	18,5	4,5
Z	10,5	14,5	6

From the table, it is evident that the results of the calculations carried out on the basis of the groundspeed are much closer to the actual results than calculations on the basis of airspeed.

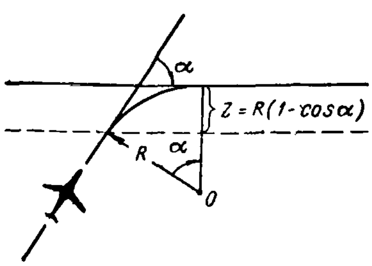

Fig. 1.67. Approach of an Aircraft to a Given Line with the Presence of an Approach Angle.

Calculations of roll on the basis of groundspeed with winds as high as 200-300 km/h, when entering a new line of flight, will be carried out with an accuracy of 1-2.5 km. Some inaccuracies arise, but only in the lateral direction. However, this is not of practical significance, since the direction of deviation coincides with the new line of flight.

With a decrease in the angle of roll, the trajectory calculated according to the groundspeed comes closer to the actual trajectory of aircraft roll. Therefore, in the future we will proceed from flight groundspeed in calculating roll.

In aircraft navigation, including maneuvering before landing, it is necessary to solve three types of problems, taking into account the roll trajectory.

1. Combination of Roll with a Straight Line

Let us assume that an aircraft is approaching a given line of flight at a definite angle (Fig. 1.67).

It is obvious that the angle of roll of the aircraft for following along the given line is equal to the approach angle (α). Let us determine the distance (Z) from the given line on which it is necessary to begin the roll so that the roll trajectory will be joined with the given line. /110

In Figure 1.67, it is evident that this distance is equal to:

$$Z = R - R\cos\alpha$$

or

$$Z = R(1 - \cos\alpha). \quad (1.109)$$

Example. An aircraft approaches a given line of flight with a groundspeed of 900 km/h at a 25° angle. Determine the lateral distance from the line of flight at which it is necessary to begin a roll for a smooth approach to the line.

Solution.

$$R = \frac{250^2}{9{,}81 \operatorname{tg} 15^\circ} = 26{,}5 \text{ km}$$

$$Z = 26{,}5(1 - \cos 25^\circ) = 2{,}46 \text{ km}$$

2. Combination of two rolls

If, during flight along a given flight line, a deviation from it occurs and it is necessary to approach the given line by the shortest trajectory, an approach maneuver is used which is a combination of two rolls (Fig. 1.68).

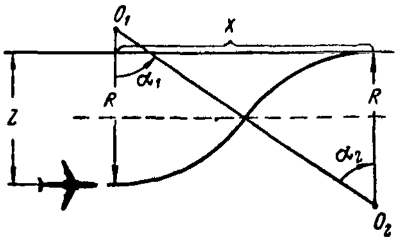

Fig. 1.68. Approach of an Aircraft to a Given Flight Line with a Parallel Flight Line.

Since the value of Z in this case is considered known, while the radius of roll is determined on the basis of the groundspeed and the given banking in the roll, it is necessary to determine the value of the angles $\alpha_1 = \alpha_2$ of the combined rolls.

It is obvious that in this case, in each of the two combined rolls, the aircraft approaches the flight path by a value $Z/2$; therefore,

$$\frac{Z}{2} = R(1 - \cos\alpha),$$

whence

$$\cos\alpha = 1 - \frac{Z}{2R}. \qquad (1.110)$$

For example, let us say that an aircraft having a groundspeed of 900 km/h has deviated from a given flight path by 5 km; to make the approach, it is necessary to execute two combined rolls with banking of 15° to angles up to 25°.

3. Linear prediction of roll (LPR)

Let us examine two solutions to problems, with a consideration of the roll trajectory of an aircraft which includes one rectilinear part of the path.

Linear prediction of roll is calculated in instances of a break in the flight path at turning points in the route (Fig. 1.69). /111

In the figure, TPR is the turning point in the route and TA is the turn angle of the flight path equal to the roll angle of an aircraft (RA).

As is clear from Figure 1.69, the radius of roll of an aircraft, at its beginning and end, is directed perpendicular to the preceding and following orthodrome segments of the path. The lines O-TPR form the bisector of the angle of roll.

Thus, we have two identical rectangular triangles with vertex angles equal to RA/2. The linear prediction of roll (LPR) is the line of tangency of the roll angle, divided in half:

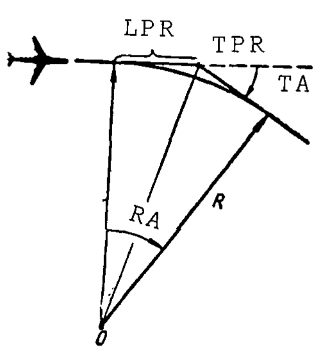

Fig. 1.69. Linear Prediction of Roll of an Aircraft (LPR).

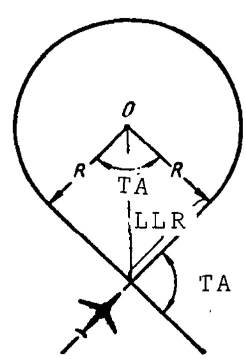

Fig. 1.70. Linear Lag of Aircraft Roll (LLR).

$$\text{LPR} = R\,\text{tg}\,\frac{\text{RA}}{2}. \qquad (1.111)$$

Example: Determine LPR with a flight groundspeed of 900 km/h and an angle of turn to the new flight path of 40° for banking in a roll of 15°.

Solution.

$$R = \frac{250^2 \cdot}{9{,}81\ \text{tg}\ 15^\circ} = 26{,}5\ \text{km}$$

$$\text{LPR} = 26{,}5 \cdot \text{tg}\ 20^\circ = 9{,}6\ \text{km}$$

Linear predictions with roll angles from 0 to 150° are given in Table 1.1. /112

W, km/hr	R, M	Prediction with roll angles from0 to 150°, km										t_{roll} to 90° sec
		15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	
400	4 600	0,6	1,2	2,0	2,7	3,5	4,6	6,0	8,0	11,0	15,0	65
500	7 300	1,0	2,0	3,1	4,3	5,7	7,3	9,7	12,8	18,0	27,5	82
600	10 600	1,4	2,8	4,2	6,1	8,2	10,6	13,8	18,3	25,5	40,0	100
700	14 700	1,9	3,8	6,0	8,3	11,0	14,4	19,0	25,0	35,0	52,0	116
800	18 500	2,4	4,8	7,7	10,7	14,0	18,5	24,0	32,0	42,0	70,0	132
900	23 500	3,1	6,2	9,7	13,5	18,0	23,5	30,0	40,0	58,0	87,0	148

In some cases, the necessity for flight above the TPR with the flight angle of the following part of the path can arise (Fig. 1.70), e.g., in flights of different kinds for testing aircraft and ground navigational equipment. In these cases, instead of linear prediction, linear lag of roll (LLR) is calculated, while the roll is carried out in the direction opposite to the turn of the new flight path by the angle

$$\text{RA} = 360^\circ - \text{TA}$$

In Figure 1.72, it is clear that the LLR is a line of the tangents of the turn angle of the flight path divided in half, i.e., with the same turn angles, the formula for the LLR remains the same as for the linear prediction of roll:

$$\text{LLR} = R\text{tg} - \frac{\text{RA}}{2}.$$

CHAPTER TWO

AIRCRAFT NAVIGATION USING MISCELLANEOUS DEVICES

1. Geotechnical Means of Aircraft Navigation

Geotechnical means of aircraft navigation constitute a portion of the navigational equipment of an aircraft which has an autonomous character and is used under all flight conditions, independently of the use of other special devices such as those employing radio engineering or astronomy, for example. /113

Such devices include those which measure the aircraft course, airspeed, and flight altitude, as well as devices for automatic solution of navigational problems.

Geotechnical devices for aircraft navigation are based on highly diverse physical principles for the use of natural geophysical fields of the Earth (magnetic, gravitational, pressure, the field of electromagnetic oscillations in the optical and infrared range, etc.). The operation of these devices is more dependent on the physical parameters of the medium in which the flight is being carried out than is the case for devices employing radio engineering or astronomy. Therefore, they have a complex mathematical basis for their regulation, especially with regard to the system for making corrections to instrument readings.

Aircraft navigation using only geotechnical devices can be carried out in cases when it is possible to check the navigational calculations (even periodically) by determining the locus of the aircraft by other means or visually.

Historically speaking, the development of radio-engineering and astronomical means for aircraft navigation has been directed toward a solution of only one problem, namely, the determination of the aircraft coordinates on the Earth's surface, which proved a necessary adjunct to the geotechnical means of aircraft navigation in flight under conditions when the ground was not visible.

In recent years, there has been a development of the radio-engineering, astronomical, and astro-inertial systems for solving problems which are inherent in geotechnical devices for aircraft /114
navigation, i.e., measurement of the aircraft course, airspeed, turn angle, altitude, etc.

2. Course Instruments and Systems

Course instruments are intended for determining the position of the longitudinal axis of an aircraft in the plane of the horizon or (what amounts to the same thing) for measuring the course of the aircraft.

It is necessary to know the aircraft course in order to determine both the flight direction and the position of the aircraft relative to orientation points on the ground.

As we have mentioned above, there are several systems for calculating the aircraft course, and the selection of the system of calculation is governed both by the requirements of aircraft navigation and by the technical possibilities for equipping the aircraft with the corresponding instruments.

At the present time, there are no course instruments which completely satisfy the requirements of aircraft control under all conditions. Therefore, aircraft usually are fitted with several different course instruments operating on different principles and using different systems of calculation; each of them is used under the conditions which are most favorable for it. In some cases, these instruments are combined into complexes, called course systems, where the operation of the individual instruments is closely related. This makes it possible to exploit the positive qualities of each of them in actual operation.

Methods of Using the Magnetic Field of the Earth to Determine Direction

Directions on the Earth's surface can be measured most accurately by astronomical methods. However, this requires optical visibility of the sky, complex and accurate apparatus, and tedious calculation. Directions on the Earth's surface can be determined more simply and in many cases quite reliably by using the magnetic field of the Earth.

The magnetic field of the Earth (Fig. 2.1) is characterized by the following parameters at every point on its surface:

(a) Directionality of the horizontal component of the field ($\bar{H}$);

(b) Directionality of the vertical component of the field ($\bar{Z}$);

(c) The direction of the plane in which the vectors $\bar{H}$ and $\bar{Z}$ lie relative to the geographic meridian at the given point.

The plane in which the vectors $\bar{H}$ and $\bar{Z}$ are located is called /115 the *plane of the magnetic meridian*. The angle between the planes of the magnetic and geographic meridians is called the *magnetic declination* and is represented by Δ_M.

The points on the Earth's surface at which the magnetic meridians intersect are called *magnetic poles*. Obviously, the horizontal component of the magnetic field is lacking at the magnetic poles, while the intensity of the vertical component reaches its maximum value.

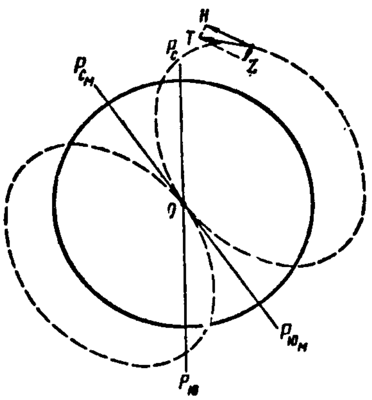

Fig. 2.1. Magnetic Field of the Earth.

The magnetic poles of the Earth do not coincide with the geographic ones. The coordinates of the North Magnetic Pole are 74°N and 100°W; those of the South Magnetic Pole are 68°S, 143°E (as of 1952).

The device of a freely rotating magnetic pointer mounted in the plane of the magnetic meridian is used to determine direction on the Earth's surface. Therefore, at every point on the Earth's surface there will be a reliable indication of the three parameters which characterize the magnetic field of the Earth.

The total intensity of the magnetic field of the Earth (vector T) is the resultant vector of $\bar{H}$ and $\bar{Z}$. Consequently,

$$\bar{T}^2 = \bar{H}^2 \quad \bar{Z}^2. \tag{2.1}$$

The oersted (Oe) is the unit of measurement for the total intensity of the magnetic field, as well as the intensity of its components; in other words, it is the intensity of a field which interacts with a unit magnetic pole with a force of one dyne.

The limits of change in the intensity of the components in the magnetic field of the Earth are the following:

(a) *Horizontal:* from zero in the vicinity of the magnetic poles to a maximum at the magnetic equator (0.4 oersteds in the vicinity of Indonesia);

(b) *Vertical:* from zero at the magnetic equator to 0.6 oersteds in the vicinity of the magnetic poles.

A smaller unit of intensity, the gamma (γ), is used for very precise magnetic measurements; it is equal to one hundred thousandth of an oersted.

The angle which characterizes the inclination of the vector of total intensity of the magnetic field of the Earth to the plane of the true horizon is called the *magnetic inclination* "θ". /116

$$\theta \operatorname{arctg} \frac{\bar{Z}}{H} \tag{2.2}$$

Charts of the magnetic fields are prepared for convenience in using the magnetic field of the Earth to determine directions on the Earth's surface.

A chart of magnetic inclinations is extremely important for aircraft navigation. Lines joining points on the Earth's surface which have the same magnetic declination are called *isogonics*. They are printed directly on flight and large-scale geographic maps.

To determine the true course, the magnetic declination determined from the chart at the locus of the aircraft (with its sign, as a correction) is entered in the readings of the magnetic compass.

Figure 2.2 shows a map of the World with the magnetic declinations entered on it; the isogonics are shown as they appear on the Earth's surface. The positive isogonics on the chart are marked by solid lines, while the negative ones are marked by dashed lines.

All of the isogonics meet at the magnetic poles of the Earth, and the compass readings (and consequently the magnetic inclination) change by 180° when passing through the magnetic pole.

In addition, the isogonics also meet at the geographic poles, since the directions of the magnetic and geographic meridians are opposite between the magnetic and geographic poles, but coincide after passing through the pole, i.e., the declination changes by 180°.

The map of the World showing the magnetic declinations has the isogonics only for the normal magnetic field of the Earth. In addition to this normal field, there is also an anomalous field, caused by the magnetization of the soil in the upper layers of the Earth. Regions and areas of changes in the declination in such regions are marked on large-scale charts.

The reliability of operation of magnetic compasses and the magnitude of the errors in their readings depend on the intensity of the horizontal component of the magnetic field of the Earth. Errors in the readings of compasses, particularly when the aircraft is rolling, depend only on the intensity of the vertical comonent.

The lines on the Earth's surface which connect points with the same intensity of the horizontal or vertical components of the magnetic field are called *isodynamic* lines.

Figure 2.3 shows a map of the World with the isodynamic lines for the horizontal component of the Earth's magnetic field, while Figure 2.4 shows those for the vertical component.

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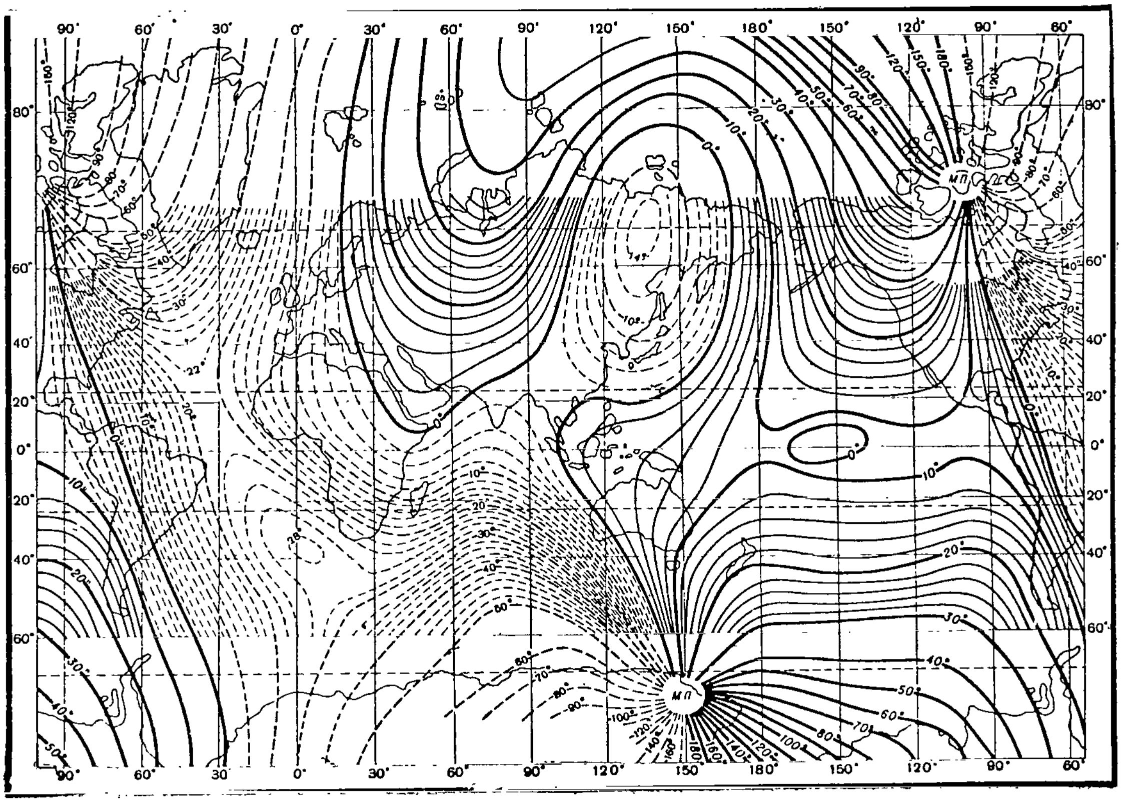


Fig. 2.2. World Chart of Magnetic Declinations.

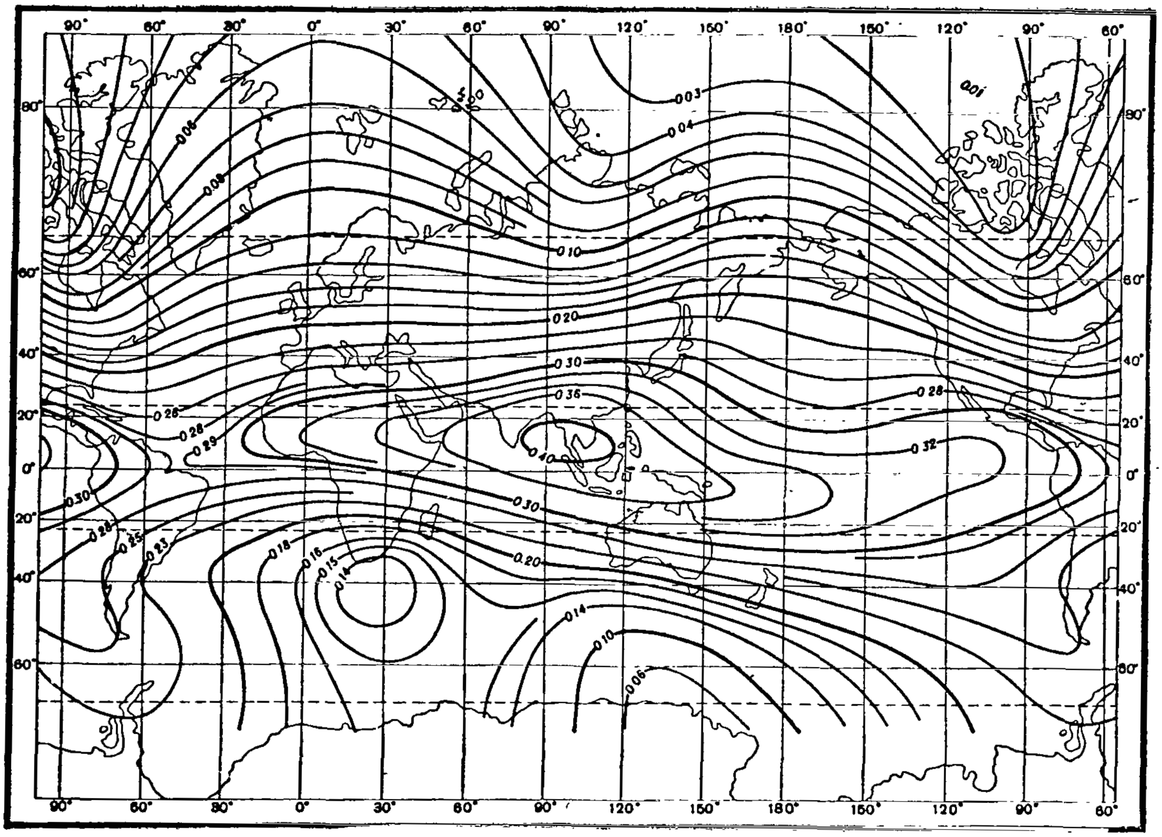


Fig. 2.3. World Chart of Isodynamic Lines for the Horizontal Component of the Earth's Magnetic Field.

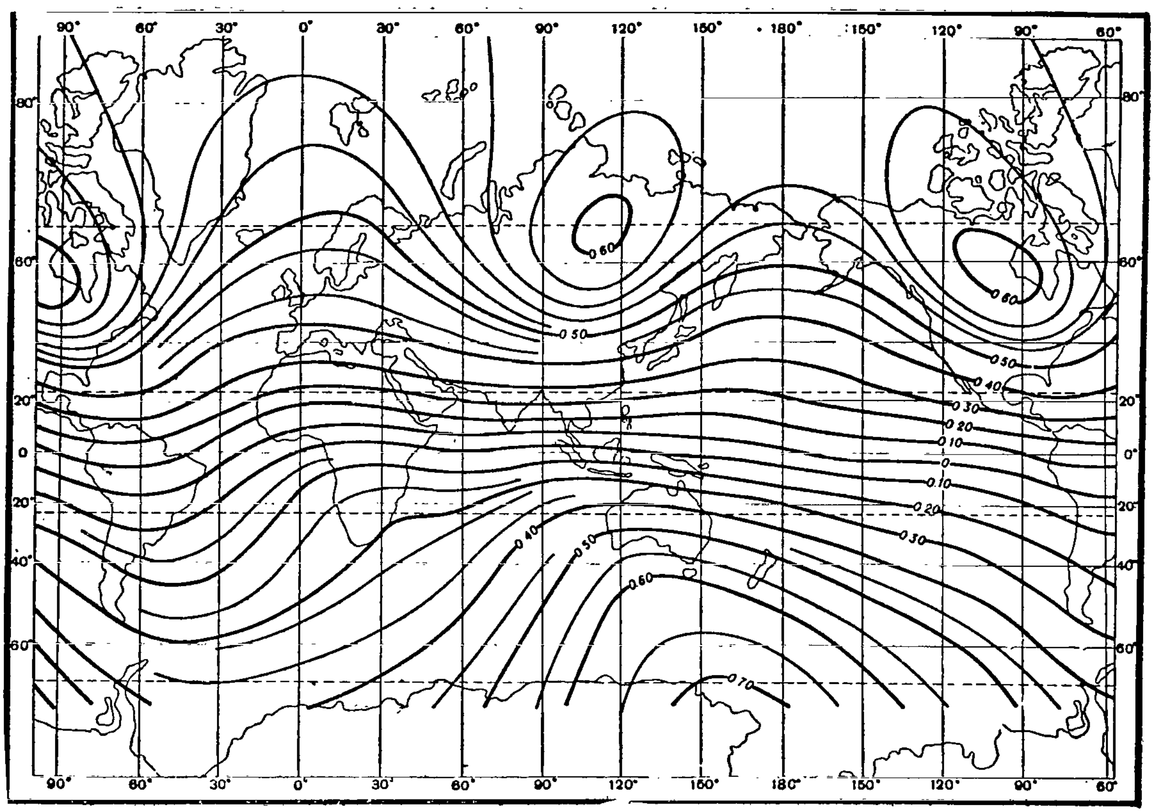


Fig. 2.4. World Chart of Isodynamic Lines for the Vertical Component of the Earth's Magnetic Field.

Only general (outline) charts of isodynamic lines are used in aircraft navigation. These lines do not appear on flight charts. /120

Lines on the Earth's surface which connect points with the same declination of the magnetic field are called *isoclines*. Formerly, outline maps of isoclines were used jointly with charts of isodynamic lines showing the total intensity of the magnetic field to determine the errors of magnetic compasses. At the present time, these charts are no longer used, since it is better to use the isodynamic lines of the horizontal and vertical components of the magnetic field.

Variations and Oscillations in the Earth's Magnetic Field

There are several hypotheses regarding the origin of the magnetic field of the Earth, but none of them has been adequately proven as of the present time. Possible factors in the formation of the magnetic field are the subsurface and ionospheric electrical currents, as well as the magnetic induction and magnetic hysteresis of the soil, composing the structure of the Earth's sphere.

Even if these factors are not primarily responsible for the formation of the magnetic field of the Earth, they are in any case important influences on its structure and stability.

An analysis of the isolines of the intensity of the components in the magnetic field of the Earth and the magnetic declinations reveals that their configuration is determined both by general laws of the distribution of magnetic forces in the field of a magnetized sphere, as well as by local disturbances in the general structure of the field. Therefore, the stationary magnetic field of the Earth is assumed to consist of a sum of fields:

(a) The field of the uniform magnetized sphere;

(b) The continental field, related to the nonuniformity of the relief and the structure of the internal layers of the Earth;

(c) The anomalous field, related to the existence of deposits of magnetic material in the upper layers of the Earth's core.

As systematic observations of the structure of the magnetic field of the Earth have shown, it does not remain strictly stationary but undergoes constant changes. Changes or variations in the magnetic field of the Earth have a diverse nature.

Annual or constant changes in the magnetic field of the Earth are called *secular variation*. These variations constitute the difference between the average annual values for the elements of the Earth's magnetism. The causes for the annual variations are changes in the components of the stationary field with time, i.e., the magnetic moment of the Earth and the continental field.

The annual variations in the declination at middle latitudes reached 10-12', and up to 40' at high latitudes; therefore, when using charts of magnetic declinations, or isogonics on flight charts, it is necessary to consider the period when they were made. If the chart of magnetic declinations is obsolete, changes must be /121
made when using it for the variation in the declination during the time which has passed since the chart was made. The desired correction is determined from special charts of the secular variations of the magnetic field of the Earth. The isolines of equal secular variations in declination on a chart are called *isopors*.

In addition to the slow systematic changes in the magnetic field of the Earth, there are also periodic and even chaotic changes which are related to the so-called internal field of the Earth, the main cause of which is ionospheric currents. These are estimated periodically or are disregarded entirely.

Magnetic Compasses

The magnetic compass is the simplest course device; in most cases, it is sufficiently reliable though not sufficiently accurate.

However, a simple magnetic compass with a freely turning magnetic needle is not suitable for use on board an aircraft, since its readings would be inaccurate and unstable. Various kinds of interference would influence the operation of the compass during flight, including:

(a) Movements of the aircraft relative to its axis;

(b) Vibrations produced by the operation of the engines and by the movement of the aircraft through the air;

(c) The effect of the magnetic field of the aircraft, which would cause deflections of the magnetic needle from the plane of the magnetic meridian, i.e., compass deviation.

Obviously, a magnetic compass which is intended for use on an aircraft must have devices for compensating the interference mentioned above.

The simplest form of an aircraft magnetic compass is the integrated compass, i.e., one in which the course transmitter (sensitive element) and the indicator are combined in a single housing.

Of the large number of types of magnetic compasses which have been devised as of the present time, the one most used nowadays is the "KI" (an abbreviation for the historic name of the magnetic compasses which were devised in the past for fighter aircraft). Compasses using other systems are called distance-magnetic or gyro-magnetic compasses.

Any integrated aviational magnetic compass consists of the following main parts (Fig. 2.5):

The bowl or container 1 of the compass, filled with a damping fluid to decrease the oscillations, usually liqroin; on the bottom of the bowl is a pivot support for the movable part of the compass, with a damping spring and pivot bearing made of agate; /122

The movable part of the compass, consisting of a card 2, which is a combination of a magnetic system (H-shaped magnet), a float to reduce the weight of the card and reduce the friction on the bearing, a needle pivot, and a rotating scale for the readings, mounted on the magnetic system;

Chamber 3, compensating for thermal expansion and contraction of the damping fluid; the expansion chamber is located above the bowl and is connected to it by holes of very small diameter. This allows air bubbles to escape from the bowl into the chamber and permits the fluid to flow back and forth with expansion and contraction. It also prevents it from splashing in the bowl as the airplane moves;

A device 4 for getting rid of deviations of the compass, which contains several bar magnets pressed into drums which rotate in mutually perpendicular planes with the aid of screws. Rotation of the drums permits them to be set to a position where the magnetic field of the bar magnets compensates for the magnetic field of the aircraft acting on the compass card.

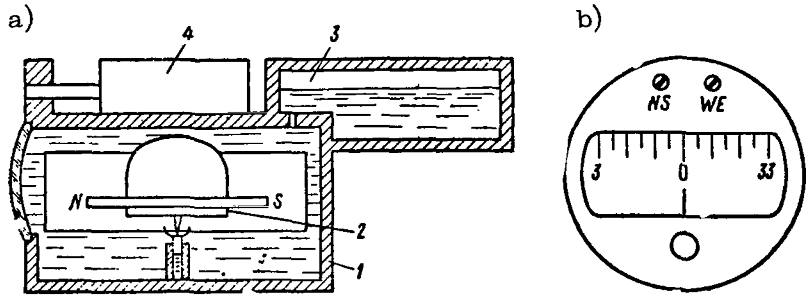


Fig. 2.5. Combined Magnetic Compass: (a) Cross Section; (b) External View.

The design of the magnetic compass described above reduces the effect of interference with its operation to a considerable degree and the compass readings are quite stable. Nevertheless, magnetic compasses (especially integrated ones) have a number of shortcomings which prevent the course from being calculated under certain conditions. The most important of these shortcomings are the following:

(1) A limitation of the choice of mounting location for the compass aboard the aircraft; the integrated compass must be located in a place which is suitable for determining the course, and therefore close to other instruments and moving parts for controlling the aircraft, which produce a large and varying deviation of the compass;

(2) The impossibility of using the compass when the aircraft is turning. When the aircraft makes a turn, several factors act on the compass card to move it from its customary position: the pressure of the damping fluid on the card, the action of centrifugal force on the southern, somewhat elongated portion of the card, as well as a change in the structure of the magnetic field of the aircraft while turning. The deviations of the compass card /123

from the plane of the magnetic meridian when the aircraft is turning are particularly noticeable when the aircraft course crosses the northern and southern directions. These deviations are called the northern and southern turning errors.

The instability of the structure of the magnetic field of the Earth at the locus of the aircraft and its changes with time are the major shortcomings of using magnetic compasses of all types.

Deviation of Magnetic Compasses and its Compensation

The cause of magnetic compass deviation is the presence of parts on board the aircraft which are made of materials exhibiting magnetic properties. Some of these parts have a constant magnetic field. Parts of this kind are called *hard magnetic iron.* Another group of parts are magnitized under the effect of the magnetic field of the Earth and are called *soft magnetic iron.*

According to Coulomb's law, the force (F) of the interaction of magnetic masses (m) is inversely proportional to the distance between them (r).

$$F = \frac{m_1 m_2}{r^2}. \quad (2.3)$$

Therefore, the deviation of the magnetic compass increases very sharply with the approach of its sensitive element to parts which have high magnetization.

According to the principle of independence of the action of forces at a given point in the aircraft, it is possible to sum the magnetic fields coming from individual parts of the aircraft and to subject them to the equivalent effect of a single magnetized bar located at a certain point. However, if we take into account the diverse nature of the action of the hard and soft magnetized iron on different courses and during different motions of the aircraft, it is better to subject this field to the equivalent action of bars which have a constant and varying magnetization.

Let us assume that the equivalent bar of hard magnetized iron is located horizontally and coincides with the direction of the longitudinal axis of the aircraft (Fig. 2.6).

With a magnetic course of the aircraft equal to zero, the vector $\bar{F}$ of the field intensity of the bar coincides in direction with the horizontal component of the magnetic field of the Earth /124
$\bar{H}$, which does not produce any deviation of the compass card from the plane of the magnetic meridian.

In the case of aircraft courses equal to 90 or 270°, the vector $\bar{F}$ of the field intensity of the bar is located at right angles to the vector $\bar{H}$, producing maximum deviation of the card from the plane of the magnetic meridian.

Hence, when the aircraft is turning around its vertical axis through 360°, the resultant vector (P_t) of the hard magnetic iron and the magnetic field of the Earth will coincide at two points with the direction of the magnetic meridian, and will be at a maximum distance from it at two other points. *Deviation* of this kind *is called semicircular,* i.e., it has zero value with every 180° rotation of the aircraft (Fig. 2.7).

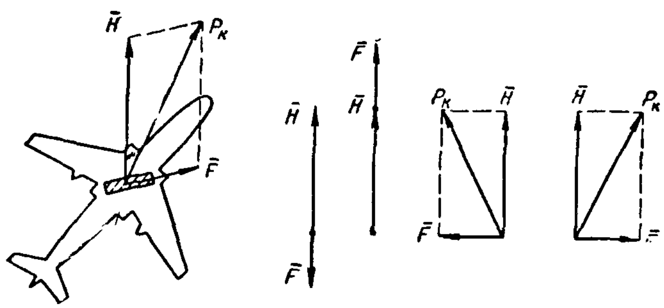


Fig. 2.6. Deviation of Compass Card by a Bar of Hard Magnetic Iron.

Finally, it cannot be expected that in the general case the equivalent bar of hard magnetic iron will coincide in direction with the longitudinal axis of the aircraft. However, this does not alter the nature of the semicircular deviation, but only shifts the graph of deviation relative to the course scale of the aircraft by an angle which is equal to that between the axis of the aircraft and the axis of the equivalent bar.

Semicircular deviation of a magnetic compass can be compensated easily. To do this, it is sufficient to make a bar of hard magnetic iron and place it near the compass installation in such a way that its field is opposite to the direction of the field of the equivalent bar of hard magnetic iron.

Let us now assume that there is no hard magnetic iron aboard the aircraft, but a field of soft magnetic iron is located horizontally and contributes to the action of the equivalent bar, coinciding in direction with the longitudinal axis of the aircraft.

The essence of the effect of the soft magnetic iron on the compass readings consists in the fact that the bar, which is located in a certain position relative to the magnetic field of the Earth, is not magnetized in the direction of the field but along the length of the bar. /125

The magnetization of the bar can be expressed by the formula

$$B = \mu H \cos \alpha, \quad (2.4)$$

where B is the magnetic induction, μ is the magnetic permeability of the bar, H is the intensity of the magnetic field, and α is the angle between the direction of the intensity vector of the field and the direction of the bar.

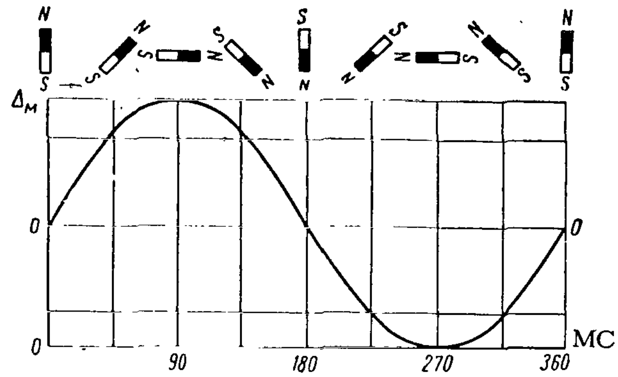


Fig. 2.7. Graph of Semicircular Deviation.

On courses 0 and 180°, in this case, the direction of the equivalent bar coincides with the direction of the horizontal component of the vector of intensity of the magnetic field of the Earth (α = 0); although the magnetic induction of the bar is maximum, there will be no compass deviation.

In changing the aircraft course from 0 to 90° or from 0 to 270°, the magnetic induction of the bar will decrease, but the angle between the vectors H and B will increase. It is obvious that the deviation will then reach its maximum at a course of 45 or 315° and will reach zero once again on courses of 90 and 270°. A similar change in deviation will occur in the flight sectors from 90 to 180° and from 180 to 270° (Fig. 2.8).

It is clear in the figure that the deviation from the soft magnetic iron during one complete turn of the aircraft around the vertical axis passes through zero four times, i.e., it has a quarternary nature.

The action of one magnetic bar of soft iron clearly illustrates the quarternary nature of the alternating magnetic field of the aircraft. In practice, however, with the exclusion of rare cases, the alternating magnetic field of the aircraft cannot amount to the effect of one bar of soft magnetic iron.

In fact, if we take two bars of soft iron and locate them at /126
90° to one another, the resultant vector of induction of the bars will coincide with the bisectrix between them ($B_1 = B_2$) only in the case when the intensity vector of the magnetic field (H_1) coincides with the bisectrix of the angle between the bars (Fig. 2.9). In all other cases, the induction vector will approach the axis of the bar which is closer to the intensity vector of the magnetic field.

If we consider the action of one bar, the vector of magnetic induction will change in value but will always coincide with the axis of the bar. This essentially explains the existence on the aircraft of both semicircular and quarternary deviation as well as deviations of higher order.

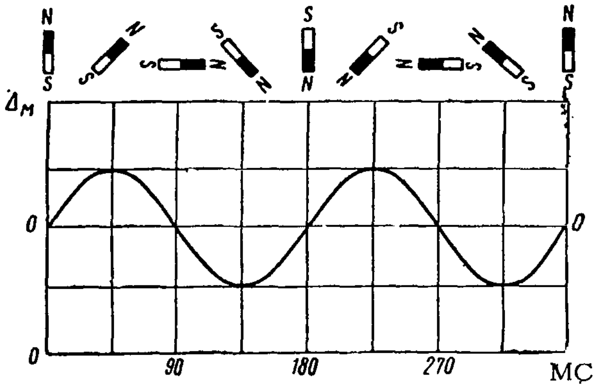


Fig. 2.8.

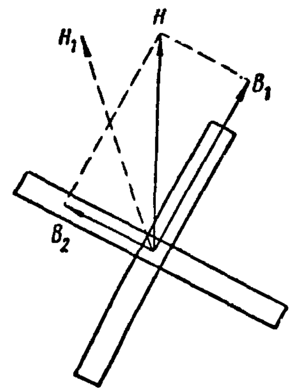


Fig. 2.9.

Fig. 2.8. Graph of Quarternary Deviation from Soft Magnetic Iron.

Fig. 2.9. Magnetic Induction of Crossed Bars of Soft Iron.

In addition, if we disregard the deviation of higher order, the deviation from soft magnetic iron cannot be eliminated by using a suitable bar of soft iron, since it will also be magnetized like all other parts of the aircraft and will not lead to a reduction but rather to an increase of the deviation.

Equalizing the Magnetic Field of the Aircraft

The cause of magnetic compass deviation on board an aircraft is generally a lack of coincidence between the resultant components of the magnetic field of the aircraft with the vector of intensity of the Earth's magnetic field.

When the aircraft rotates around its axis, the alternating magnetic field of the aircraft not only rotates along with it, but simultaneously changes in magnitude and sign. Therefore, in order to determine the magnitude and sign of the deviation for various aircraft courses, it is advisable to express its field components in the form of forces acting along the axes of the aircraft. /127

Obviously, the magnitude of these forces (with the exception of the components made of hard magnetic iron) will vary with changes in the magnetic course of the aircraft (γ_M).

Depending on the nature and character of the action of the components of the magnetic field on the sensitive element of the compass, we can divide them into three groups:

(1) Components of the magnetic field of the Earth along the axes of the aircraft; their designations coincide with the designations for the aircraft axes *X*, *Y*, *Z*. The resultant vector of these components is $\overline{T}$.

(2) The components of the magnetic field of the aircraft made of hard magnetic iron have the designations: *P* along the *X*-axis of the aircraft; *Q* along the *Y*-axis, and *R* along the *Z*-axis.

(3) Components of soft magnetic iron of the aircraft. As follows from what has been said above, they cannot be viewed as a simple part of the resultant vector along the axes of the aircraft.

For convenience in mathematical operations, these components lead to an equivalent effect of nine bars of soft magnetic iron, of which three bars coincide with each of the axes of the aircraft. This means that each of the three bars which coincide with a given axis of the aircraft is magnetized by a component of the magnetic field of the Earth which is located only along some one axis of the aircraft.

Equivalent bars *a, b, c* are located along the *X*-axis of the aircraft; bar *a* is magnetized by the component of the magnetic field of the Earth *X*, bar *b* by component *Y*, and bar *c* by component *Z*.

Equivalent bars *d, e, f* are located along the *Y*-axis, and bars *g, h, k* are located along the *Z*-axis; they are magnetized by the same components of the vector $\overline{T}$.

The contribution of the magnetic field of the soft iron in the aircraft to the equivalent effect of nine bars acquires physical significance in summing the magnetic induction of the components of the vector $\overline{T}$, along the axes of the aircraft.

For example, the *X*-component of the magnetic field of the Earth acts on bars *a, d, g*, and the resultant induction from these three bars shows how the vector of the magnetic field from the soft magnetic iron of the aircraft lX would be located if the components of the magnetic field of the Earth *Y* and *Z* were equal to zero.

In other words, the equivalent bars are equivalent to the vectors of division of the magnetic induction from the components of the magnetic field of the Earth along the axes of the aircraft (Table 2.1).

In summing the magnetic forces along the axes, we obtain the equations for the magnetic field of the aircraft: /128

$$\left.\begin{aligned} \overline{X'} &= \overline{X} + \overline{P} + \overline{aX} + \overline{bY} + \overline{cZ}; \\ \overline{Y'} &= \overline{Y} + \overline{Q} + \overline{dX} + \overline{lY} + \overline{fZ}; \\ \overline{Z'} &= \overline{Z} + \overline{R} + \overline{gX} + \overline{hY} + \overline{kZ}. \end{aligned}\right\} \quad (2.5)$$

These fields will be used as a basis for deriving formulas for the deviation of magnetic compasses on an aircraft.

TABLE 2.1

axis of the aircraft	Resultant forces				
	T	E	lX	mY	nZ
OX	X	P	aX	bY	cZ
OY	Y	Q	dX	eY	fZ
OZ	Z	R	gX	hY	kZ

The sum of the vectors $\overline{X}'$, $\overline{Y}'$ and Z' gives a total vector $\overline{T}'$ acting on the sensitive element of the compass.

Deviation Formulas

In the equations of the magnetic field of the aircraft, the constant terms are only the components of the field of the hard magnetic iron, P, Q, R. However, to calculate the deviation in horizontal flight, we can consider that the magnetic induction Z from the vector $\overline{T}$ is constant along the vertical axis of the aircraft (terms cZ, fZ, kZ).

In addition, horizontal flight will not involve the third equation in (2.5), determining Z'.

If we also consider that the sum of the vectors X and Y constitutes the horizontal component of the magnetic field of the Earth H,

$$X = H\cos\gamma;$$
$$U = H\sin\gamma,$$

the first two equations in (2.5) can be rewritten to read as follows:

$$\left.\begin{aligned} X' &= H\cos\gamma + aH\cos\gamma - bH\sin\gamma + cZ + P; \\ U' &= H\sin\gamma + dH\cos\gamma - eH\sin\gamma + fZ + Q, \end{aligned}\right\} \qquad (2.6)$$

where γ is the magnetic course of the aircraft.

The vectors X', Y' are the components of the magnetic field along the longitudinal and transverse axes of the aircraft at the locus of the compass.

The magnetic compass deviation (δ) is expressed by the angle between the direction of the horizontal component of the magnetic field of the Earth H and the horizontal component of the total magnetic field on the aircraft H' (Fig. 2.10). /129

Obviously, tgδ is equal to the ratio of the projection of vector H' in a direction perpendicular to the magnetic meridian H'', to its projection on the magnetic meridian H''':

$$\operatorname{tg}\delta = \frac{H'''}{H''} = \frac{X'\sin\gamma + Y'\cos\gamma}{X'\cos\gamma - Y'\sin\gamma}. \tag{2.7}$$

If we substitute into Equation (2.7) the values of X' and Y' from Equation (2.6), and also reduce similar terms, replacing the values $\sin\gamma\cos\gamma$, $\sin^2\gamma$ and $\cos^2\gamma$ by their obvious homologues $\frac{1}{2}\sin^2\gamma$, $\frac{1}{2}(1-\cos^2\gamma)$ and $\frac{1}{2}(1+\cos^2\gamma)$, we will have:

$$\operatorname{tg}\delta = \frac{\frac{d-b}{2}H + (cZ+P)\sin\gamma + (fZ+Q)\cos\gamma + \frac{a-e}{2}H\sin 2\gamma + \frac{d+b}{2}H\cos 2\gamma}{H + \frac{a+e}{2}H + (cZ+P)\cos\gamma - (fZ+Q)\sin\gamma + \frac{a-e}{2}H\cos 2\gamma - \frac{d+b}{2}H\sin 2\gamma}. \tag{2.8}$$

The terms in Equation (2.8), with a coefficient equal to unity, have a constant character, i.e., they are independent of the aircraft course at a given magnetic latitude. The terms which have the coefficients 2 sin γ and 2 cos γ have a quarternary character. The terms with coefficients sin γ and cos γ have a semicircular character.

All of the forces designated by values located in the numerator of Equation (2.8) are directed at an angle of 90° to the magnetic meridian while those in the denominator coincide with it.

The force $\frac{d-b}{2}H$ is independent of the aircraft course; it is proportional to the horizontal component of the magnetic field of the Earth and is directed at an angle of 90° to the magnetic meridian. This force is related to the magnetization of the soft iron of the aircraft by the magnetic field of the Earth and varies as a function of the magnetic latitude of the locus of the aircraft. We will designate this force by $A_0\lambda H$.

The force $cZ+P$ is directed along the longitudinal axis of the aircraft; it is the result of the longitudinal component of the field from the hard magnetic iron P and the induction from the vertical

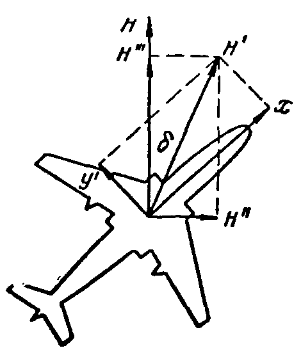


Fig. 2.10. Deviation of Magnetic Compass Aboard an Aircraft.

component of the magnetic field of the Earth. This force is designated $B_0\lambda H$ and changes with the magnetic latitude of the aircraft location only in accordance with the first term. The projection of the force on the normal to the magnetic meridian is proportional to the sine of the magnetic course of the aircraft. /130

The force $fZ+Q$ is designated by $C_0\lambda H$, and is analogous in the nature and character of its changes to the force $B_0\lambda H$, but is directed along the transverse axis of the aircraft. Consequently, its projection on the normal to the magnetic meridian is proportional to the cosine of the magnetic course of the aircraft.

The forces $\frac{a-b}{2}$ and $\frac{d+b}{2}$ are related to the soft magnetic iron on the aircraft, magnetized by the magnetic field of the Earth. The former is designated $D_0\lambda H$ and coincides with the direction of the double course of the aircraft; the latter is $E_0\lambda H$ and is perpendicular to the double course of the aircraft.

The force $H+\frac{a+b}{2}H$ is designated λH. In Equation (2.8), it is in the denominator, and therefore coincides with the direction of the magnetic meridian.

If we substitute into Equation (2.8) these designations for the forces and divide the numerator and denominator by λH, the latter will give us

$$\operatorname{tg}\delta=\frac{A_0+B_0\sin\gamma+C_0\cos\gamma+D_0\sin 2\gamma+E_0\cos 2\gamma}{1+B_0\cos\gamma-C_0\sin\gamma+D_0\cos 2\gamma-E_0\sin 2\gamma}. \quad (2.9)$$

Expression (2.9) is called the point-deviation formula, and the coefficients A_0, B_0, C_0, D_0 and E_0 are the point coefficients of deviation.

The point-deviation formula is inconvenient to use, so it has been simplified for practical purposes.

Since it is almost always necessary to select a place for mounting the compass on the aircraft where the deviation does not exceed 8-10°, we can let $\operatorname{tg}\delta=\delta$.

The denominator of Formula (2.9) can be expressed in the form of a binomial:

$$[1+(B_0\cos\gamma-C_0\sin\gamma+D_0\cos 2\gamma-E_0\sin 2\gamma)]^{-1}=(1+a)^{-1}.$$

We know that with $a<1$, the expansion of the binomial gives the converging series:

$$(1+a)^{-1}=1-a+a^2-a^3\ldots$$

For practical purposes, we can limit ourselves to the first two terms of the series

$$(1+a)^{-1} \approx 1-a,$$

so that Equation (2.9) assumes the form:

$$\delta=(A_0+B_0\sin\gamma+C_0\cos\gamma+D_0\sin 2\gamma+E_0\cos 2\gamma)(1-B_0\cos\gamma+ \\ +C_0\sin\gamma-D_0\cos 2\gamma+E_0\sin 2\gamma). \quad (2.10)$$

Having carried out the multiplication of the multipliers, reduced the similar terms, and carried out simple trigonometric conversions, Equation (2.10) assumes the form: /131

$$\delta=A+B\sin\gamma+C\cos\gamma+D\sin 2\gamma+E\cos 2\gamma+F\sin 3\gamma+G\cos 3\gamma+ \\ +H\sin 4\gamma+K\cos 4\gamma\ldots \quad (2.11)$$

Here the coefficients *A*, *B*, *C*, *D*, *E* have a somewhat different value than in the point-deviation formula:

$$A=A_0;\quad B=B_0+A_0C_0;\quad D=D_0+\frac{B_0^2}{2}+\frac{C_0^2}{2}+A_0E_0;$$

$$C=C_0-A_0B_0;\quad E=E_0-B_0C_0-A_0D_0.$$

The coefficients of deviation of higher orders, i.e., proportional to the sines and cosines 3γ, 4γ,..., can be disregarded, since they are much smaller than any of the first five coefficients. Then Formula (2.11) assumes the form:

$$\delta=A+B\sin\gamma+C\cos\gamma+D\sin 2\gamma+E\cos 2\gamma, \quad (2.12)$$

where *A* is the coefficient of constant deviation, *B*, *C* are the coefficients of semicircular deviation, and *D*,*E* are the coefficients of quarternary deviation.

Formula (2.12) is called the approximate formula of deviation, and its coefficients are the approximate deviation coefficients. However, it is completely satisfactory for practical applications, especially if we recall that other factors are acting on the compass which are very difficult to allow for.

Calculation of Approximate Deviation Coefficients

We will assume that we know the deviation of a magnetic compass at eight symmetrical points: 0, 45, 90, 135, 189, 225, 270 and 315°.

According to Equation (2.12), the deviation at these points must have the values:

$$\delta_0=A+B\sin 0°+C\cos 0°+D\sin 0°+E\cos 0°.$$

Since $\sin 0° = 0$, $\cos 0° - 1$, then $\delta_0 = A + C + E$;

$$\delta_{45} = A + B\sin 45° + C\cos 45° + D\sin 90° + E\cos 90°$$

or, if we consider the values $\sin 90° = 1$, $\cos 90° = 0$,

$$\delta_{45} = A + B\sin 45° + C\cos 45° + D.$$

Similarly, we can obtain a system of equations for the deviation of the eight points: /132

$$\left.\begin{aligned}
\delta_0 &= A + C + E;\\
\delta_{45} &= A + B\sin 45° + C\cos 45° + D;\\
\delta_{90} &= A + B - E;\\
\delta_{135} &= A + B\sin 45° - C\cos 45° - D;\\
\delta_{180} &= A - C + E;\\
\delta_{225} &= A - B\sin 45° - C\cos 45° + D;\\
\delta_{270} &= A - B - E;\\
\delta_{315} &= A - B\sin 45° + C\cos 45° - D.
\end{aligned}\right\} \quad (2.13)$$

Summing Equation (2.13), we obtain:

$$\delta_0 + \delta_{45} + \delta_{90} + \delta_{135} + \delta_{180} + \delta_{225} + \delta_{270} + \delta_{315} = 8A$$

or

$$\sum_{i=0}^{8} \delta_i = 8A,$$

consequently,

$$A = \frac{\sum_{i=0}^{8} \delta_i}{8}.$$

To find the approximate deviation coefficient B, we multiply each of the Equations (2.13) by the coefficient at B, depending on the aircraft course. Then, keeping in mind the fact that $\sin 45° = \cos 45°$, the equations for δ_0 and δ_{180} become zero and the remainders assume the form:

$$\begin{aligned}
\delta_{45}\sin 45° &= A\sin 45° + B\sin^2 45° + C\sin^2 45° + D\sin 45°;\\
\delta_{90} &= A + B - E;\\
\delta_{135}\sin 45° &= A\sin 45° + B\sin^2 45° - C\sin^2 45° - D\sin 45°;\\
-\delta_{225}\sin 45° &= A\sin 45° + B\sin^2 45° + C\sin^2 45° - D\sin 45°;\\
-\delta_{270} &= A + B + E;\\
-\delta_{315}\sin 45° &= A\sin 45° + B\sin^2 45° - C\sin^2 45° + D\sin 45°.
\end{aligned}$$

In summing the six remaining equations, the sum of the terms containing coefficient A becomes zero, since three of them have a plus sign and the remaining three, symmetrical to the first, have a minus sign.

The sum of the terms containing coefficient B is equal to

$2B + 4B \sin^2 45°$, but since $\sin^2 45° = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$, this sum will be equal to $4B$.

The sum of the terms containing coefficient C, as well as the sum of the terms containing coefficient D, is equal to zero. /133

Consequently,

$$\sum_{i=0}^{8} \delta_i \gamma_i = 4B$$

or

$$B = \frac{1}{4} \sum_{i=0}^{8} \delta_i \sin \gamma_i.$$

Similarly, we can find the formulas for determining the remaining three coefficients:

$$\left.\begin{aligned} C &= \frac{1}{4} \sum_{i=0}^{8} \delta_i \cos \gamma_i; \\ D &= \frac{1}{4} \sum_{i=0}^{8} \delta_i \cos 2\gamma_i; \\ E &= \frac{1}{4} \sum_{i=0}^{8} \delta_i \cos 2\gamma_i. \end{aligned}\right\} \qquad (2.14)$$

Change in Deviation of Magnetic Compasses as a Function of the Magnetic Latitude of the Locus of the Aircraft

The deviation of a magnetic compass determined for a given point on the Earth's surface, does not remain fixed for other points, but changes depending on the magnetic latitude of the locus of the aircraft.

Obviously, a change in deviation cannot take place as a result of changes in the magnetic induction of soft magnetic iron from the horizontal component of the magnetic field of the Earth.

By the same token, the induction from the component producing the deviation will change in the same proportion with a change in the horizontal component of the magnetic field of the Earth, as the principal directional position of the compass card. Consequently, the compass deviation remains constant.

The deviation from magnetic induction of the horizontal component of the field of the Earth has a constant and quarternary character:

$$A_0\lambda H=\frac{d-b}{2}H;\quad D_0\lambda H=\frac{a-e}{2}H;\quad E_0\lambda H=\frac{d+b}{2}H.$$

Hence, we reach the conclusion that the constant and quarternary deviation at various magnetic latitudes remains constant.

Essentially, the change in the deviation with a change in the magnetic latitude is the result of the influence of hard magnetic iron and partially as a result of induction with soft magnetic iron from the vertical component of the Earth's magnetic field. This takes place because the magnitude of the vectors $\bar{P}$, $\bar{Q}$ remains constant with a change in the directional vector H. Consequently, with an increase in the magnetic latitude, the semicircular deviation must increase. /134

In addition, with an increase in the magnetic latitude, the induction of the soft magnetic iron from the vertical component of the Earth's field increases with a simultaneous decrease in the directional force H. However, if we consider the predominant influence on the aircraft produced by the hard magnetic iron, we can consider in approximation that the semicircular deviation is inversely proportional to the horizontal component of the magnetic field of the Earth.

$$B_0=\frac{cZ+P}{\lambda H};\quad C_0=\frac{fZ+Q}{\lambda H},$$

which gives the following for the approximate coefficients of deviation B and C:

$$B_2=B_1\frac{H_1}{H_2};\quad C_2=C_1\frac{H_1}{H_2}, \tag{2.15}$$

where B_1, C_1, H_1 are the approximate coefficients and the horizontal component of the Earth's field at the point where the deviation is measured; B_2, C_2, H_2 are the same values at a point with a different magnetic latitude.

With known coefficients $B + C$, the semicircular deviation at a given point on the Earth's surface can be determined by the formula

$$\delta=B\frac{H_1}{H_2}\sin\gamma+C\frac{H_1}{H_2}\cos\gamma. \tag{2.16}$$

Elimination of Deviation in the Magnetic Compasses

Modern magnetic compasses are fitted with a device for compensating only semicircular deviation, resulting from hard magnetic iron.

In addition, by a suitable rotation of the compass housing in its mountings, we can compensate for the constant component of deviation along with the adjustment error of the compass.

Elimination of quarternary deviation by magnetic means encounters considerable technical difficulty. Therefore, if we keep in mind the relatively low value of the quarternary deviation relative to the semicircular deviation, as well as its constant value at various latitudes, we will not be able to get rid of the latter but will enter it on special graphs for compass correction.

Modern remote control magnetic compasses have devices for mechanical compensation of deviation of all orders. /135

The device for compensating semicircular deviation consists of a system of four cylinders mounted in pairs, with permanent magnets installed in them (Fig. 2.11).

The cylinders, intended for compensating for deviation in 0 and 180° courses, in which the field of hard magnetic iron produces the deviation, are arranged along the transverse axis of the aircraft, and placed parallel to the axis of the aircraft in such a way that when they are rotated, the small magnets can turn from a vertical position to one which is coincident with the transverse axis of the aircraft. With a vertical position of the magnets, their field does not have any influence on the position of the compass card (neutral position, Fig. 2.11, a).

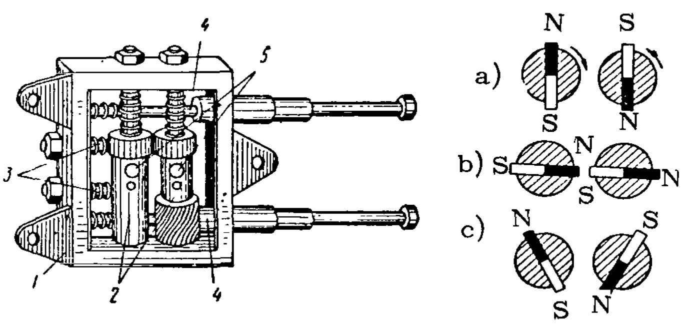


Fig. 2.11. Device for Correcting Semicircular Deviation of the Compass. (1) Frame; (2) Transverse Cylinders; (3) Longitudinal Cylinders; (4) Actuating Cylinders; (5) Embedded Magnets.

When the magnets are tilted (Fig. 2.11, c), the horizontal component of their field appears, and can be set so that it is equal but directed opposite to the magnetic field of the aircraft (horizontal component), located along its transverse axis. The maximum effect of the small magnets will be observed when they are in the horizontal position (Fig. 2.11, b).

The cylinders for compensating deviation at courses of 90 and 270° are mounted in the transverse axis of the aircraft in such a way that the small magnets can be used to compensate for the component of the magnetic field of the aircraft which is directed along its longitudinal axis.

The rotation of the longitudinal and transverse cylinders is accomplished by means of special handles made of diamagnetic material.

To determine and get rid of deviations, the aircraft is placed on a specially prepared stand, made of concrete (for heavy aircraft) /136 but without a metal core.

The stand must be of sufficient size so that aircraft of any kind can be rotated in a circle and the distance from the stand to other aircraft and metal structures is at least 200 m.

The accuracy of the setting of the aircraft on a given course for determining and getting rid of deviation can be checked in one of the following two ways:

1. **Direction finding of landmarks from on board the aircraft.** In the center of the area where the aircraft is to turn, a magnetic direction finder or theodolite is mounted on a stand so that the indicating dial is located exactly in a horizontal position, and the zero reading on the dial coincides with the direction of the magnetic meridian. For this purpose, these instruments are fitted with a bubble level and orienting magnetic needle.

Then two or three distinct and prominent landmarks on the horizon are selected (towers and chimneys are best for this purpose), and their magnetic bearings (MB) are determined with the aid of a sight, rotating on the dial used for determining the bearings.

The landmarks should be located as far as possible from the area so that the shifting of the aircraft from its center during rotation will not produce any noticeable changes in the bearings of the landmarks. For light aircraft, this distance should be at least 2-3 km; for larger aircraft with a greater radius of turn on the ground, it should be at least 5-6 km.

After determining and recording the magnetic bearings of the landmarks, the aircraft is mounted on the stand. The direction finder is placed in front of or behind the aircraft at a distance of 20-100 m, depending on the length of the aircraft, exactly along its longitudinal axis so that the forward and rear points on the axis of the aircraft will be projected on the sight, e.g., the centers of the nose and keel. Then the dial on the direction finder is set to the magnetic meridian, and the direction of the longitudinal axis of the aircraft is measured, and its initial course is set.

It is necessary to recall that the minimum distance for the direction finder from the aircraft is limited by the effect of the aircraft on the magnetic needle of the deviation direction finder, and the maximum distance is set by the length of the aircraft, since at a distance of more than 100 m, with an aircraft which is not very long, this method will be insufficiently precise.

After the direction finder has been moved to the aircraft, the magnetic needle is fixed and set so that one of the selected landmarks (Fig. 2.12) appears at a course angle (CA) equal to

$$CA = MBL - Mc \tag{2.17}$$

where MBL equals the magnetic bearing of the landmark and MC is the initial magnetic course of the aircraft.

If the above condition is satisfied, the zero point on the direction finder dial will coincide exactly with the longitudinal axis of the aircraft. /137

To set the aircraft on definite courses, a table of course angles for landmarks for each aircraft course is compiled.

For example, if the deviation has been determined at eight points, but the selected landmarks have magnetic bearings of 115 and 328°, then the course angles for the courses which we require will have the values shown in Table 2.2.

TABLE 2.2.

MC	Cal_1(MBL=115°)	Cal_2(MBL=328°)	CC	Δ_K
0	115	328	358	+2
45	70	283	42	+3
90	25	238	91	−1
135	340	193	133	+2
180	295	148	178	+2
225	250	103	224	+1
270	205	58	271	−1
315	160	13	313	+2

When using this table, the sight of the direction finder is set to a given course angle for a landmark and the aircraft is then turned until the axis of the sight lines up with the direction of the selected landmark. It is clear that the aircraft is then set precisely on the desired course.

The second landmark is an extra one in case the first is obstructed by some part of the aircraft such as the empennage or wing.

The method of setting an aircraft on course by the method described above for obtaining the course angles of landmarks is the most precise and reliable one, especially since a fixed area can be set up at an airport for correcting deviations and doing other work to set the bearings of landmarks and compiling tables of course angles for given aircraft courses.

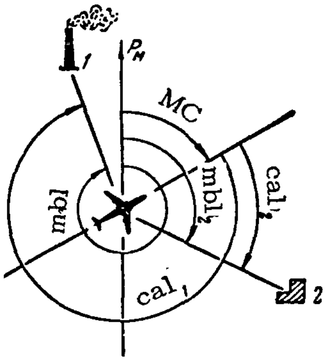


Fig. 2.12. Determination of Aircraft Course by the Course Angle of a Landmark.

However, this method is not always practicable. In some cases, it may be impossible to select suitable landmarks, and in other cases the visibility may be inadequate for them to be seen. In some aircraft, there may be difficulty in fastening the direction /138 finder on board the aircraft in a place where there would be a clear field of vision for observing the landmarks.

2. **Direction finding of an aircraft from the nose or tail.** This method is used in cases when it is impossible to set the aircraft on courses of 0, 45, 90°, etc., by the method described above.

In this case, the aircraft is set each time(e.g., according to the readings of the magnetic compass) to a given course. Then the direction finder is located along the extension of the longitudinal axis at a distance of 20-100 m from the aircraft, depending on the type of the latter; the correcness of the setting of the aircraft on course is then determined as in the first case before mounting the direction finder on board the aircraft. It may be necessary to turn the aircraft for a secondary check.

This method is less convenient than the first, since it is necessary to shift the direction finder for each course, set it exactly along the extension of the aircraft axis, adjust the zero on the dial along the magnetic meridian, and make the dial level, in addition to measuring the distance to the aircraft. Under unfavorable conditions aboard the aircraft, this operation may have to be repeated after moving the aircraft. The advantage of this method is its independence of the existence of landmarks, meteorological visibility, and peculiarities of aircraft design.

Semicircular deviation of magnetic compasses is corrected and eliminated at four basic points: 0, 180, 90 and 270°.

It is clear from (2.13) that semicircular deviation at the 0 and 180° points is equal in value, but opposite in sign, and expressed by the maximum value of coefficient C. Deviation from coefficient *B* is equal to zero on these courses.

However, all of these courses are subject to the action of a constant deviation in the coefficient *A* and quarternary deviation *E* in addition to the semicircular deviation. This means that the values of the constant and quarternary deviation are equal in value and sign.

Consequently, if the deviation on course 0° is set to zero by turning the cylinder of the deviation-correcting apparatus with the marking "N-S", the semicircular deviation will be compensated for and the constant and quarternary deviation will simultaneously be compensated for. It will change with the same sign to a course of 180°, where its value doubles. Therefore, after setting the aircraft to a course of 180°, it is necessary to set the deviation not to zero, but to half the rotation of that cylinder, and in the reverse direction.

Hence, the semicircular deviation from coefficient *C* can be eliminated completely and precisely without disturbing the constant and quarternary deviations.

Analogously, by turning the cylinder of the deviation-correcting apparatus with the marking "E-W", it is possible to reduce the deviation to zero for a 90° course and by half for a 270° course, /139
which completely gets rid of the semicircular deviation from coefficient *B* without disturbing the constant and quarternary deviations.

TABLE 23

MC	Deviation Shown	Up to
0	12	0
180	+4	+2
90	+7	0
270	—2	—1

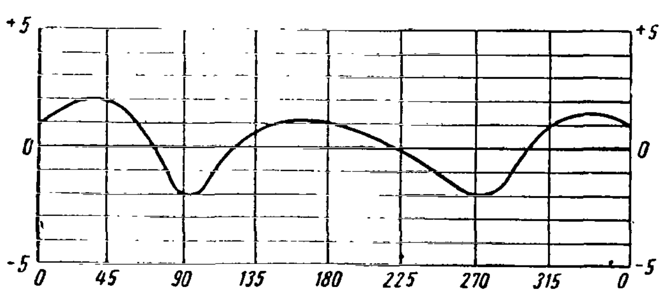


Fig. 2.13. Graph of Deviation of a Magnetic Compass.

The operation with semicircular deviation is described in a special table (Table 2.3).

Obviously, the remaining deviation at these points will be equal to +2° for courses of 0 and 180° and -1° for courses of 90 and 270°.

After getting rid of the semicircular deviation, the aircraft is set to courses at 45° intervals and the remaining deviation is measured. An example of the recording is shown in Table 2.2.

After summing the remaining deviation for eight courses (Graph 5, Table 2.2) and dividing the sum by eight, we obtain the value of the constant deviation

$$A = \frac{2+3-1+2+2+1-1+2}{8} = +1.25°.$$

The bowl of the compass must be set in its mounting to this value. If we disregard the value of 0.25° produced by turning the bowl of the compass through 1°, the remaining deviation for the eight courses will have a value of +1, +2, -2, +1, +1, 0, -2, +1 so that the graph of the corrections can be compared with the readings of the compass (Fig. 2.13).

If the aircraft is intended for use on flights at magnetic latitudes where there will only be small changes, this will mark the end of the work with deviation.

In preparing for long distance flights, with considerable changes in the magnetic latitudes, the coefficients of the semicircular /140
deviation B and C must also be found with determination of their changes with magnetic latitude.

In this case, the coefficient B will be equal to:

$$B = \frac{+1\sin 0° + 2\sin 45° - 2\sin 90° + 1\sin 135° + 1\sin 180° + 0 - 2\sin 270° + 1\sin 315°}{4} =$$

$$= \frac{0 + 1.4 - 2 + 0.7 + 0 + 0 + 2 - 0.7}{4} = 0.35,$$

and coefficient C will be

$$C = \frac{1 + 1.4 - 0 - 0.7 + 1 + 0 - 0 + 0.7}{4} = 0.85.$$

Gyroscopic Course Devices

Regardless of the fact that measures have been employed for a long period of time which are directed toward increasing the accuracy of readings and the stability of operation of integrated magnetic compasses, their shortcomings have not been completely overcome.

In addition, magnetic course devices are difficult to use in a flight along an orthodrome for long distances, due to the complexity of the calculation of the magnetic declination as it changes along the route.

All of this has made it necessary to seek new ways of devising course instruments and systems which will satisfy the requirements of aircraft navigation at all stages and all conditions of flight.

The first steps in this direction were made by the remote control magnetic compasses, containing a magnetic transmitter (a sensitive element) located at any convenient point in the aircraft, whose readings were transmitted by means of special potentiometric transmitters to dials mounted in the cockpit.

This made it possible to mount the compass in the pilot's field of vision and ensure optimum conditions for operation of the compass from the standpoint of deviation. However, there were still considerable shortcomings in the operation of the compass, such as instability of the readings with movement of the aircraft and the impossibility of using it when the aircraft was turning.

In addition, the reliability of operation of the compass decreased, since the potentiometric connection with reliable contacts produced an additional delay in the turning of the sensor card to a significantly greater degree than was the case for the rotation of a freely moving card on its bearing in an integrated compass.

The next steps in increasing the accuracy and reliability of /141
operation of course devices was made by the gyroscopic semicompasses and magnetic course sensors linked with gyroscopic dampers. This made it possible to use the course instruments while the aircraft was turning and to achieve stability of course readings under any flight conditions. Analysis of the induction course sensors, free of friction during turning of an aircraft, significantly increased the reliability of magnetic compasses.

However, the greatest reliability and accuracy in course measurements for aircraft has been achieved by the building of complexes of course instruments (course systems), combining the operation of gyroscopic, magnetic, and astronomic sensors. The principle of these systems is a stable and prolonged maintenance of the system for estimating the course with a gyroscopic assembly having periodic correction of the readings by means of a magnetic or astronomical sensor, or input of corrections manually as desired by the crew.

Principle of Operation of Gyroscopic Instruments

The *gyroscope* is a massive balanced body, rotating around its axis of symmetry at a high angular velocity.

Gyroscopes are usually made in a form such that they have relatively low weight and small size, yet have a maximum inertial moment which is reached relative to the basic mass of the gyroscope as far as possible from the center of rotation within the given dimensions of the gyroscope.

Let us recall that the inertial moment J in mechanics is the product of the mass times the square of the distance to the axis of rotation:

$$J = mr_1^2, \tag{2.18}$$

where r_1 is the distance from the mass to the axis of rotation.

For a complete cylinder, which constitutes the basic mass of a gyroscope (Fig. 2.14), the inertial moment is

$$J = \frac{m\left(r_2^2 - r_1^2\right)}{2}. \tag{2.19}$$

The gyroscope has two interesting properties which are used in a number of devices for pilotage and navigation:

(1) *Axial stability*, i.e., the ability to maintain the direction of its axis of rotation in space in the absence of moments of external forces tending to change this direction;

(2) *Axial precession* of rotation under the influence of moments of external force, i.e., a slow rotation of the axis in a plane which is perpendicular to the applied force, with maintenance of the direction in the plane of the application of the force.

The first property of the gyroscope is usually used for stabilizing the directions of the axes of the coordinates for determining the required values, the banking of the aircraft, the angle of pitch, and the course. The second property is used to set the axis of the gyroscope in the desired position, e.g., to the vertical of the locus of the aircraft, to the plane of the true horizon, for compensation of the apparent rotation of the axis due to the diurnal rotation of the Earth, etc. In addition, the property of precession is sometimes employed in devices which integrate the /142 action of the forces with time, e.g., in the construction of inertial navigation devices.

To explain the principles of operation of gyroscopic devices, let us consider the physical significance of the two properties of a gyroscope mentioned above.

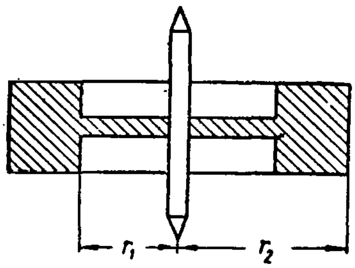


Fig. 2.14. Gyroscope Rotor.

For the sake of simplicity, we shall assume that the mass of the gyroscope is located along the circumference around the axis of rotation (Fig. 2.15), and we shall select an element of this mass at some point on the circumference.

Let us assume that under the influence of a force F, the axis of the gyroscope has been tilted to an angle $\Delta\phi$.

Obviously, the direction of rotation of the element of mass of the gyroscope does not change when it passes through points A and B, since the motion of the element at points A, A_1 and B, B_1 tangent to the circumference remain parallel. The tangents to the direction of motion at the point C and diametrically opposite to it are at an angle equal to $\Delta\phi$.

Consequently, at these points there arises a difference in the velocities

$$\Delta\bar{V} = V \sin \Delta\phi. \qquad (2.20)$$

The greater the angular velocity of rotation of the gyroscope and the radius of the ring, the greater will be the circumferential speed of the element of mass and the magnitude of the vector $\Delta\bar{V}$.

Obviously, the reaction of the mass of the gyroscope must produce resistance to the vector of velocity change at the points C and C_1, i.e., the forces F_p and F_{p_1} arise at these points, directed opposite to vector $\Delta\bar{V}$ and producing the precession of the gyroscope axis.

Hence, the inertia of the mass of the gyroscope will cause precession of the gyroscope axis. The forces producing the precession will in turn cause a tilting of the axis in a plane perpendicular to the action of the external force, thus generating inertial forces analogous to the precession forces but directed against the external force.

It is easy to see that the inertial forces directed against the external force will be exactly equal to the latter, so that no rotation of the axis of the gyroscope in the plane of the action of the external force will be observed. /143

The precession rate of the gyroscope can be determined easily if we know the moment of inertia of the rotor and the moment of the applied external force.

A change in the moment of inertia of the gyroscope with time will be proportional to the moment of the external force

$$\frac{d(J\omega)}{dt} = J\omega\frac{d\varphi}{dt} = J\omega\omega_1 = M, \tag{2.21}$$

whence

$$\omega_1 = \frac{M}{J\omega}, \tag{2.22}$$

where M equals the moment of the external force, J equals the moment of inertia of the gyroscope, ω is the angular velocity of gyroscope rotation, and ω_1 is the angular velocity of precession.

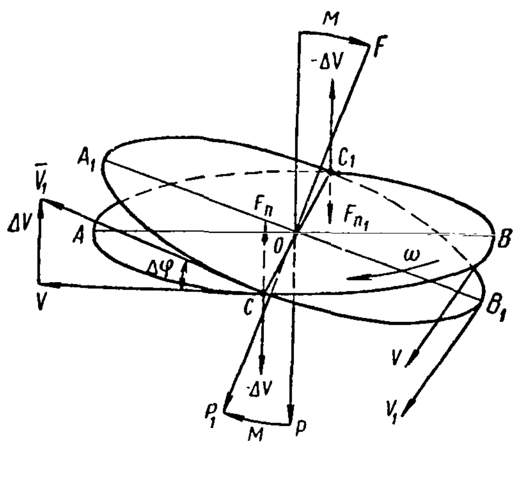


Fig. 2.15. Precession of a Gyroscope Axis.

By the change in the moment of inertia of the gyroscope, we mean here the change in the direction of the vector of inertia.

At the same time, the rotation of the axis of the gyroscope through 180° produces an opposite motion of all points on the rotor, which amounts to a braking of the gyroscope from its initial angular velocity to zero, with a subsequent speeding up in the opposite direction to the same angular velocity.

Degree of Freedom of the Gyroscope

By degrees of freedom in mechanics, we mean the directions of free motion of a body which is not limited by connections of any sort. For example, an object sliding along a given line (rail) has one degree of freedom; an object moving in any direction in a plane has two degrees of freedom, and an object which is moving in three dimensional space has three degrees of freedom.

Besides the degrees of freedom of linear motion, there are /144
also degrees of freedom of rotational motion of a body around its three axes.

Hence, a completely free body has six degrees of freedom.

The rotors of gyroscopes in navigational and pilotage instruments have supports which limit their linear motion in a certain direction relative to the axes of the aircraft, so that when we are talking about the degrees of freedom of a gyroscope we are referring only to the degrees of rotational motion.

A gyroscope is considered to be free if all three degrees of rotational motion are free (Fig. 2.16).

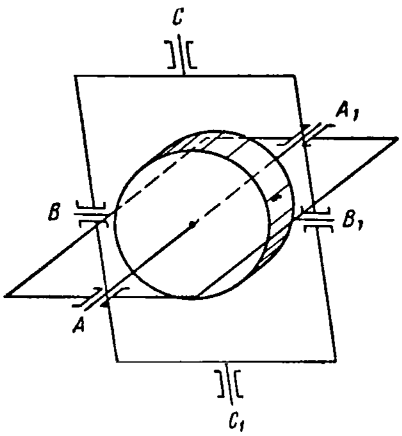


Fig. 2.16. Gyroscope with Three Degrees of Rotational Freedom.

The first degree of freedom of a gyroscope is the rotation of its rotor around the axis in bearings A, A_1. If these bearings are tightly fastened to the body of the machine, as is done for example for the flywheels in machinery, the gyroscope will have only one degree of freedom. However, if these bearings can move around an axis perpendicular to A, A_1 (bearings B, B_1), then there will be two degrees of freedom.

If bearings B, B_1 can also have the freedom to move around still another (third) axis, perpendicular to B, B_1 (bearings C, C_1), the gyroscope will have three degrees of freedom and its axis can be set readily to any direction in space.

As we can see from Figure 2.16, the degrees of freedom of the gyroscope are ensured by pairs of bearings and (with the exclusion of the first) rotating frames.

A gyroscope usually has two rotating frames, internal and external. In course gyroscopic instruments, the internal frame, together with the rotor and the bearings of the gyroscope, serves to set the gyroscope axis in the plane of the true horizon. The same frame contains a sensitive element for correcting the gyroscope axis for this plane. The internal frame of the gyroscope along with the rotor and sensitive element for correction are called the gyro assembly.

The external frame ensures free motion of the axis of the gyroscope in the plane of the horizon; from its position in the unit, we can get an idea of the direction of the gyroscope axis relative to the axis of the aircraft, or vice versa, thus making it possible to determine the aircraft course. /145

Direction of Precession of the Gyroscope Axis

The direction of the precession of the gyroscope axis under the influence of the moment of external forces can be seen in Figure 2.15.

For a rapid and error-free determination of the direction of the precession of the gyroscope axis, we use the concepts of "pole of the gyroscope" and "pole of the external force", and use the rule of the right-hand screw.

For example, in observing the rotation of a gyroscope which is turning clockwise as viewed from the top (turning the screw inward), the pole of the gyroscope will be considered as being located at the lower end of its axis (Points P and P_1); with left-hand rotation of the gyroscope, at the upper end of the axis. Analogously, with a right-hand direction of the moment of external force, the pole of the moment is considered as being directed along the screw, in its rear portion as shown in our diagram (Point C_1). With a left-hand direction of the moment of external force, its pole is located in the front part of the picture (Point C).

The precession of the gyroscope is always directed in such a manner that the pole of the gyroscope attempts to reach the pole of the external force by the shortest path.

In our diagram, the lower end of the gyroscope axis will tilt backward, and the upper one forward, i.e., if we look at the drawing from left to right, the axis of the gyroscope will rotate clockwise.

Apparent Rotation of Gyroscope Axis on the Earth's Surface

A freely moving gyroscope, with an ideally stabilized external and internal support and the lack of noticeable friction in the bearings, tends to keep the position of the axis of rotation of the rotor in space.

On the Earth's surface, however, due to the diurnal rotation of the Earth and partially due to the curvilinearity of its motion around the Sun, there arises an apparent rotation of the gyroscope axis in the vertical and horizontal planes.

The apparent rotation of the gyroscope due to the motion of the Earth around the Sun is expressed as a slight deviation of the rotation of the gyroscope axis from the apparent diurnal rotation of the Earth, as a result of the fact that the Earth makes a complete rotation around the Sun along its orbit in the course of a year. This conditional rotation amounts to a total of about 1/365 of the apparent rotation of the gyroscope due to the diurnal rotation of the Earth. Hence, this value will not be considered in future.

Let us consider the apparent rotation of the gyroscope axis at various points on the Earth's surface, which appears as a result of the rotation of the Earth around its axis. We will assume that /146 we have a freely mounted gyroscope, whose axis at the initial moment coincides with the vertical of the locus (Fig. 2.17, a).

Obviously, if such a gyroscope is placed on a pole of the Earth, the axis of its rotation will coincide with the axis of rotation of the Earth and there will be no apparent rotation of the gyroscope axis (position *A* in the diagram).

If the gyroscope with a vertical axis is placed on some latitude ϕ (position *B* in the diagram), its axis will be at an angle to the axis of rotation of the Earth, equal to $90°-\phi$. As we can see from the diagram, the apparent rotation of the gyroscope axis will describe a cone with an aperture angle at the vertex equal to $2\ (90-\phi)$.

In the case when the latitude of the locus is equal to zero (position *C* in the diagram), the aperture angle of the cone will be equal to 180°, i.e., it will turn in the plane of rotation.

Now let us examine the case when the axis of the gyroscope at the initial moment is located horizontally at various points on the Earth's surface (Fig. 2.17, b) and coincides in direction with the meridian of the Earth.

It is obvious that the axis of the gyroscope located on the pole (position *A*) will remain horizontal and will rotate in the plane of the horizon with the angular velocity of the Earth. The axis of a gyroscope located at some latitude (position *B*) will describe a cone with an aperture angle equal to 2ϕ. The axis of the gyroscope located on the Equator will remain horizontal and will have no apparent diurnal rotation.

It is important to note in this regard that if there is any kind of correcting force which acts constantly on the gyroscope axis in the plane of the true horizon, the angular velocity of the rotation of the gyroscope axis in the plane of the horizon will be equal to (Fig. 2.17, c): at the pole, the angular velocity of rotation of the Earth; at the Equator, zero; at any other point,

$$\omega = \Omega \sin \phi, \qquad (2.23)$$

where Ω is the angular velocity of the Earth's rotation and ω is the angular velocity of the apparent rotation of the gyroscope axis.

From the examples which we have seen, it is clear that a freely moving gyroscope can be used to determine the position of the aircraft axis only in the following cases:

(a) To determine the position of the vertical axis (banking, pitch) only at the poles;

(b) To determine the direction of the longitudinal axis (course of the aircraft) only at the Equator.

In order to render the gyroscope useful for determining the position of the aircraft axis at any other point on the Earth's surface, we used devices which compensate for the apparent rotation of the axis of the gyroscope due to the diurnal rotation of the Earth, as well as its own drift, which arises as a result of imperfect balance, friction in the bearings, etc. /147

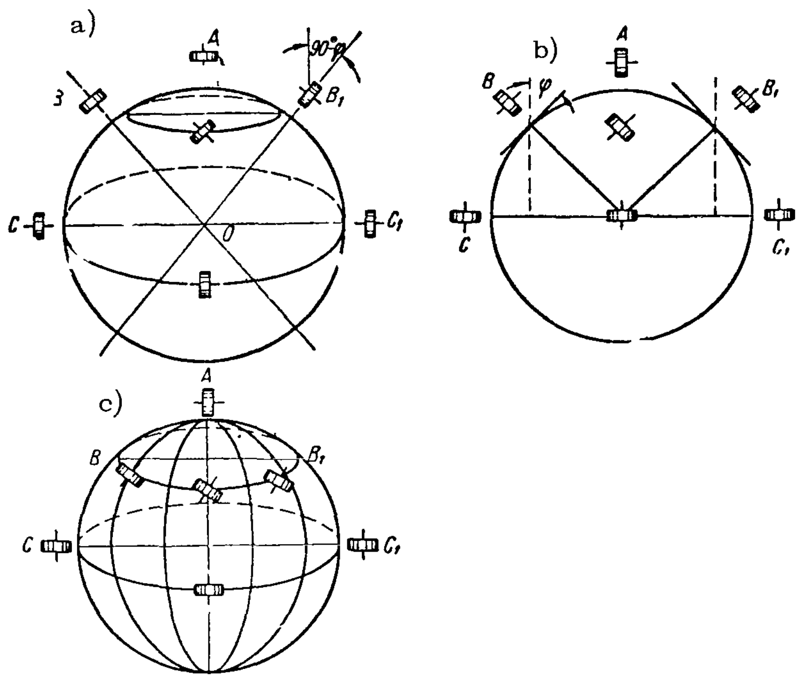


Fig. 2.17. Apparent Rotation of a Gyroscope on the Earth's Surface: (a) With Vertical Axis; (b) With Horizontal Axis; (c) With Constant Correction of the Axis in the Horizontal Plane.

To keep the axis of a gyroscope constantly in the vertical position, pilotage devices (gyrohorizon, gyrovertical), or in the horizontal position in the case of course instruments, are usually fitted with pendulum devices which act as sensitive elements reacting to any deviations which may arise.

The signals from these devices are converted to air currents in pneumatic devices and to moments of special electric motors in electrical devices.

Electrolytic gravitational correction (Fig. 2.18) is most widely used at the present time. This device consists of a bubble level attached to the lower part of the gyro assembly. Unlike a conventional level, its chamber is filled with an electrically conductive liquid (electrolyte), while on the top of the spherical surface are mounted four current-carrying contacts.

When the gyro assembly is in a vertical position (Fig. 2.18, a), the bubble level is located so that all four contacts are covered half-way by electrolyte, so that the moment applied to the frame of the gyro assembly by the correcting motor is equal to zero. /148

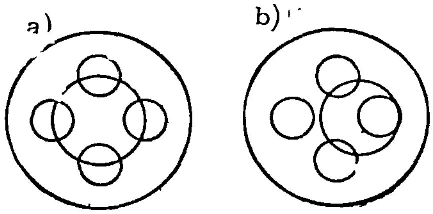


Fig. 2.18. Electrolytic Gravitational Correction.

If for some reason the gyro assembly varies from the vertical position, the current-carrying contacts will not be uniformly covered by the fluid (Fig. 2.18, b), resulting in a suitable distribution of currents to the windings of a small motor and in a moment which is applied to the axis of the gyroscope in such a way that the precession which is produced brings the gyro assembly to a given vertical position. For course devices which have a vertical external frame and a horizontally located axis of the gyroscope, in order to correct the latter to the plane of the horizon, it is sufficient to have one pair of current-carrying contacts with a gravitational level, in order to regulate the moment of the forces acting on the external frame.

Obviously, for those devices which measure direction on the Earth's surface, in addition to devices for correcting the axis of the gyroscope in the plane of the true horizon, there must also be other devices which compensate for the apparent rotation of the axis of the gyroscope in the horizontal plane due to the diurnal rotation of the Earth.

Gyroscopic Semicompass

In principle of operation, the gyrosemicompass (GSC) is a gyroscope with three degrees of freedom and its axis of rotation located in the horizontal, a vertical external frame, and a fluid gravitational corrector, attached to the gyro assembly. The rotation of the gyroscope rotor is produced by alternating three-phase current, while the correction of the axis in the horizontal position is achieved by an electromagnetic moment applied to the external frame.

The gyrocompass has has a very sensitive balance and low friction in the axes of the supports, which ensures a low intrinsic shift of the gyroscope (called *"drift"*). In addition, in order to compensate for this "drift", the gyroscope is fitted in the horizontal plane with a special balancing potentiometer and motor, which apply a moment to the external frame of the gyroscope in the vertical plane.

This same motor is used for compensating the apparent diurnal rotation of the axis of the gyroscope, and is therefore fitted with a special latitudinal potentiometer, which regulates the moment of the motor in such a way that the rate of precession of the gyroscope axis is equal to and coincides in direction with the rate /149 of rotation of the Earth's meridian in the plane of the true horizon at the given latitude.

By comparing the formula for the precession of the gyroscope axis (2.22) and the formula for the angular velocity of rotation of the Earth's meridian (2.23), we can determine the moment which is required to be applied to the gyroscope axis to compensate for the diurnal rotation of the Earth

$$M = \Omega J \omega \sin \phi, \quad (2.24)$$

where M is the moment applied to the gyroscope axis, Ω is the angular rotational velocity of the Earth, J is the inertial moment of the rotor of the gyroscope in the plane of its rotation, ω is the angular velocity of rotation of the rotor, and ϕ is the latitude of the aircraft's location.

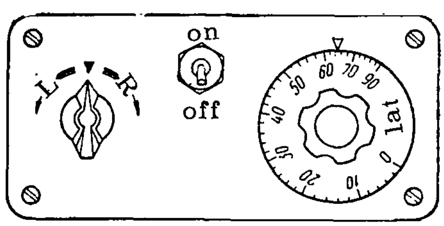


Fig. 2.19. Control Panel of KPK-52 Gyrosemicompass.

With a constant rate of rotation of the rotor of the gyroscope, all of the coefficients which enter into the right-hand side of (2.24), with the exception of $\sin \phi$, are constants. The latter must be regulated in flight. Therefore, the potentiometer which regulates the moment according to the latitude of the aircraft, as well as the balancing potentiometer, are mounted on the control panel of the gyrocompass (Fig. 2.19).

The external frame of the gyroscope is fitted with a scale for estimating the gyroscopic course and a selsyn-transmitter for transmitting the course to the indicators.

The indicating dial and the selsyn-transmitter are free to rotate along with the external frame and can also be set with the aid of a motor to any angle relative to the frame. The setting of the indicator dial to the zero position is accomplished manually by turning a special handle on the control panel marked "L-R" (left - right), see Figure 2.19.

Hence, the gyrocompass is a sort of "keeper" for the course calculation set by hand: the direction of the zero setting of the course on the GSC remains constant in the plane of the horizon, so that the gyrocompass is an *orthodromic course device*, and is capable of guiding a flight along an orthodrome over any distance. The advantage of a gyrocompass is its independence of operation from the magnetic field of the Earth, and consequently the fixed accuracy and stability in operation at any point on the Earth's surface, as well as the ease of determining the course without any kind of methodological corrections; this is particularly important for automatic navigational devices. /150

However, the gyrosemicompass is not a measuring device, but one which retains the course setting (this is where it gets its name of semicompass); therefore, it cannot be used alone without other course sensors. Nevertheless, it does not reduce the value of the gyrosemicompass, since the use of other course sensors becomes necessary only in the initial setting of the readings of the GSC and at various points to make corrections for the accumulated errors in its operation.

It is relatively easy to eliminate errors in the operation of the GSC, which arise in the form of "drift". For this purpose, the operation of the GSC is tested on the ground for a period of one to two hours with an attempt being made to use the rotation of the balancing potentiometer to set the minimum excursions of the needle with time from the true settings.

If a considerable deviation of the needle from the correct readings of the gyroscope is noticed during flight, this can be corrected by shifting the latitude scale on the control panel relative to the average latitude of the given path segment. This means that the degree by which the scale is shifted for each degree at the time that the drift occurs will be the following at various flight latitudes:

Range of Latitudes, Degrees	Magnitude of Scale Deviation, Degrees
0 - 32	4
32 - 42	5
42 - 60	6
60 - 70	10
70 - 90	20

The latitude on the scale must be increased if the tendency of the GSC is directed toward a reduction of the readings for the course with time, and it must be reduced if the course readings increase with time.

It should be mentioned that all shifting mentioned above with regard to the gyrosemicompass is in reference to northern latitudes. In southern latitudes, the latitudinal compensations for the apparent rotation of the axis of the gyroscope must be reversed, since the rotation of the meridian takes place in the opposite direction relative to the northern latitudes. In addition, the system for introducing corrections to the movement of the needle of the gyrosemicompass must also be shifted to the opposite direction.

Shortcomings of the gyrosemicompass include the fact that it is necessary to set its readings manually at the beginning of a flight and to make corrections en route. During flight, especially in rough air, this involves a certain amount of difficulty, since it is impossible to separate the movement of the indicator needle due to course variations from those motions which are caused by setting the course manually, i.e., the value of the course to which /151 the GSC must be set becomes variable.

In addition, the GSC is subject to Cardan errors during turns.

The essence of the Cardan errors is the shift in the reading of the indicator dial during banking. When the aircraft is banking less than 8°, these errors do not have any practical significance, but they rapidly increase with the degree of banking and can reach 6-8°.

The Cardan errors have a quaternary nature. They are equal to zero in banking in the plane of rotation of the rotor of the gyroscope and in the plane of the position of the axis of its rotation. Maximum errors arise when the gyroscope axis is then at an angle of 45° to the plane of the banking.

Therefore, the axis of the gyroscope can assume any position relative to the axes of the aircraft, and also with respect to the zero point on the course indicator scale, and the graph of the banking error is "floating", i.e., its maxima and minima can assume any position on the indicator dial while retaining the values and periodicity of the errors.

These errors automatically disappear when the aircraft comes out of the turn; however, they do constitute certain shortcomings in the pilotage of an aircraft, i.e., they disturb the correct estimation of the moment when the aircraft begins to stop banking in making a turn.

Distance Gyromagnetic Compass

The distance gyromagnetic compass (DGMC) has significant advantages over the integrated and distance magnetic compasses, since it is suitable for use when the aircraft is banking at a certain angle and completely damps the oscillations of the magnetic card in flight in a turbulent atmosphere.

The gyromagnetic compass is a combination of magnetic and gyroscopic course devices, in which the role of the course sensor is played by the magnetic transmitter and the role of the stabilizer of the readings is played by the gyro assembly.

Let us consider the combined system which is presently used for distance gyromagnetic compasses, e.g., the DGMC-7 (Fig. 2.20).

The basic parts of the distance gyromagnetic compass are the magnetic sensor, the gyro assembly, and the main course corrector.

In addition to these main parts, the compass must be fitted with a power supply (not shown in the diagram), as well as compensating and regulating devices:

(a) Compensating mechanism (combined with the gyro assembly);

(b) Rapid compensation button;

(c) A mechanism for compensating the remaining deviation (com- /152
bined with the main course indicator);

(d) Outputs for course repeaters and other indicators;

(e) Two-channel amplifier.

The magnetic transmitter of the compass has a card whose axis carries a dial for showing the course directly on the transmitter (it can be used to get rid of semicircular deviation), as well as the brushes for the wires leading to the potentiometer on the transmitter.

The transmitter potentiometer has a three-wire circuit connecting it to the gyro-assembly potentiometer, through which it receives alternating current from the power supply.

The transmitter in the damping suspension is mounted in the aircraft at a location where there is a minimum influence on the cards of the magnetic and electromagnetic fields of the aircraft.

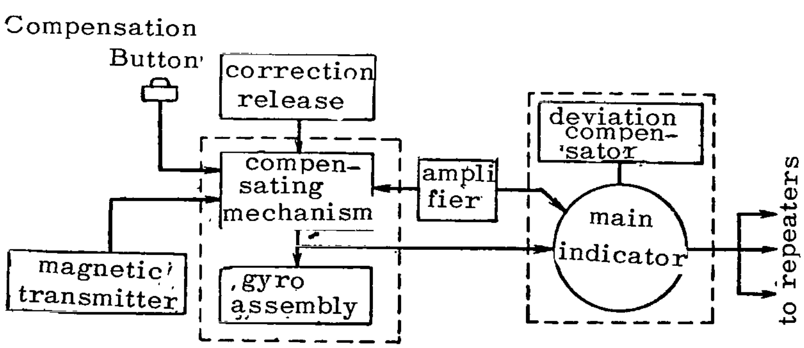


Fig. 2.20. Functional Diagram of Distance Gyromagnetic Compass (DGMC).

The transmitter housing carries a device for correcting semicircular deviation. If the semicircular deviation at the point where the magnetic sensor is mounted does not exceed 1-2°, the deviation device is not used, since in this case it would not improve but would rather detract from the operating conditions of the transmitter.

The gyro assembly consists of the gyroscope with a horizontal axis and a Cardan support, which ensures three degrees of freedom for the gyroscope rotation. The external frame of the gyro assembly rotates around the vertical axis.

The gyroscope is set in motion by means of a three-phase motor, whose stator is mounted on the internal frame of the gyro assembly and whose short-circuited rotor is the rotor of the gyroscope.

For correction of the gyroscope axis in the horizontal position, the lower part of the gyro assembly is fitted with a two-contact gravitational corrector, whose activating mechanism is a motor which produces a moment of force that is applied to the ex- /153
ternal frame of the gyroscope and acts in the horizontal plane.

If for some reason the axis of the gyroscope varies from the plane of the true horizon, the contacts of the corrector will be covered nonuniformly by the shifting conducting fluid, thus resulting in a distribution of currents passing through the corrector. This in turn transmits a signal for a correcting moment of force to be applied to the external frame. As a result of the precession of the gyroscope axis, it is shifted to a horizontal position.

The external frame of the gyro assembly carries a master selsyn for connecting to the principal indicator of the compass (the pilot's indicator, PI) and a three-conductor cord for connection to the magnetic transmitter.

The master selsyn and cable are connected closely together and can rotate together with the external frame of the gyro assembly. However, they can also rotate relative to the external frame by means of a special coordination mechanism.

The coordination mechanism consists of a small motor with a reduction gear for the slow-coordination regime, in which the rate of rotation of the selsyn is 1-4° per minute.

When it is necessary to carry out a rapid coordination, the motor is switched to reduced reduction by means of the rapid-coordination button and a special relay. The rate of rotation of the selsyn in this case is raised to 15-16° per second.

The potentiometer of the gyro assembly is firmly fastened to the housing.

The coordination of the magnetic transmitter with a gyro assembly is accomplished as follows (Fig. 2.21).

The alternating current passes through contacts A and B to reach the potentiometer of the gyro assembly and is picked up by pickups 1, 2, 3 mounted on the external frame of the gyro assembly, from which it passes to the pickups on the transmitter potentiometer, 1a, 2a, 3a.

It is clear from the figure that if the position of the brushes of the current pickups on the transmitter A_1,B_1 relative to the current leads of the potentiometer 1a, 2a, 3a differs from the position of the current pickups of potentiometer A, B relative to their current connections 1, 2, 3 by 90°, there will be a current in the pickups of the transmitter.

At the same time, between the current connection A and the current pickup A_1 in this case, there will be a portion of the potentiometer in the gyro assembly A-1 and a portion of the transmitter potentiometer 1a-A_1, represented as a sum of the four circumferences. Such a length of winding of potentiometer will be placed between current connection A and current pickup B_1 (segments A-2 and 2a-B_1). Consequently, a potential difference will develop between points A_1 and B_1.

We can reach an analogous conclusion if we consider the path of the current from connection B to pickups A_1 and B_1.

If the position of the brushes of current pickups A_1 and B_1 differs from the position of connectors A and B by an angle which is not 90° (considering their relationship to the potentiometer /154
sections), there will be a current in pickups A_1 and B_1. This current is fed to the first channel of the amplifier, and then to the motor of the coordination mechanism. The potentiometer brushes in the gyro assembly, along with the selsyn-transmitter, begin to rotate at a very low speed until there is an equilibrium of the currents on pickups A_1 and B_1.

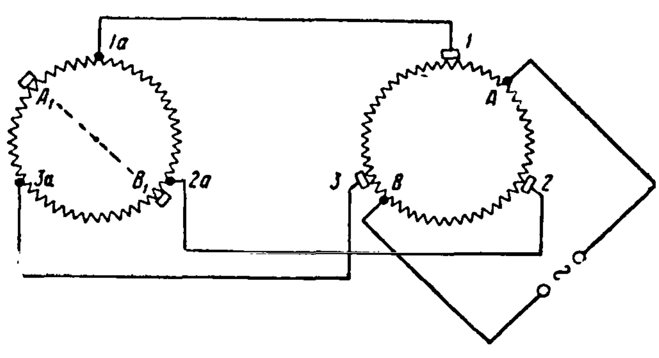


Fig. 2.21. Potentiometric Transmitter of Position Signal.

Thus, the position of the master of the gyro assembly constantly shifts to agree with the position of the transmitter card, regardless of the apparent rotation of the gyroscope axis due to the rotation of the Earth and the natural changes in the gyroscope axis.

Inasmuch as the agreement of the readings of the selsyns of the transmitter and gyro assembly takes place at an angular velocity which does not exceed 4° per minute, the readings of the gyro assembly cannot show the influence of rapid changes in the position of the transmitter card, i.e., the mechanism for coordination is a damper which averages out the readings of the compass for an average position of the card.

In order that no transmitter errors be transmitted to the gyro assembly when the aircraft is making a turn, the DGMC complex includes a correction switch which automatically shuts off the correction mechanism of the gyro assembly from the compass card when the aircraft is turning. Estimation of the readings of the aircraft's course during turns is made with a purely gyroscopic operation regime of the DGMC.

Inasmuch as the apparent diurnal rotation of the gyroscope axis cannot exceed 1° in four minutes of turn, while the turning time of the aircraft at an angle up to 90° as a rule does not exceed 1-3 minutes, no great errors in the compass readings are produced during the turn and the gyromagnetic compass can be used successfully for turning an aircraft at a desired angle.

Agreement of the gyro assembly with the basic course indicator is accomplished by means of a master selsyn (Fig. 2.22).

Winding AB rotates inside the housing of the master selsyn, allowing alternating current to flow in the windings of the selsyn 0-1, 0-2, 0-3. Currents which are symmetrical in phase also arise in the windings of the slave selsyn 0_1-1_1, 0_1-2_1, 0_1-3_1. Hence, the magnetic field of the resultant currents of the slave selsyn will be parallel to the magnetic field of the supply winding AB. Therefore, if winding A_1B_1 of the slave selsyn occupies a position which is perpendicular to the supply winding AB, the current in it will be equal to zero. /155

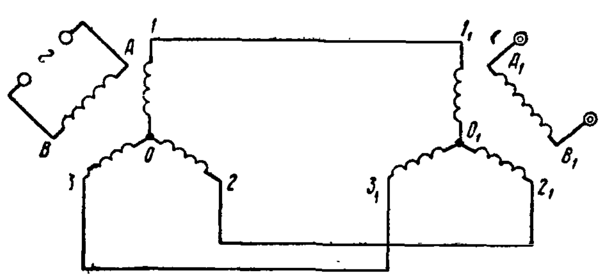


Fig. 2.22. Master Selsyn for Transmitting Position Signal.

If the angle between windings AB and A_1B_1 differs from a right angle, there will be a current in winding A_1B_1; this current passes through the second channel of the amplifier to a motor which turns winding A_1B_1, with an indicator scale showing readings up to a position where AB is perpendicular.

The potentiometric and selsyn systems, with amplification of currents and analysis of signals by means of small motors, give very precise agreement of readings and transmit them with high mechanical moments and good damping. This permits us not only to obtain precise and stable readings with the compass, but also to apply an additional stress to the course indicators or the intermediate links. For example, they can be used to set the mechanical compensators for deviation, and to take readings from other indicators or devices which use course signals.

The device for mechanical compensation of the residual deviation consists of a circular curved strip with special bends, which operates by means of a lever and pinion to produce an additional turning of the needle on the scale for showing the magnetic course. The adjustment screws are mounted along the edge of the strip, usually at every 15°, thus making it possible to compensate for the residual deviation practically down to zero.

However, it is not recommended that residual deviation greater than 2-3° be compensated, if it is possible to get rid of it by a deviation device with a magnetic transmitter, for the following reasons:

(a) Not getting rid of, but compensating for, semicircular /156
deviation leads to considerable changes in it, depending on the magnetic latitude of the locus of the aircraft.

(b) When the aircraft is turning and the magnetic correction is switched off while the compass is operating in a regime of gyroscopic stabilization, the mechanical compensation for deviation (if it is shown on the indicator) causes errors in the course readings in the form of overshooting and lagging, equal to the value of the compensated deviation, thus making it more difficult to turn the aircraft at a given angle.

In addition to the mechanical compensator for the residual deviation, the main indicator has a declination scale whose revolution to the value of the magnetic declination of the locus of the aircraft converts the compass readings from magnetic to true.

To link it with other devices, the main indicator has both a master and a slave selsyn, whose indications can be transmitted either with the aid of the activating motors or by a direct selsyn connection.

In the case of direct selsyn connection, the windings of the selsyn in transmitter AB and the selsyn of the indicator A_1B_1 are connected in parallel with the alternating current source. In this case, the winding A B of the slave selsyn attempts to set itself according to the regulation of the magnetic field, produced by windings $O_1 1_1$, $O_1 2_1$, $O_1 3_1$, i.e., it automatically assumes the position of the power winding AB of the master selsyn.

The direct selsyn connection has a lower sensitivity for the matching of the selsyns and a smaller working moment, so that there is a reduced accuracy of transmission. Hence, it is used for transmissions where there are no particularly high demands made on accuracy, e.g., for pilotage course repeaters connected to the main indicator.

Gyroinduction Compass

In the preceding paragraph, it was mentioned that the distance gyromagnetic compass has considerable advantages over the integrated compass. However, the magnetic transmitter of this compass has a serious shortcoming.

The fact is, that the magnetic moment which moves the transmitter card to the plane of the magnetic meridian is itself very small, and while it is sufficient for turning the floating card, it is frequently insufficient for overcoming the friction of the brushes on the current pickups, especially in flight at high magnetic latitudes. Therefore, this transmitter is unstable in operation and frequently goes out of order.

To overcome this shortcoming, new types of induction magnetic transmitters have been developed; in addition to having an increased threshold of sensitivity, they do not have the ability to move in the horizontal plane (in the azimuth); consequently there are no errors due to splashing of the fluid over the sensitive element or obstruction; they are less sensitive to the influence of accelerations when the aircraft is yawing, and the size of the transmitter is smaller. /157

The operating principle of the induction-type sensitive element is the dependence of the value of the alternating magnetic induction of the core upon the presence of its constant component, exerted in the core by the horizontal component of the terrestrial magnetism.

For example, if the core has a constant component of magnetic induction in the direction of the vector OA (Fig. 2.23, a), then in order to bring it up to complete saturation in this same direction we will require an additional vector AB. The change in induc-

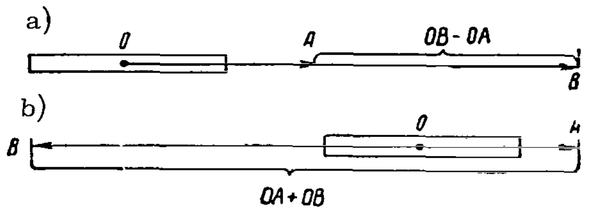


Fig. 2.23. Induction Saturation of the Core of the Sensitive Element: (a) Induction Vector Coincides with Saturation Vector; (b) Induction Vector and Saturation Vector are in Opposite Directions.

tion in this case is expressed by the difference between the vectors OB-OA.

As we see from Figure 2.23, b, when the magnetic induction is brought up to full saturation, the change in induction in the opposite direction will be equal to the sum of the vectors OA + OB.

The transmitter of an induction compass has three sensitive elements, each of which is made as follows: Two parallel magnetic cores made of permalloy (a material with a high magnetic permeability and a very low value of magnetic hysteresis) have separate primary windings, connected in opposite phase, and a common secondary winding around both cores (Fig. 2.24, a). Alternating current /158 flows through the primary windings of the cores.

Obviously, if the constant component of the magnetic induction of the cores from the horizontal component of the Earth's magnetic field is zero, the vectors of its change with passage of an alternating current through the winding will be the same in both cores, but in opposite directions, and there will be no alternating current in the secondary winding.

If the cores have a constant component of magnetic induction, the vector of the change in magnetic induction will be greater in one and smaller in the other; this will produce pulses of alternating current as shown in the graph in Figure 2.24, b. The magnitude of the current pulses will be proportional to twice the value of the constant component of the magnetic induction of the cores.

The sensitive elements in the transmitter are arranged in the form of a triangle and their secondary windings form a sort of master selsyn (Fig. 2.25). The rotating winding of the slave selsyn is connected to the amplifier and mounted in a position perpendicular to the resultant vector of the electromagnetic field of the slave selsyn by means of an activating motor with reduction gearing.

The primary winding of this transmitter is mounted in an intermediate element between the transmitter and the gyro assembly in a correction mechanism which has a device for mechan compensation of residual deviation and is used as a correction mechanism for the following system.

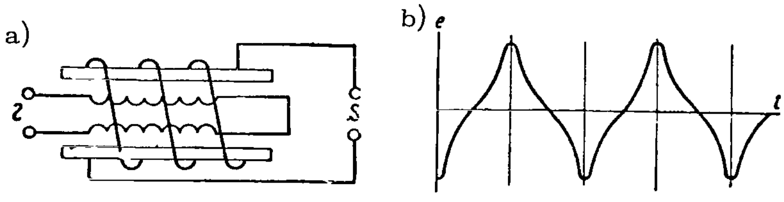


Fig. 2.24. Sensitive Element of Induction Transmitter: (a) Winding; (b) Graph of Current.

The induction transmitters for the course are reliable and stable in operation, but their accuracy of operation drops when the transmitter is tilted to a sufficiently greater degree than is the case for magnetic transmitters.

At the same time, if the tilting of the transmitter takes place in the plane perpendicular to the magnetic meridian, the vertical component of the magnetic field of the Earth, projected on the plane of the sensitive element, forms a magnetic induction normal to the magnetic meridian; the banking deviation will then be determined by the formula

$$\text{tg}\ \delta = \frac{Z}{H} \sin i \sin \theta, \qquad (2.25)$$

where i is the banking of the transmitter, θ is the angle between the plane of the magnetic meridian and the banking plane of the transmitter, and Z, H are the vertical and horizontal components of the Earth's field, respectively.

For example, with the ratio $\frac{Z}{H} = 3$ and the angle $\theta = 90°$, each banking radius of the transmitter will produce an error of approximately 3° in the operation of the compass.

The ratio $\frac{Z}{H} = 3$ corresponds (e.g.) to the latitude of Moscow /159 and increases rapidly with an approach to the polar regions. Therefore, the banking errors in the induction transmitter can take on very significant values.

In order to reduce the errors in the induction transmitter, its sensitive element is mounted on a float mounted in a Cardan support. The body of the transmitter is filled with fluid to reduce the pressure on the axis of the frame of the Cardan suspension (a mixture of ligroin and methylvinylpyridine oil). The Cardan suspension ensures the horizontal position of the sensitive element during banking and pitching to within 17°.

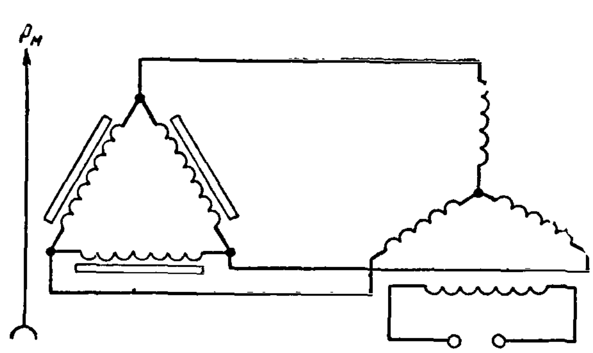


Fig. 2.25. Diagram Showing Connection of Elements in Sensor of Gyroinduction Compass.

The induction transmitter, like the magnetic one, is mounted aboard the aircraft in a position such that it is exposed to the smallest magnetic field of the aircraft and one which is as constant as possible; a deviation mechanism is mounted on it to record the semicircular deviation of the transmitter.

However, the curvilinear trajectory of flight (although the radius of curvature is very great), in addition to the acceleration produced by Coriolis forces, produces a constant tilting of the sensitive element of the transmitter, the deviation from which is transmitted to the main indicator and its repeaters.

For example, at the latitude of Moscow and an airspeed of 800 km/hr, the tilting of the sensitive element of the transmitter due to the acceleration of the Coriolis forces will be equal to approximately 20', which undergoes deviation equal to 1° in a flight in the northerly and southerly directions.

The gyroscopic induction compass (with the exception of the induction transmitter) is built in a manner similar to that of the distance magnetic compass.

Its principal components are the induction transmitter, the gyro assembly and the course indicator.

In addition to the principal units, there is also a power supply, amplifiers, correction mechanism with a curved device for getting /160 rid of residual deviation, a connecting chamber, a button with a mechanism for rapid coordination, a correction switch, and repeaters from the main course indicator.

The correction mechanism is the intermediate link between the induction transmitter and the gyro assembly. The connection between the induction transmitter and the correction mechanism is made with a selsyn, while the connection between the correction mechanism and the gyro assembly, the gyro assembly with the main indicator, and the main indicator with the repeaters is made by potentiometers.

The main indicator also has a curved device for getting rid of errors in the distance transmission of the course indications from the gyro assembly to the indicator at the factory.

The correction switch is a two-stage gyroscope which serves for automatically disconnecting the gyro assembly from the correction mechanism; this disconnects the circuit for azimuth correction from the induction transmitter and disconnects the correction of the horizontal position of the axis of the gyroscope rotor when the aircraft is making turns with an angular velocity greater than 36 deg/min.

Disconnecting the induction transmitter during turns gets rid of the considerable errors which arise due to the influence of the vertical component of the Earth's magnetic field Z. In order to ensure that the gyroscope correction will not be disconnected in a turbulent atmosphere when the aircraft is bumping and yawing, the correction switch has a delay mechanism which disconnects the correction only after 5-15 sec have elapsed following the moment when the aircraft reaches an angular velocity of 36 deg/min.

The course repeaters are simple in design and consist of three-phase magnetoelectric lagometers whose accuracy for determining the course is lower than that of the main indicator.

Despite the numerous advantages of distance gyromagnetic and gryoinduction compasses over integrated compasses, they do not completely satisfy the requirements of aircraft navigation, particularly with regard to automation of its processes, since the following shortcomings of compasses still persist:

(a) The dependence of the accuracy with which the course is measured upon the magnetic latitude and the impossibility of using the instrument at high magnetic latitudes.

(b) The difficulty of maintaining an orthodromic direction of flight, since the magnetic flight angles which are then obtained vary.

(c) The magnetic loxodrome along which a flight can be carried out with a constant magnetic flight angle is a complex curve, since it depends on the intersection of meridians and magnetic declinations, which limit the length of the straight-line flight segments, /161 along which the flight angle can be assumed constant.

(d) Regardless of all the measures which have been taken to get rid of and correct for deviations, as well as the consideration of magnetic declinations, the accuracy of the measurements of the magnetic course still remain low(within the limits of 2-3°).

The majority of these shortcomings can be overcome by using gyroscopic semicompasses with high accuracy, or course systems which make it possible to fly in a regime using highly sensitive gyro-semicompasses (the GSC regime).

Details of Deviation Operations on Distance Gyromagnetic and Gyroinduction Compasses

Deviation operations on distance compasses are carried out using the same method as for integrated compasses, with certain changes necessitated by features of the design and mounting of these compasses.

In several types of aircraft, the semicircular deviation at the point where the transmitters are mounted can be very low. In these cases, the deviation devices must be removed from the transmitters and all forms of deviation are compensated for by a mechanical compensator on the main course indicator or on the correction mechanism.

The compensation for the residual deviation, using a mechanical compensator, is carried out on 24 courses: 0, 15, 30, ..., 345°, in which the aircraft is set to the desired courses, and a screw is turned (corresponding to the course of the aircraft) in order to bring the remaining deviation to zero. The graph of the remaining deviation on the main course indicator is not plotted. However, if differences in readings between the main indicator and its repeaters are noticed, it is necessary to plot a graph of the corrections for the readings on the repeaters.

After each two intermediate settings of the aircraft on course (at the points 0, 45, 90, 135, 180, 225, 270 and 315°), it is necessary to mark the readings of the compass transmitter on the scale of the compass course on the main indicator (for induction transmitters, on the scale of the correction mechanism), and use this to determine the coefficients of semicircular deviation *B* and *C*. The form shown in Table 2.4 is recommended for convenience in determining these coefficients.

The coefficients are calculated according to the formulas:

$$B = \frac{\sum_{i=0}^{8} \delta_i \sin \mathrm{MC}}{4}; \quad C = \frac{\sum_{i=0}^{8} \delta_i \cos \mathrm{MC}}{4},$$

where δ_i is the compass deviation on individual courses.

TABLE 2.4. /162

MC °	δ°	sin MC	δ sin MC	cos MC	δ cos MC
0		0		1	
45		0,7		0,7	
90		1		0	
135		0,7		—0,7	
180		0		—1	
225		—0,7		—0,7	
270		—1		0	
315		—0,7		0,7	

The calculated coefficients must be in the form of tables, attached to the instrument panel along with the main course indicator. In addition to the coefficients on the table, it is also necessary to show the place where the deviations were corrected or the horizontal component of the magnetic field of the Earth at the point where the correction was carried out.

Since the semicircular deviation, as well as all its other forms, can be made by a mechanical compensator at the magnetic latitude of the point where the correction was made, Formula (2.16) for calculating the deviation for other magnetic latitudes assumes the form

$$\delta = B\left(\frac{H_1}{H_2} - 1\right)\sin\gamma + C\left(\frac{H_1}{H_2} - 1\right)\cos\gamma. \qquad (2.26)$$

Course Systems

The most complete devices for measuring the course of an aircraft are the course systems. Course systems are combinations or complexes of various course transmitters mounted on the aircraft, with their readings displayed on general indicators. Such transmitters include the following:

Magnetic induction (MC regime);
Astronomical (AC regime);
Gyroscopic (GSC regime).

In principle, the course system consists of a combination of the design features of a gyroinduction compass, gyrosemicompass and astronomical course transmitter, whose operating principle will be discussed in the chapter devoted to astronomical means of aircraft navigation.

The primary feature of the design of the gyroscopic portion of the course system is the presence of a third frame for the gyroscope with a horizontal axis, coinciding with the longitudinal axis of the aircraft. The purpose of the third frame is to select the Cardan errors in the readings of the gyrosemicompass when the air- /163 craft is turning.

The use of this third frame completely excludes Cardan errors from the transverse rolling of the aircraft, since the second frame of the gyroscope (with a master selsyn) will always be in a vertical position.

The setting of the second frame of the gyroscope in a vertical position is accomplished by means of an electrical circuit and a mechanical device for matching it with the so-called gyrovertical, mounted on aircraft for pilotage purposes.

The second feature of course systems is the use (as a rule) of two gyro assemblies, a main one and a standby, which improve the reliability of the system and ensure reciprocal control of the readings.

Figure 2.26 shows the control panel and the indicator of the course system. The course system operates on the main indicator in a regime in which the switch for the operating regime is set at the top part of the panel (MC, AC, or GSC).

When switching the course system from the GSC regime to the MC or AC regimes, in order to correct the readings, it is necessary to press the button for rapid correlation in order to adjust the readings of the gyro assembly to the readings of these transmitters. After correlation, the switch is again returned to the GSC position.

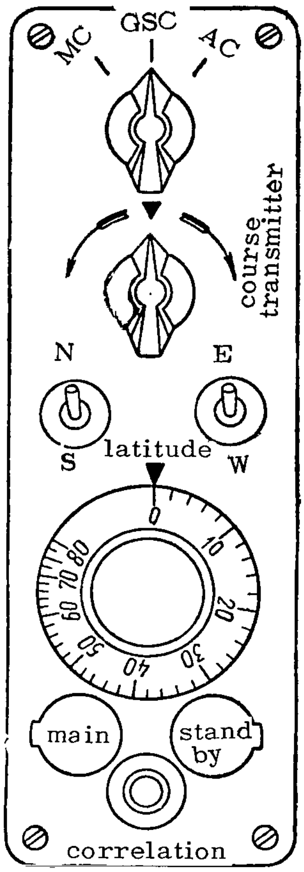


Fig. 2.26. Control Panel of Course System.

The pushbutton course control serves for manual setting of the values for the course system only in the GSC regime. The switch on the left-hand side of the panel, marked "N-S", is used to switch the polarity of the latitudinal potentiometer in order to compensate for the rotation of the Earth in the Northern or Southern Hemisphere. The covers at the bottom of the panel, marked "main" and "standby", cover adjustments for the balancing potentiometers of the main and standby gyro assemblies.

Methods of Using Course Devices for Purposes of Aircraft Navigation /164

The methods of using course equipment depend upon the resolving powers of the complex of course devices mounted on the aircraft, the presence of other equipment for purposes of aircraft navigation, and also on the distance, geographic and meteorological conditions of flight.

While the meteorological flight conditions along a given route (path) change in the course of time and can vary depending on altitude and distance of flight, the remaining conditions for a given type of aircraft and a given route (air route) remain constant.

In discussing the methods of using course devices in flight, the constant conditions listed above can be divided into three groups.

(1) The aircraft is equipped with an integrated or distance gyromagnetic (induction) compass. Flights are carried out over long or medium distances without significant changes in magnetic latitude. The equipment for constant measurement of the airspeed, drift angle, and automatic calculation of the path are lacking on the aircraft.

(2) The aircraft is fitted with a distance gyromagnetic or gyroinduction compass and a gyrosemicompass or course system of average accuracy. Flights are made over long distances with considerable changes in magnetic latitude. There is no equipment for automatic measurement of the drift angle or airspeed, or calculating the flight according to these parameters on board the aircraft.

(3) The aircraft is fitted with a course system of high accuracy, as well as devices for automatically measuring the drift angle, the airspeed, and calculating the path. Flights are made at any geographical latitude and for any distance.

Methods of Using Course Devices Under Conditions Included in the First Group

Under the conditions in the first group, i.e., when flights are being made over short distances in aircraft which have simple navigation equipment, the following methods are used to prepare the calculated data and use the course devices in flight.

In preparing for a flight, the route of the flight to be made is entered on a flight chart. If the flight chart is one which is in an international or diagonal cylindrical projection, the straight-line portions of the flight between the turning points along the route are plotted as straight lines by means of a ruler. When using charts which are plotted with an isogonal cylindrical projection (Mercator), the straight-line portions of a flight which /165 is very long are plotted as a curved line on the basis of the intermediate points along the orthodrome, calculated by analytical means.

Since the magnetic compass is a loxodromic course-measuring device, and the parts of the routes which have been plotted are very nearly orthodromic, in order to avoid overly high deflections in the loxodrome from the given line of flight, the length of the flight segments with a constant flight path angle are selected so that the initial and final flight path angles under conditions of following an orthodrome do not differ by more than 2-3°, i.e., so that the total correction for the readings of the magnetic compass at the end of the segment relative to its readings at the beginning of the segment is no more than 3°:

$$\Delta = (\lambda_1 - \lambda_2) \sin \varphi_m + (\Delta_{M_2} - \Delta_{M_1}) < 3^\circ.$$

If the indicated correction is more than 3° in the straight-line portion of the flight, this segment is divided into two, three or more parts and the flight path angle is determined for each. This is usually not done by simple division of a straight line into equal parts, but by selecting characteristic orientation points along the section of the route, the flight between which can be made at the constant flight path angle.

If we consider the low accuracy of the indications of the magnetic compasses in a relatively short length of flight segment for a flight with a given flight path angle, the latter are determined not by analytical means, but by simple measurement of the direction of the segment on the chart by means of a protractor.

Measurement of the loxodromic flight path angle can be made relative to the meridian which intersects the segment at a point which is closest to its center, considering the magnetic declination of this point. However, to increase the accuracy of the measurements, it is recommended that it be done at two points, at the beginning and end of the segment, considering the average declination of these points.

Obviously, in the first case the magnetic flight angle of the segment will be

$$MFA = \alpha_m - \Delta_{M_m},$$

while in the second case

$$MFA = \frac{\alpha_b + \alpha_e - \Delta M_b - \Delta M_e}{2},$$

where α_b, α_m, α_e are the azimuths of the orthodrome at the beginning, the middle, and end, respectively.

An advantage of the second method is the double measurement of the angles and the averaging of the declinations, since the accuracy of two measurements and the averaging of their result is always /166 higher than the accuracy of a single measurement.

For the first group of conditions, it is possible to have some simplified preparation for the course equipment of the aircraft for the flight. Since the flights are made with relatively low measurements of magnetic latitude, there is no need to determine the coefficients of semicircular deviation *B* and *C* or to consider their changes during the flight.

If the deviation is compensated by a mechanical compensator, it is assumed to be zero during the flight. In considering the residual deviation, a value is assigned to it as shown on the graph.

During the flight, the course of the aircraft is checked so that its value together with the drift angle of the aircraft will be equal to a given magnetic flight path angle of the flight segment.

$$MFA_a = MC + US = MFA_g.$$

On the other hand, since the magnetic course of the aircraft is equal to the compass course, it is necessary to add the compass deviation:

$$MFA_a = CC + \Delta_c + US = MFA_g.$$

Problems

1. The direction of a flight segment measured along the average meridian is equal to 48°; the magnetic declination in the middle of the segment is +7°. Determine the given magnetic flight path angle of the segment.

Answer: $MFA_g = 41°$.

2. The direction of a flight segment measured along the initial meridian is equal to 136°, 132° at the final meridian, with an initial magnetic declination of +7° and a final one of +5°. Determine the MFA_g.

Answer: $MFA_g = 128°$.

3. The given magnetic path flight angle of a segment is equal to 84°, the drift angle was equal to -6°, the deviation of the magnetic compass is +4°. Determine the required compass course for following the flight lines.

Answer: CC = 86°.

4. The compass course of an aircraft is equal to 54°, the compass deviation is +3°, the drift angle is +6°. Determine the actual flight path angle.

Answer: $MFA_a = 63°$.

Methods of Using Course Devices Under Conditions of the Second Group

When flights are being made over long distances using distance gyromagnetic and gyrosemicompasses or course systems, but without any automatic course calculation, the use of course instruments in flight and preparation of charts for a flight is accomplished by devices which are somewhat different from those which are recommended for the conditions of the first group.

The most important of these devices is the plotting of the orthodromic course along the straight-line segments of the flight with a gyrosemicompass or a course system in the "GSC" regime, with periodic correction of the gyroscope course by means of a magnetic or astronomic transmitter.

As a rule, in flights over long distances, the flight chart /167
is one with a scale of 1:2,000,000 on the international projection. If a straight line within the limits of one sheet of this map, with distances up to 1200-1500 km, can be assumed with insignificant error to be an orthodrome, then when two or more sheets are combined

and the route does not run along a meridian or when sheets of this chart are used separately at great distances, the orthodrome must be located along points which are determined by calculation. When splicing two adjacent sheets along the meridian, the orthodrome has a significant break in it, and in this case (when it crosses the adjacent sheets) a straight line cannot be taken as the orthodrome.

On the charts of all other projections, except the central polar and special route maps in a diagonal, cylindrical projection, when the line of the tangent (cross-sectional strip) of the cylinder coincides with the axis of the route, the orthodrome is calculated analytically and plotted on the chart according to the calculated intermediate points. The distances for the sections of the orthodrome are also determined by analytical means.

The orthodromic flight path angles of the route segments under these conditions are measured or calculated analytically relative to the initial meridian of each flight segment. If the straight-line segments of the flight have a very short length, the flight path angles calculated from the initial meridians of the segments can be applied to the system relative to the selected reference meridian (Fig. 2.27) according to the following formula:

$$\mathrm{OFA} = \mathrm{TFA} + \delta = \mathrm{TFA} + (\lambda_{ref} - \lambda_{init}) \sin \phi m,$$

where δ is the angle of convergence between the reference and initial meridians of the segment.

Since the condition for the second group assumes flights over long distances with considerable changes in the magnetic latitudes, the preparation of the magnetic compasses must be made with a consideration of determination of the changes in the semicircular deviation during the flight.

Course devices intended for flights under conditions of the second group have devices for mechanical compensation of the residual deviation. Therefore, the graph of the deviation for them is not plotted. However, in getting rid of the deviation, it is necessary to determine and write down the coefficients of deviation *B* and *C*:

$$B = \frac{\sum_{i=0}^{8} \delta_i' \sin \gamma_i}{4}; \quad C = \frac{\sum_{i=0}^{8} \delta_i \cos \gamma_i}{4}.$$

It is then necessary to write down the intensity of the horizontal component of the Earth's magnetic field at the point where the deviations were corrected. /168

To calculate the changes in the semicircular deviation during flight, the corrections for the magnetic course at different segments of the route must be determined when preparing for a flight. They are determined for a number of points along the flight path, on the basis of the magnetic flight angles of the route at these points with a frequency such that the difference between two adjacent corrections along a straight line path does not exceed 1° and after each turning point on the route.

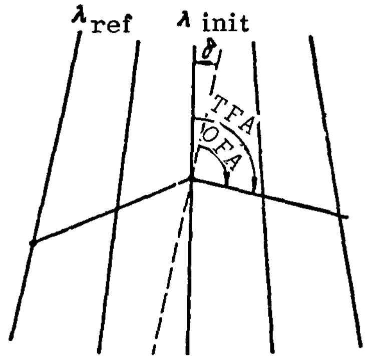


Fig. 2.27. Calculation of Flight Path Angles from Reference Meridian.

In fact, the changes in the semicircular deviation at corresponding points along the route will differ only slightly from the calculated corrections, since the course which is followed will be prepared with a consideration of the drift angle of the aircraft. However, the errors which arise in this process will be small and can be disregarded.

During the flight, the gyrosemicompass or the course system is corrected for the magnetic or astronomical transmitter when flying along the reference meridians or the turning points of the route (TPR). If the correction is made on the basis of the magnetic transmitter, then the main indicator will have the required correction entered on its dial. This correction is equal to the sum of the magnetic declination and the change in the semicircular deviation along the magnetic latitude.

For correction, the course system is switched to the "MC" regime and the button is pushed to match the readings. The system operates for a period of 1-2 min in the slow coordination regime and is then switched to the "GSC" regime.

In this manner, the systems are corrected for the astronomical transmitter. Having determined the latter on the basis of the coordinates of a star and the locus of the aircraft, the system is switched to the "AC" regime, the coordination is carried out, and then switched back to the "GSC" regime. This means that at the turning points of the route, no corrections are required on the scale of the declinations.

The correction of the gyrocompass is made in the same manner, except that the course is set on the gyrosemicompass not by comparing the readings of the transmitters, but by manual setting on the basis of the readings of the magnetic or astronomical transmitters.

After correction, the flight is carried out with an orthodromic course up to the next turning point of the route or reference meridian.

When it is necessary to make a correction for the orthodromic course between two reference meridians, the correction is set on the main indicator and is equal to: /169

for the magnetic transmitter,

$$\Delta = \Delta_M + (\lambda_{ref} - \lambda_{MC}) \sin \phi_m ;$$

for the astronomical transmitter

$$\Delta = (\lambda_{ref} - \lambda_{MC}) \sin \phi_m .$$

Then the readings are matched in the manner described above.

Problems

1. The east longitude of the reference meridian is 40°, the north latitude of the reference point is 52°. The coordinates of the setting point of the route are: longitude 43°, latitude 54°. The true flight path angle of the segment at the starting point is 67°. Determine the orthodromic flight path angle calculated from the reference meridian.

Answer: 64.5°.

2. The intensity of the horizontal component of the Earth's magnetic field at the point where the deviations are corrected is 0.24 oersteds, while at a certain point along the flight route it is 0.08 oersteds. Determine the corrections for the magnetic course of the aircraft at this point, if the magnetic flight path angle of the flight segment is equal to 60°, coefficient $B = +1.5$, and coefficient $C = +0.9$.

Answer: +3.5°.

3. The east longitude of the reference meridian is equal to 70°, the north latitude of the reference point is 58°. The aircraft is located at the point $\lambda = 76°$, $\phi = 60°$; the magnetic declination of the location of the aircraft is equal to +11°, while the correction for the change in the semicircular deviation $\Delta_{BC} = +2°$. Determine the correction for the readings of the magnetic compass for correction of the orthodromic course.

Answer: $\Delta = +8°$.

Methods of Using Course Devices Under the Conditions of the Third Group

The third group of conditions for using course devices refers to flights in aircraft which are fitted with precise course systems, apparatus for automatic measurement of the airspeed of the aircraft, the drift angle, and automatic calculation of the flight path of the aircraft.

The conditions of the third group assume a prolonged autonomic aircraft navigation with no visibility of the ground or over water, with correction of the aircraft coordinates only at individual points located significant distances apart. This places particularly strict requirements on the accuracy of the plotting of the orthodrome on the charts, the determination of the flight path angles, and the retention of systems for calculating the aircraft course, since the course is a basis for the automatic calculation of the flight in terms of direction.

From the theoretical standpoint, a more precise and convenient form for using the course devices under conditions of the third group is the following: /170

In preparing the flight charts for each orthodrome section of the flight between the turning points on the route, regardless of their length, we determine the conditional shift in the longitude (λ_s), i.e., the difference between the longitude calculated from the point where the given orthodrome intersects the Equator (λ_0) and the geographical longitude (λ):

$$\lambda_s = \lambda_0 - \lambda.$$

Here, the orthodromic longitude λ_{0_1} is determined for the starting point of each segment by the formula

$$\operatorname{ctg} \lambda_{0_1} = \operatorname{tg} \varphi_2 \operatorname{ctg} \varphi_1 \operatorname{cosec} \Delta\lambda - \operatorname{ctg} \Delta\lambda.$$

After determining the change in the longitude, the longitude of any point along the route can be converted easily to the orthodromic system, thus making it possible to determine relatively easily all of the required elements of the orthodrome for these points:

(a) The azimuth of the point of intersection of the orthodrome with the Equator (α_0)

$$\operatorname{tg} \alpha_0 = \frac{\sin \lambda_{0_1}}{\operatorname{tg} \varphi_1};$$

(b) The coordinates of intertermediate points for plotting the orthodrome on the map:

$$\operatorname{tg} \varphi_i = \frac{\sin \lambda_{0_i}}{\operatorname{tg} \alpha_0};$$

(c) The initial, intermediate, and final azimuths of the orthodrome

$$\operatorname{tg} \alpha_i = \frac{\operatorname{tg} \lambda_{0_i}}{\sin \varphi_i};$$

(d) The distance to any point along the orthodrome (S_i) from the point of its intersection with the Equator

$$\cos S_i = \cos \lambda_{0_i} \cos \phi_i;$$

(e) The distance between any two points along the orthodrome as a difference in the distance from the point of intersection with the Equator

$$S_{1-2} = S_2 - S_1.$$

Considering the necessity of precisely calculating the course for automatic computation of the path in terms of the direction, and the difficulty of an exact setting of the course in flight relative to the new reference meridians, it is desirable for the conditions in the third group to retain a single system for calculating the courses over the entire length of the flight from takeoff to landing.

In this case, the path angle of the first orthodromic flight segment is considered to be equal to the azimuth of this segment relative to the meridian of the airport from which the aircraft took off. The path angles of all subsequent segments are obtained by combining the orthodromic flight angle (OFA) of the previous section with the turn angle (TA) of the line of flight at the turning points along the route (Fig. 2.28): /171

$$OFA_1 = \alpha_1;$$

$$OFA_n = \alpha_1 + TA_1 + TA_2 \ldots TA_{n-1}.$$

The turn angles along the line of flight are found as the differences of the azimuths of the orthodrome, intersecting at the turning points of the route, determined according to the formula

$$\operatorname{tg} \alpha_i = \frac{\operatorname{tg} \lambda_{0_i}}{\sin \varphi_i}.$$

Obviously, the latitude of the turning points will be common for the two orthodromes; for one it will be final, for the other it will be initial. As far as the longitude is concerned, it is determined on the basis of the geographical longitude of the turning point of the route, considering the shift in longitude of the previous and subsequent segments.

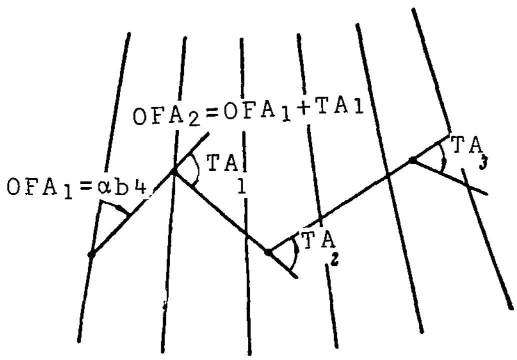


Fig. 2.28. System for Calculating Path Angles by Combining the Turn Angles along the Flight Path.

When flying above a continent, the best method of correction for the orthodromic course under conditions of the third group is to introduce corrections into the course as a result of calculations of the aircraft path.

For example, if the readings of the calculating devices on board the aircraft at both the initial and final points indicate that it is on the line of flight, but has undergone a lateral deviation ΔZ during the flight, then obviously

$$\operatorname{tg} \Delta\gamma = \frac{\Delta Z}{S},$$

where $\Delta\gamma$ equals the error in the readings of the orthodromic course, and S is the length of the control section of the flight.

The sign of the correction to the compass reading coincides with the sign of ΔZ.

With positive values of ΔZ, (a shift from the line of the desired flight to the right), the readings of the compass will be reduced and the correction must be positive; in the case of deviation to the left, the correction must be made with a minus sign.

In flights over water, when determination of the correctness of the calculation of the aircraft path in terms of direction is more difficult, the correction of the gyroscope course must be made by astronomical methods. This means that the difference between the orthodromic and true courses at any point will be equal to the difference between the orthodromic path angle of the segment and the running azimuth of the orthodrome at a given point: /172

$$\text{OC} - \text{TC} = \text{OFA} - \alpha.$$

If the positive difference of the courses turns out to be greater (or if it is negative, turns out to be smaller) than the difference between the path angles, the reading for the orthodromic course will be increased and it will be necessary to reduce it manually by the course detector. When the readings of the orthodromic course are low, it must be increased.

In this manner, but with reduced accuracy, the orthodromic course can be corrected magnetically:

$$OC - (MC + {}_{M}) = OFA - \alpha.$$

For the conditions of the third group, the preparation of the magnetic compasses must be carried out according to the rules given above for the conditions of the second group. However, the use of magnetic transmitters for correction of orthodromic course during flight is limited to cases when the readings of the orthodromic course cannot be checked on the basis of the results of calculations of the path or by means of astronomical course transmitters.

The meteorological conditions of a planned flight, especially over long distances, call for careful preparation of all course equipment on the plane, since it may become necessary to use devices for measuring the courses which belong to all three groups of conditions.

3. Barometric Altimeters

The principal method of measuring flight altitude for navigational purposes is the barometric method. It is based on the measurement of the atmospheric pressure at the flight level of the aircraft.

For special purposes, such as aerial photography or aerial geodesic studies, as well as for signaling dangerous approaches to the local relief when coming in for a landing under difficult meteorological conditions, electronic devices for measuring altitude are used, which are more accurate in principle than the barometric method. However, they are not widely employed for navigational purposes because they are used only for measuring the true flight altitude. On the basis of the barometric method of measuring altitude, it is the law of change of atmospheric pressure with increase in height which means that the calibration of the altimeter dial must be made on the basis of the conditions of the international /173
standard atmosphere.

The conditions of the standard atmosphere are as follows:

(a) The pressure at sea level is equal to 760 mm Hg, or 1.0333 kg/cm^2.

(b) The air temperature at sea level is +15° C with a linear decrease for flight altitudes up to 11,000 m of 6.5° for each 1000 m of altitude. Beginning at 11,000 m, the air temperature is considered constant and equal to -56.5°.

To understand the operating principle of the barometric altimeter, let us recall the familiar equations from physics which describe this state of gases and the conditions of their change.

Thus, according to the Boyle-Mariotte law, with isothermal compression (i.e., fixed temperature), the pressure of a gas changes in inverse proportion to its volume so that the product of the volume times the pressure remains constant:

$$pv = \text{const},$$

where p is the pressure of the gas and v is the volume of the gas at temperature t.

According to the Gay-Lussac law, heating a gas by 1° C at constant pressure causes the gas to expand to 1/273.1 of the volume which it occupied at zero temperature:

$$v - v_0 = \frac{v_0}{273,1} t,$$

where v_0 is the volume at zero temperature and the same pressure.

By combining the Boyle-Mariotte and Gay-Lussac laws, we obtain the state equation of a gas:

$$pv = \frac{p_0 v_0}{273,1} (t + 273,1).$$

This equation is known as the Clapeyron equation. The temperature (t +273.1° C) is called the *absolute temperature* (T), i.e., calculated relative to absolute zero (-273.1° C)[1], and the constant value of $\frac{p_0 v_0}{273.1}$ is called the *gas constant*.

A gram molecule of any gas (gram mole, or simply mole), i.e., the number of grams of a gas which is equal to its molecular weight, always occupies exactly the same volume (22.41 liters) at zero temperature and a pressure of 1 atm.

The gas constant for one mole of gas is called the universal gas constant (R): /177

$$R = \frac{p_0 v_0}{273,1}.$$

With P = 1 atm, v = 22.41 liters.

The Clapeyron equation for one mole of gas in this case assumes the form

[1] This value is usually assumed to ba approximately 273° in calculations.

$$pv = RT.$$

The numerical value of the universal gas constant is

$$R = \frac{1{,}033 \cdot 22\,410}{273{,}1} = 84{,}8 \text{ kg/cm(degrees/mole)}.$$

In technical calculations, the weight of the gas is usually expressed in kilograms. Therefore, we do not use the universal gas constant but rather the characteristic gas constant

$$B = \frac{1000}{M} R,$$

where M is the number of grams of gas per mole, or its molecular weight.

Then

$$pv = BT.$$

The constant B for air is 29.27 m/degree.

By using the gas constant, we can find the weight density of air (γ) at a given pressure p and absolute temperature T.

$$\gamma = \frac{p}{BT}.$$

Let us define an area on the Earth's surface measuring 1 cm^2, and erect a vertical column on it which extends upward to the limits of the Earth's atmosphere (Fig. 2.29).

Obviously, the drop in pressure with increased altitude to the distance ΔH at a certain height will be equal to:

$$\Delta p = \gamma \Delta H = \frac{p}{BT} \Delta H$$

or

$$\frac{\Delta p}{p} = \frac{\Delta H}{BT}. \qquad (2.27)$$

By using Equation (2.27) and the altitude temperature gradient, /175 we obtain the so-called barometric formula

$$p_H = p_0 \left(1 - \frac{t_{gr}}{T_0} H\right)^{\frac{1}{Bt_{gr}}}, \qquad (2.28)$$

where T_0 is the temperature on the ground under standard conditions equal to 281° K, and t_{gr} is the vertical temperature gradient.

Formula (2.28) is obtained from Formula (2.27), switching to infinitely small values:

$$\frac{dp}{p} = \frac{dH}{BT}. \tag{2.27a}$$

Integrating (2.27a) and keeping in mind that $T_H = T_0 - {}_{gr}H$, we obtain:

$$\int_{p_0}^{p_H} \frac{dp}{p} = -\frac{1}{B}\int_0^H \frac{dH}{T_0 - t_{gr}H};$$

$$\ln\frac{p_H}{p_0} = \frac{1}{Bt_{gr}}\lg\frac{T_0 - t_{gr}H}{T_0}$$

or

$$p_H = p_0\left(1 - \frac{t_{gr}}{T_0}H\right)^{\frac{1}{Bt_{gr}}}$$

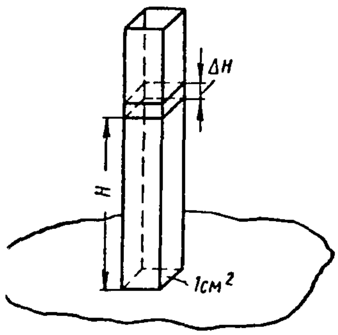


Fig. 2.29. Column of Air on the Earth's Surface.

Solving Equation (2.28) for H, we obtain the standard hypsometric formula for the troposphere:

$$H = \frac{T_0}{t_{gr}}\left[1 - \left(\frac{p_H}{p_0}\right)^{Bt_{gr}}\right] \tag{2.29}$$

Substituting into Formula (2.29) the numerical values of T_0, t_{gr} and B, we obtain:

$$H = 44\,308\left[1 - \left(\frac{p_H}{p_0}\right)^{0.19028}\right]. \tag{2.30}$$

We can use Formula (2.30) to calculate the hypsometric tables which relate the flight altitude up to 11,000 m to the atmospheric pressure; these tables are used to adjust and correct altimeters.

Under the conditions of a standard atmosphere, the air temperature at altitudes greater than 11,000 m is considered to be constant, so that the barometric formula for these altitudes can be written as follows:

$$\ln\frac{p_H}{p_{11}} = -\frac{H - 11\,000}{BT_{11}}. \tag{2.31}$$

We obtain Formula (2.31) by integrating Equation (2.27) for /176
11,000 m and consider T_H equal to T_{11}:

$$\int_{p_{11}}^{p_H} \frac{dp}{p} = -\frac{1}{BT_{11}} \int_{11}^{H} dH$$

or

$$\ln \frac{p_H}{p_{11}} = -\frac{H - 11\,000}{BT_{11}}.$$

Solving Equation (2.31) for *H*, we obtain the standard state formula of the hypsometric table (Table 2.5) for altitudes greater than 11,000 m.

$$H = 11\,000 + BT_{11} \ln \frac{p_{11}}{p_H}. \quad (2.32)$$

TABLE 2.5.

H, м	p_H, мм Hg	T_H, °K	*a*, m/sec	*H*, м	p_H, мм Hg	T_H, °K	*a*, m/sec
—500	806,2	291,25	342,1	10 500	183,40	219,25	297,2
0	760,2	228,00	340,2	11 000	169,60	216,50	295,0
500	716,0	284,75	338,3	12 000	144,87	216,50	295,0
1 000	674,1	281,50	336,4	13 000	123,72	216,50	295,0
1 500	634,2	278,25	334,4	14 000	105,67	216,50	295,0
2 000	596,2	275,00	332,5	15 000	90,24	216,50	295,0
2 500	560,1	271,75	330,5	16 000	77,07	216,50	295,0
3 000	525,8	268,50	328,5	17 000	65,82	216,50	295,0
3 500	493,2	265,25	326,5	18 000	56,21	216,50	295,0
4 000	462,2	262,00	324,5	19 000	48,01	216,50	295,0
4 500	432,9	258,75	322,5	20 000	41,00	216,50	295,0
5 000	405,1	255,50	320,5	21 000	35,02	216,50	295,0
5 500	378,7	252,25	318,4	22 000	29,90	216,50	295,0
6 000	353,8	249,00	316,3	23 000	25,54	216,50	295,0
6 500	330,2	245,75	314,3	24 000	21,81	216,50	295,0
7 000	307,8	242,50	312,2	25 000	18,63	216,50	295,0
7 500	286,8	239,25	310,1	26 000	15,91	216,50	295,0
8 000	266,9	236,00	308,0	27 000	13,59	216,50	295,0
8 500	248,1	232,75	305,9	28 000	11,60	216,50	295,0
9 000	230,5	229,50	303,7	29 000	9,91	216,50	295,0
9 500	213,8	226,25	301,6	30 000	8,46	216,50	295,0
10 000	198,2	223,00	299,4				

Note: The table for adjusting and correcting the barometric altimeters is given in abbreviated form. The value *a* represents the speed of sound at flight altitude under standard conditions, given in the fourth column of the table.

Substituting the value of *B* and *T* = 216.5°, and shifting to the log ten (ln *N* = 2.30259 lg*N*), this formula assumes the form:

$$H = 11\,000 + 14\,600 \lg \frac{p_{11}}{p_H}. \quad (2.33)$$

Formulas (2.30) and (2.33), suitable for compiling hypsometric tables and calibrating altimeters, are not completely suitable for calculating the methodological errors in the altimeter, related to a failure of the actual air temperature at heights from zero to the flight altitude of the aircraft to agree with the conditions of the standard atmosphere. /177

Since the accuracy of altitude measurement is affected by the air temperature not only at the flight altitude but at all intermediate layers from the one on the ground up to that at the flight altitude, it is better to use the formula which relates the flight altitude not to the temperature gradient, but to the average temperature of the column of air which we have selected, and to use this to calculate a hypsometric table for adjusting and correcting barometric altimeters. This formula has the form:

$$H = BT_{av} \ln \frac{p_0}{p_H}. \tag{2.34}$$

Formula (2.34) is obtained by integrating Equation (2.27a) at a constant average temperature:

$$\int_{p_0}^{p_H} \frac{dp}{p} = \frac{1}{BT_{av}} \int_0^H dH,$$

whence

$$H = BT_{av} \ln \frac{p_0}{p_H}.$$

If we consider that

$$T_{av} = 273 + t_{av} = 273 \left(1 + \frac{t_{av}}{273}\right),$$

and the value $B = 29.27$, by using the coefficient for transition from natural logarithms to the log 10, Formula (2.34) assumes the form:

$$H = 18\,400 \left(1 + \frac{t_{av}}{273,1}\right) \lg \frac{p_0}{p_H}.$$

This formula is known as the Laplace formula.

Description of a Barometric Altimeter

The sensitive element in the barometric altimeter is a corrugated manometric (aneroid) box 1 (Fig. 2.30), made of brass. The box has two rigid points (on the top and bottom corrugated surfaces), one of which is fixed or tightly fastened to the casing of the apparatus, while the other is movable.

In principle, the aneroid box can be either evacuated or filled /178 with a gas.

Usually, the space within the box is filled with a gas to a pressure such that when the box is heated, the thermal losses of its elastic properties will be roughly compensated by an increase in gas pressure within the box when it is heated.

The casing of the altimeter is hermetically sealed and connected by a nipple to a sensor of the atmospheric (static) pressure.

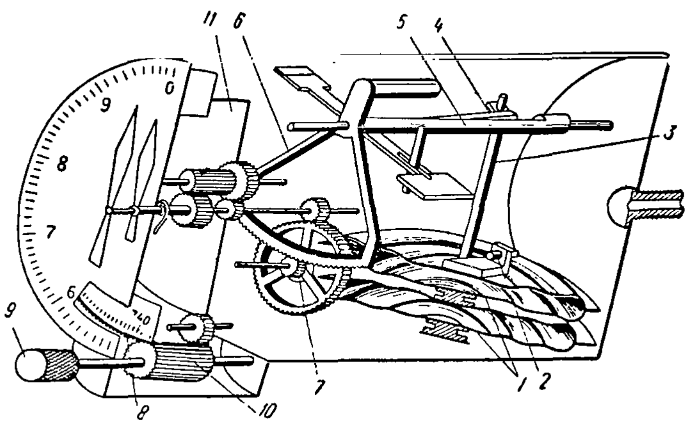


Fig. 2.30. Schematic Diagram of Barometric Altimeter.

When the aircraft is located at sea level, the aneroid box is compressed to the maximum degree, since the atmospheric pressure acting on it has a maximum value.

With a gain in altitude, the atmospheric pressure in the chamber decreases and the aneroid box expands due to its elastic properties, shifting its movable center (with bimetallic shaft 2) upward.

As it moves, the center displaces rod 3, which in turn acts through a lever 4 to convey a rotary motion to shaft 5.

Shaft 5 carries a toothed sector 6 with a counterweight, fitted with a cog wheel 7, which transmits the movement to the pointer through another gear.

Thus, the motion of the center of the box is used to indicate the flight altitude on the scale of the instrument.

In addition to the parts listed above, the kinematic portion of the instrument includes elements intended for regulating the instrument and adjusting the backlash in the transmission mechanism.

1. **Zero-point bimetallic compensator,** This device is intended /179
for compensating the temperature changes in the elastic properties of the box for zero altitude. If the atmospheric pressure in the casing of the instrument is set to zero altitude, but the temperature of the box increases, the loss of elastic properties of the material in the box creates additional compression, causing the indicator needle to shift from the zero altitude reading. The bimetallic strip bends as the temperature changes, due to different coefficients of linear expansion for the two materials of which it is made. By rotating the strip in its socket, it is possible to set it in a position such that the deflection in the direction of the shift of the center of the box will exactly correspond to the additional travel of this center, but in the opposite direction. Then rod 3 remains in place and the indicator needle will not move from the zero position.

2. **The regulating mechanism of the device.** This consists of strip 4 and an adjustment screw.

Turning the screw pushes the strip away from rod 5, changing the arm of the lever. This is used to regulate the angular velocity of rotation of the shaft, i.e., the transmission ratio of the apparatus. The transmission ratio of the rotation of the shaft is set so that the readings of the needle correspond to the atmospheric pressure in the casing of the apparatus.

3. **High temperature compensator.** When the elastic properties of the box change due to the effect of temperature, this not only causes an additional compression at zero altitude but also changes the amount by which its center moves with a change in altitude. For compensation of this error, strip 4 is of bimetallic construction. When the instrument is heated, and the travel of the center of the box increases, the end of the strip bends away from the shaft, thus reducing the transmission ratio for rotating shaft 5 and compensating for the increase in sensitivity of the box.

It is important not to confuse the instrumental temperature errors of the instrument, which are compensated by the zero and altitude bimetallic compensators, with the methodological temperature errors in the altimeter.

The instrumental errors are related to the temperature in the casing of the instrument, acting on the properties of the material from which the sensitive element is made, and can be overcome by compensators .

The methodological errors which are related to the nature of the changes in pressure with flight altitude can only be corrected by special formulas. The building of a compensator for methodological errors is impossible, since in the general case the temperature of the casing is not equal to the average air temperature from zero altitude up to the flight altitude of the aircraft.

In order to increase the accuracy of the altitude readings, altimeters are made with two pointers. This means that the aneroid box is made double, increasing the travel of the movable center by a factor of two. Between the toothed sector and the axis of the pointer, there are additional gears which increase the transmission ratio of the mechanism several times. The main pointer of the instrument makes several revolutions; the number of revolutions of the pointer is equal to the change in altitude in thousands of meters. /180

In addition, there is a pressure scale 8 for setting the altimeter readings relative to a desired level.

The altimeter mechanism, along with the axis of the main pointer, is rotated within the housing by means of a rack and pinion 9, consisting of a driving gear 10 and a driven gear 11. Thus, the main pointer of the instrument can be set to any division on the scale.

Simultaneously, by means of driving gear 10, the pressure scale 8 is set in motion, which can be used in conjunction with the main scale to determine the pressure at the level at which the flight altitude is calculated.

In the VD-10 and VD-20 altimeters, a movable ring is mounted around the main scale; it is rotated by means of a rack and pinion and driving gear 10 at an angular velocity equal to the rate of turn of the mechanism. It is used for shifting a movable index along the circular scale of the instrument, and can be set to the barometric altitude of the airport where the landing is to be made. This serves the same purpose as the pressure scale. However, the latter can only be used over a range of pressures from 670 to 790 mm Hg, while the movable index can be set to any airport altitude.

In cases when pressure scales are not sufficient for airports located at high altitudes, the pressure at the level of the airport is not measured aboard the aircraft, but rather the barometric altitude of the aircraft is used for setting the movable index of the altimeter.

Errors in Measuring Altitude with a Barometric Altimeter

The errors in measuring the flight altitude with barometric altimeters can be divided into instrumental and methodological errors:

Instrumental errors. These are related to incorrect adjustment of the altimeter, friction (wear) in the transmission mechanism, as well as temperature effects on the material of the sensitive element. The errors from so-called hysteresis are particularly important, i.e., the residual deformation of the sensitive box with changes in flight altitude of the aircraft over wide limits.

In addition, instrumental errors include errors in sensing the static pressure, related to dynamic flight of the aircraft. /181

Methodological errors. In the barometric method of measuring altitude, these include errors in correspondence of the initial atmospheric pressure, the pressure along the flight route, and the average air temperature with the calculated data.

Under flight conditions encountered in civil aircraft, methodological errors in measuring altitude in approaching aircraft are extremely rare, so that these errors do not disturb the mutual position of the aircraft and are not taken into account. However, they do have significant value in determining the safe flight altitude above the relief, as well as in making special flights (for purposes of aerial photography, e.g.).

In practice, the baric stage at low flight altitudes (the difference in altitude which corresponds to a drop in pressure of 1 mm Hg) is considered roughly equal to 11 m. However, at flight altitudes of 20,000 m, the baric stage is equal to 155 m, i.e., 14 times greater than on the ground.

The increase in the baric stage with flight altitude, as well as the errors in measuring static pressure due to aerodynamic processes, complicate a precise measurement of the barometric altitude at great altitudes and high speeds.

In a flight according to a table of corrections, it is relatively easy to compensate for instrumental errors in the apparatus, related only to its regulation. Consideration of all other instrumental errors presents greater difficulty, so that all measures are usually taken to reduce them to a minimum by carefully preparing the apparatus, selecting the point of calibration, and designing the static pressure sensor.

Methodological errors in altimeters are estimated by determining the true altitude of the aircraft above the relief for special purposes and in calculating safe flight altitudes above the relief. Changes in atmospheric pressure along the flight route, relative to sea level, are calculated in baric stages, so that the lowest flight altitude oscillates as follows:

$$\Delta H = \Delta p \cdot 11.$$

For example, if the pressure measured at sea level at the point where the altitude is measured differs from 760 mm Hg to 15 mm, the methodological error in measuring the altitude from the level of 760 mm will be $15 \cdot 11 = 165$ m.

Hence, if the corrected pressure is greater than 760 mm Hg, i.e., equal to 775 mm in our example, the readings of the altimeter will be reduced and the correction will have to carry a plus sign, while if the corrected pressure is lower than the calculated pressure, it will have a minus sign. /182

Methodological errors in the altimeter, which arise due to a failure of the actual mean air temperature to coincide with the calculated temperature, are accounted for by means of a navigational slide rule, a description of which is given below. Proceeding from the fact that the instrument indicates a flight altitude on the basis of the calculated mean temperature of the air, and the corrected altitude must be determined on the basis of the actual altitude, the equation reads as follows:

$$H_{inst} = BT_{av.c.} \ln \frac{p_0}{p};$$

$$H_{corr} = BT_{av.a.} \ln \frac{p_0}{p},$$

where $T_{av.c.}$ is the average calculated temperature and $T_{av.a.}$ is the average actual temperature.

Whence

$$H_{corr} = \frac{H_{inst}}{T_{av.c.}} T_{av.a.}$$

Therefore,

$$T_{av.a.} = \frac{T_0 + T_H}{2}$$

$$\lg H_{corr} = \lg \frac{T_0 + T_H}{2} + \lg \frac{H_{inst}}{T_{av.c.}} \qquad (2.35)$$

where T_0 and T_H are the temperatures on the ground and at flight altitude, respectively.

By using Formula (2.35), we can calculate the scales of the navigational slide rule NL-10 for making corrections in the readings of altimeters for air temperature up to altitudes of 12,000 m.

For altitudes above 12,000 m, the corrected altitude is found by the formula

$$H_{corr} - 11{,}000 = \frac{T_{H_a}}{T_{H_c}} (H_{inst} - 11{,}000),$$

where T_{H_a} and T_{H_c} are the actual and calculated temperatures at the altitude.

The navigational slide rule for these altitudes is also provided with logarithmic scales according to the formula

$$\lg(H_{corr}-11{,}000) = \lg T_{H_a} + \lg \frac{H_{inst}-11{,}000}{216.5°}. \qquad (2.36)$$

4. Airspeed Indicators

/183

The flight of an aircraft takes place in the medium of air, so that a simplest and easiest method from the technical standpoint for measuring airspeed would be to measure the aerodynamic pressure or so-called velocity head of the incident airflow.

For purposes of aircraft navigation, it is better to measure the speed of the aircraft relative to the surface of the ground, since the air mass practically always has its own movement relative to the latter. At the present time, there are radial and inertial methods of measuring the speed relative to the ground, but the measurement of airspeed does not lose its significance even in the presence of such equipment.

The fact is that the stability and maneuverability of an aircraft depends on the airspeed. In addition, the operational regime of the motors on the aircraft and the fuel consumption depend on the airspeed.

The operating principle of airspeed indicators is based on a measurement of the aerodynamic pressure of the incident airflow.

The relationship between the rate of motion of a liquid or gas and its dynamic and static pressure was first established by the St. Petersburg Academician Daniel Bernoulli (1738), working with incompressible liquids or gases (Fig. 2.31).

According to the principle of inseparability of flow, the product of the speed of an air current (V) multiplied by the cross sectional area of a tube (S) must be uniform everywhere within its cross section. Consequently, in a narrow part of the tube, the speed of the flow must be greater than in a wide section.

In the general case, if the tube is not horizontal, a mass of gas m enters the tube during a time Δt which introduces an energy consisting of three components: the potential energy of the gas

$$mgh$$

the kinetic energy

$$\frac{mV_1^2}{2};$$

and the work of influx into the tube

$$p_1 S_1 V_1 \Delta t,$$

where g is the acceleration due to the Earth's gravity, h is the difference in the gas levels, and p is the gas pressure inside the tube.

These components determine the energy of the gas flowing out of the tube. Therefore /184

$$\frac{mV_1^2}{2} + p_1 S_1 V_1 \Delta t + mgh_1 = \frac{mV_2^2}{2} + p_2 S_2 V_2 \Delta t + mgh_2.$$

The product $SV\Delta t$ is the volume of fluid flowing through the cross section of the tube in a time Δt. Therefore, dividing the mass into the volume gives us the density (ρ), which is

$$\frac{\rho V_1^2}{2} + p_1 + \rho g h_1 = \frac{\rho V_2^2}{2} + p_2 + \rho g h_2.$$

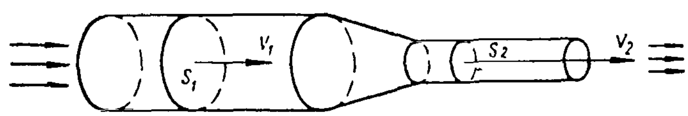


Fig. 2.31. Flow in a Tube with Varying Cross Section.

If the tube through which the current is flowing is horizontal, $h_1 = h_2$, therefore

$$\frac{\rho V_1^2}{2} + p_1 = \frac{\rho V_2^2}{2} + p_2, \quad (2.37)$$

i.e., the sum of the dynamic and static pressures at any point in the tube remains constant, since the dynamic component is proportional to the gas density (fluid density) and the square of the speed of flow.

For adiabatic compression, i.e., when the process takes place with compression of the gas (air) without exchange of heat energy with the surrounding medium, which almost always can be considered valid for high speed events, this equation takes the form:

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma_1} + U_1 + E_1 = \frac{V_2^2}{2g} + \frac{p_2}{\gamma_2} + U_2 + E_2, \quad (2.38)$$

where γ is the unit weight (weight density) of the gas, V is the internal (thermal) energy of the gas, and E is the potential energy of the gas.

Therefore, a change in the rate of airflow during flight due to the flow being retarded is usually negligible; the component E can be considered constant and may be omitted from the equation. Then each of the remaining terms of the equation, if we multiply them by mg, will characterize the component energy included in a unit mass of gas flow: $V^2/2g$ equals the kinetic energy of the flow (for a unit mass $mV^2/2$), p/γ is the energy of the pressure, and U is the thermal energy.

For measurements of airspeed, we can use sensors which allow us to separate the dynamic air pressure from the static pressure. /185

Figure 2.32 shows the operation of an air pressure sensor (Pitot tube).

In the cross section of the airflow, the speed V_1 will correspond to the airspeed, and the pressure p_1 will correspond to the static pressure of the air at flight altitude.

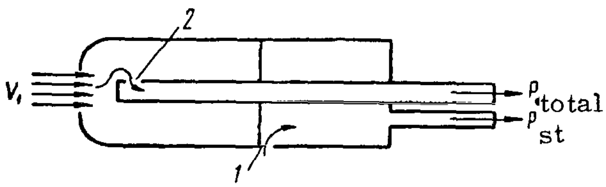


Fig. 2.32. Air-Pressure Sensor (Pilot Tube).
(1) Static Pressure p_{st}; (2) Total Pressure p_{total}.

Within the limits of the opening in the sensor for total pressure, the rate of flow will be equal to zero (the critical current or current of complete braking).

Obviously, at this point the pressure p_2 will correspond to the total pressure (the velocity head plus the static pressure), and Equation (2.38) acquires the following form for this case:

$$\frac{V^2}{2g} + \frac{p_{st}}{\gamma_1} + U_1 = \frac{p_{total}}{\gamma_2} U_2. \qquad (2.39)$$

Let us consider that for airspeeds up to 400 km/hr the compression of the air can be disregarded, i.e., the values γ and U are constants. Then Equation (2.39) assumes the form:

$$\frac{V^2}{2g} = \frac{p_{total} p_{st}}{\gamma_H} \qquad (2.40)$$

where γ_H is the unit weight of gas at a given altitude.

Since $\gamma_H = \rho_H g$ (where ρ_H is the mass density), the difference between the total and static pressures (velocity head) will be equal to

$$\frac{\rho V^2}{2} = p_{total} - p_{st}$$

whence

$$V = \sqrt{\frac{2(p_{total} - p_{st})}{\rho_H}}. \qquad (2.41)$$

The total pressure along the tube is admitted to the interior of a flexible box. The static pressure reaches the hermetically sealed chamber of the indicator through an opening made in the side of the pressure sensor and through a nipple. As a result, there will be a pressure drop between the internal space of the box and the medium surrounding the box, which will be equal to the velocity head. /186

This drop causes movement of the top of the box, which can transmit its movement by means of a system of gears similar to the mechanism in a single-pointer altimeter, eventually moving a pointer on an axis to show the airspeed on a scale which is graduated in kilometers per hr.

Formula (2.41) can be used to describe airspeed indicators for low speeds, such as the US-350. If we introduce the weight density to this formula in the form

$$\gamma_H = \frac{p_H}{BT_H},$$

whence

$$V = \sqrt{\frac{2(p_{total} - p_{st})}{p_{st}} gBT_H}. \qquad (2.42)$$

It is clear from this formula that in order to determine the true airspeed, it is necessary to know not only the value of the velocity head, but also the atmospheric pressure and the temperature of the air at flight altitude.

The airspeed, which is measured only on the basis of the velocity head, is called the aerodynamic or indicated speed. In view of the fact that calibration of the speed indicator is made for flight conditions at sea level at standard temperature and air pressure, during flight under these conditions the indicated speed will be equal to the true airspeed. Under other conditions, however, the indicated speed must be converted to the true airspeed.

At high altitudes and speeds, the difference between the airspeed and the indicated speed becomes so significant that it becomes difficult to use the latter for navigational purposes. In addition, for airspeeds above 400 km/hr, it becomes necessary to take the compression of the air into account as well. Therefore, for aircraft operating at high altitudes and speeds, a combined speed indicator "CSI" has been developed, which measures both the indicated and true airspeed.

In terms of its design, this indicator differs from the usual speed indicators in that the speed is measured in two ways:

(a) The first method consists of the conventional system for indicating speed and is used to measure the indicated airspeed (the large pointer on the dial);

(b) The second system incorporates a special compensator for changes in air density with altitude by means of a system of gears /187
and is used to measure the airspeed.

The compensator is an aneroid box, which changes the length of the arm of a control lever, increasing the latter's mechanical advantage when the atmospheric pressure (as well as the density of the air at flight altitude) is reduced, and vice versa.

It should be mentioned that in the case of high speed aircraft, the sensor for total pressure is usually separated from the static pressure indicator, so that it is possible to select the most suitable position for mounting them on the aircraft. This means that the role of the static pressure indicator is played by openings which are made on the lateral surface of the fuselage of the aircraft and are linked to the instrument itself by tubing.

In addition to the details of design described above, the regulation of the systems in the CSI are made by taking the compression of the air into account when the flow is retarded in the detector for total pressure.

Therefore, compression of the air on braking will be accompanied by heating, and therefore by an increase in its internal energy.

The relationship between the internal energy of the gas, its pressure, and weight densities is expressed by the formula:

$$U = \frac{1}{K-1} \cdot \frac{p}{\gamma}, \qquad (2.43)$$

where $K = \frac{C_p}{C_V}$ is the ratio of the specific heats of the gas when it is heated, with retention of constant pressure and constant volume.

For air, this coefficient is $K = 1.4$.

By substituting the value U into Formula (2.39), we can change it to read as follows:

$$\frac{V^2}{2g}+\frac{p_{st}}{\gamma_1}+\frac{1}{K-1}\cdot\frac{p_{st}}{\gamma_1}=\frac{p_{total}}{\gamma_2}+\frac{1}{K-1}\cdot\frac{p_{total}}{\gamma_2}.$$

or

$$\frac{V^2}{2g}+\frac{p_{st}}{\gamma_1}\cdot\frac{K}{K-1}=\frac{p_{total}}{\gamma_2}\cdot\frac{K}{K-1}. \tag{2.44}$$

After making some simple conversions,

$$\frac{V^2}{2g}=\frac{K}{K-1}\left(\frac{p_{total}}{\gamma_2}-\frac{p_{st}}{\gamma_1}\right)$$

taking p_{st}/γ_1 out of the parentheses, we will have:

$$\frac{V^2}{2g}=\frac{K}{K-1}\cdot\frac{p_{st}}{\gamma_1}\left(\frac{p_{total}}{p_{st}}\cdot\frac{\gamma_1}{\gamma_2}-1\right). \tag{2.45}$$

For the adiabatic process, there is an equation which is known /188 as the Mendeleyev-Clapeyron equation:

$$\frac{p_1}{\gamma_1^K}=\frac{p_2}{\gamma_2^K},$$

from which we obtain for our case

$$\frac{\gamma_1}{\gamma_2}=\left(\frac{p_{st}}{p_{total}}\right)^{\frac{1}{K}}.$$

Substituting the value γ_1/γ_2 in Formula (2.45), we obtain

$$\frac{V^2}{2g}=\frac{K}{K-1}\cdot\frac{p_{st}}{\gamma_1}\left[\left(\frac{p_{total}}{p_{st}}\right)^{\frac{K-1}{K}}-1\right].$$

Assuming that $\gamma_1 = \gamma_H$, so that $P_{st}/\gamma_1 = BT_H$, we can rewrite this equation in the form:

$$V^2=\frac{2K}{K-1}\,gBT_H\left[\left(\frac{p_{total}-p_{st}}{p_{st}}+1\right)^{\frac{K-1}{K}}-1\right]$$

and finally obtain the formula which can be used to calibrate the combined speed indicator by the airspeed in the channel for subsonic airspeeds:

$$V=\sqrt{\frac{2K}{K-1}\,gBT_H\left[\left(\frac{p_{total}-p_{st}}{p_{st}}+1\right)^{\frac{K-1}{K}}-1\right]}, \tag{2.46}$$

The temperature at flight altitude (T_H) is assumed to be standard according to the flight altitude (or p_{st}), i.e., up to 11,000 m, $T_H = 288° - 6.5° H$, and above 11,000 m $T_H = 216.5°$ K (-56.5° C).

To calibrate the airspeed indicator, it is necessary to know the pressure in its manometric box and in the housing of the apparatus, corresponding to the pressure in the sensors of total and static pressure under the given flight conditions. Therefore, (2.46) is solved relative to the pressures and assumes the form:

$$\frac{p_{total} - p_{st}}{p_{st}} = \left[1 + \frac{(K-1)V^2}{2KgBT}\right]^{\frac{K}{K-1}} - 1 \qquad (2.47)$$

or, if we insert the numerical values of K, g, B,

$$\frac{p_{total} - p_{st}}{p_{st}} = \left[1 + \frac{V^2}{2060T}\right]^{3,5} - 1. \qquad (2.47a)$$

As we have already pointed out, (2.46) is valid for subsonic airspeeds. At speeds which exceed the speed of sound, the flow of the particles differs from their flow at subsonic speed. /189

The local compression produced in the air by the aircraft cannot propagate itself in the atmosphere faster than the speed of sound. Therefore, at supersonic speeds, local interruptions in density are produced, in which the rate of flow decreases sharply while the pressure increases sharply.

We know that the rate at which sound travels (a) in air depends only on the temperature of the medium and is expressed by the formula

$$a = \sqrt{KgBT}.$$

In other words, if $g = 9.81$ m/sec^2, and the coefficient for air is equal to 1.4, while $B = 29.27$ m/degree,

$$a = \sqrt{412T} = 20,3\sqrt{T} \text{ m/sec.}$$

The ratio of the airspeed to the rate of propagation of sound in air is called the *Mach number:*

$$M = \frac{V}{a}.$$

If we replace $KgBT$ in Formula (2.46) by a^2, we will have the expression for M (Mach number) for subsonic airspeeds:

$$M = \sqrt{\frac{2}{K-1}\left[\left(\frac{p_{total} - p_{st}}{p_{st}} - 1\right)^{\frac{K-1}{K}} - 1\right]}. \qquad (2.48)$$

The latter formula indicates that in order to determine the Mach number, it is necessary to know only the velocity head and the static pressure at flight altitudes. There is no necessity to measure air temperature for this purpose.

For subsonic airspeeds, the relationship between the total pressure, the static pressure, and the Mach number is expressed as follows:

$$\frac{p_{total}-p_{st}}{p_{st}}=\left(\frac{K+1}{2}\right)^{\frac{K+1}{K-1}}\left(\frac{2}{K-1}\right)^{\frac{1}{K-1}}\frac{M^{\frac{2K}{K-1}}}{\left(\frac{2K}{K-1}M^2-1\right)^{\frac{1}{K-1}}}-1. \qquad (2.49)$$

In this formula, if we replace M^2 by its value as obtained in Equation (2.48), we will obtain the formula for calibrating the airspeed indicator for supersonic airspeeds:

$$\frac{p_{total}-p_{st}}{p_{st}}=\frac{\left(\frac{K+1}{2}\right)^{\frac{K+1}{K-1}}\left(\frac{2}{K-1}\right)^{\frac{1}{K-1}}\left(\frac{V^2}{KgBT}\right)^{\frac{K}{K-1}}}{\left(\frac{2K}{K-1}\cdot\frac{V^2}{KgBT}-1\right)^{\frac{1}{K-1}}}-1. \qquad (2.50)$$

If we substitute the numerical values of K for air, equal to 1.4, in Equation (2.50), we can convert it to the simpler form: /190

$$\frac{p_{total}-p_{st}}{p_{st}}=\frac{166.7V^7}{a^2(7V^2-a^2)^{2.5}}-1.$$

Errors in Measuring Airspeed

Errors in measuring airspeed, like those involved in measuring flight altitude, can be divided into instrumental and methodological ones. Instrumental errors include those which are related to improper adjustment of the apparatus and instability of its operation with changes in the temperature of the mechanism in the device. In addition, instrumental errors also include errors in sensing dynamic and especially static pressures with sensors which depend on the mounting location on the aircraft.

Instrumental errors are corrected by correction charts, which are compiled when the apparatus is tested, taking into account the errors in indicating the static pressure for a given type of aircraft.

Methodological errors include those involving failure of the actual air temperature at flight altitude to correspond with the calculated temperature for combined indicators of speed, and with the temperature and pressure for other speed indicators.

Strictly speaking, the methodological corrections which must be taken into account in converting the indicated speed to the airsoeed, are not instrument errors, since the indicated speed has its own independent value. However, from the navigational standpoint, it is convenient to consider them methodological errors.

In aircraft navigation, it is possible to use both the single pointer dial for indicated speed (Type US-350 or US-700), as well as the combined indicator (Type CSI-1200) and others, so that the methods of calculating the methodological errors can be viewed separately.

It should be mentioned first of all that the dials of speed indicators are calibrated to take into account the compressibility of the air for a true airspeed equal to the indicated speed.

In fact, at high altitudes, the true airspeed is almost always much greater than the indicated speed, so that it is necessary to consider that there is an error in the difference between the compression of the air at the actual and calculated airspeeds:

$$\Delta V_{comp} = \Delta V_{comp.a.} - \Delta V_{comp.c.}$$

There are special, precise formulas for determining the corrections for ΔV_{comp} for use with indicators of instrument speed at subsonic and supersonic airspeeds, and they are used to draw up a table of corrections (Fig. 2.33); we will limit ourselves to /191
discussing only the simple approximate formula

$$\Delta V_{comp} \approx \frac{1}{12}\left(\frac{V_{ind}}{100}\right)^3\left(\frac{p_0}{p_H} - 1\right), \qquad (2.51)$$

where V_{ind} is the indicated airspeed and ΔV_{comp} is the correction for the indicated speed.

In the approximate formula given above, as well as in the graph which is the result of accurate formulas, the corrections for compression are determined only as a function of flight altitude. It is clear that this involves an error which is related to a failure of the actual air temperature at the given altitude to correspond to standard temperature. However, if we recall that large variations in temperature usually occur only at low altitudes, where high-speed aircraft practically never fly, these errors can be disregarded.

After making the corrections in the indicated speed for the compression of the air, conversion of the latter into airspeeds is done on an navigational slide rule.

Since the dial of the indicated airspeed is calibrated by the formula

$$V_{inst} = \sqrt{\frac{2(p_{total} - p_H)}{p_0} gBT_0},$$

and the airspeed is

$$V_{true} = \sqrt{\frac{2(p_{total} - p_H)}{p_H} gBT_H},$$

then if we divide the second formula by the first we will obtain:

$$V_{true} = V_{inst} \sqrt{\frac{p_0}{p_H} \frac{T_H}{T_0}}. \qquad (2.52)$$

Let us substitute into Formula (2.28) the following values: t_{gr} = 6.5 deg/km, T_0 = 288° K, and B = 29.27. We will then obtain:

$$p_H = p_0 (1 - 0.0226H)^{5.256},$$

and if we let the value p_H be substituted into Formula (2.52), we will obtain:

$$V_{true} = V_{inst} \sqrt{\frac{T_H}{T_0} \cdot \frac{1}{(1 - 0.0226H)^{2.628}}}$$

or

$$\lg V_{true} = \lg V_{inst} + \frac{1}{2} \lg (273 + t_H) - \frac{1}{2} \lg 288 - 2.628 \lg (1 - 0.0226H). \qquad (2.53)$$

According to Formula (2.53) we can convert the logarithmic scales of navigational slide rules for converting the indicated airspeed into the true airspeed.

Calibration of the combined speed indicator on the basis of the true airspeed is performed by taking into account the compressibility of the air over the entire range of the scale. The methodological error in the reading is related only to the differences between the actual air temperature and the calculated temperature at the flight altitude.

Since the airspeed, as shown by a combined speed indicator under standard temperature conditions, is expressed by the formula /192

$$V_{CSI}=\sqrt{\frac{2K}{K-1}BgT_{H_p}\left[\left(\frac{P_{total}-P_{st}}{P_{st}}+1\right)^{\frac{K-1}{K}}-1\right]}$$

and the corrected value for the airspeed at flight altitude in accordance with the actual temperature is

$$V_{corr}=\sqrt{\frac{2K}{K-1}BgT_{H_a}\left[\left(\frac{P_{total}-P_{st}}{P_{st}}+1\right)^{\frac{K-1}{K}}-1\right]},$$

if we divide the second formula into the first, we will have:

$$V_{corr}=V_{CSI}\sqrt{\frac{T_{H_a}}{T_{H_p}}},$$

or

$$V_{corr}=V_{CSI}\sqrt{\frac{273+t_H}{288-0.0065H_{inst}}}. \qquad (2.54)$$

After looking up the logarithm of the latter, we will obtain a formula which can be used to construct the logarithmic scale on the NL-10M for a combined speed indicator:

$$\lg V_{corr}=\lg V_{CSI}+\frac{1}{2}\lg(273+t_H)-\frac{1}{2}\lg(288-0.0065H_{inst}) \qquad (2.54a)$$

Relationship Between Errors in Speed Indicators and Flight Altitude

In describing the errors in barometric altimeters and airspeed indicators, instrumental errors of aerodynamic origin are found, which are related to errors in recording the static pressure by the air pressure sensors.

Experience has shown that aerodynamic errors in the speed indicators due to incorrect recording of the dynamic pressure are negligibly small by comparison with the errors in incorrect recording of static pressure. This is explained by the fact that it it immensely easier to measure the pressure of a retarded airflow with a sensor that is aimed into the airflow, than it is to select a location on an aircraft for a static-pressure sensor, such that the latter will not be distorted by the airflow over the body of the aircraft.

In connection with the fact that the static pressure from the sensor is transmitted simultaneously to the hermetic chambers of the speed and altitude indicators, there must be a mutual relationship between the errors in the measurement of altitude and speed owing to errors in recording the pressure. /193

At the same time, the velocity head according to which the dial of the speed indicator is calibrated is equal to

$$p_{total} - p_{st} = \frac{\rho_0 V^2}{2} . \tag{2.55}$$

Since the errors in measuring the velocity head are equal to the errors in measuring the static pressure, then

$$\Delta p_{st} = \frac{\rho_0}{2} \Delta(V^2). \tag{2.56}$$

Under standard conditions, ρ_0 = 0.125 $kg/sec^2/m^4$. The static pressure is usually given in mm Hg. The specific gravity of mercury is 13.6, so that the pressure of 1 kg/cm^2 would equal 10,000/13.6 = 735 mm Hg.

On the other hand, since the parameter ρ has m^4 in the denominator, the pressure expressed by (2.56), relative to an area of 1 m^2, must be divided by 10,000 to determine the value for 1 cm^2, so that we finally obtain

$$\Delta p_{st} = \frac{735 \cdot 0.125}{2 \cdot 10\,000} \Delta(V^2) \approx 0.0048\,\Delta(V^2).$$

Example: At an indicated speed of 396 km/hr (110 m/sec), at a flight altitude of 5000 m, the aerodynamic correction for the speed indicator is 36 km/hr, (10m/sec). Find the aerodynamic error in the altimeter.

Solution:

$$\Delta p_{st} = 0.0048(110^2 - 100^2) = 0.0048 \cdot 2100 = 10.8 \text{ mm Hg}.$$

According to the hypsometric table, the baric stage at a flight altitude of 5000 m is equal to 18.5 mm Hg; hence, the aerodynamic component in the altimeter error is

$$\Delta H = -10.8 \cdot 18.5 = -200 \text{ m}.$$

Formula (2.56) is an approximate one, but it yields sufficiently accurate results up to an indicated airspeed of 400 km/hr. The altimeter error can be determined more precisely if we know the dynamic pressure and take into account the compression of the air at different instrument readings.

Table 2.6 shows the velocity head at various indicated airspeeds, and can be used to determine the aerodynamic corrections of the altimeter. The third column in Table 2.6 shows the manometric stage, i.e., the change in pressure with change in airspeed by 1 km/hr. If we multiply the aerodynamic correction of the speed indicator by the manometric stage and then use the hypsometric table, it will be easy to determine the aerodynamic correction for the altimeter for a given flight altitude.

TABLE 2.6. /194

V_{inst}	$P_{total}-P_{st}$	Δp for 1 km/hr	V_{inst}	$P_{total}-P_{st}$	Δp 1 km/hr
50	0,89	0,054	700	188,3	0,65
100	3,57	0,089	800	252	0,8
150	8	0,126	900	322	0,96
200	14,3	0,162	1000	418	1,04
250	22,37	0,2	1100	522,8	1,23
300	32,4	0,24	1200	645,8	1,42
350	44,27	0,28	1300	787,2	1,6
400	58,25	0,32	1400	947,2	1,78
450	74,23	0,36	1500	1125,4	1,92
500	92,35	0,41	1600	1317,6	2,08
550	112,7	0,46	1700	1525,7	2,23
600	135,7	0,53	1800	1748,8	2,4

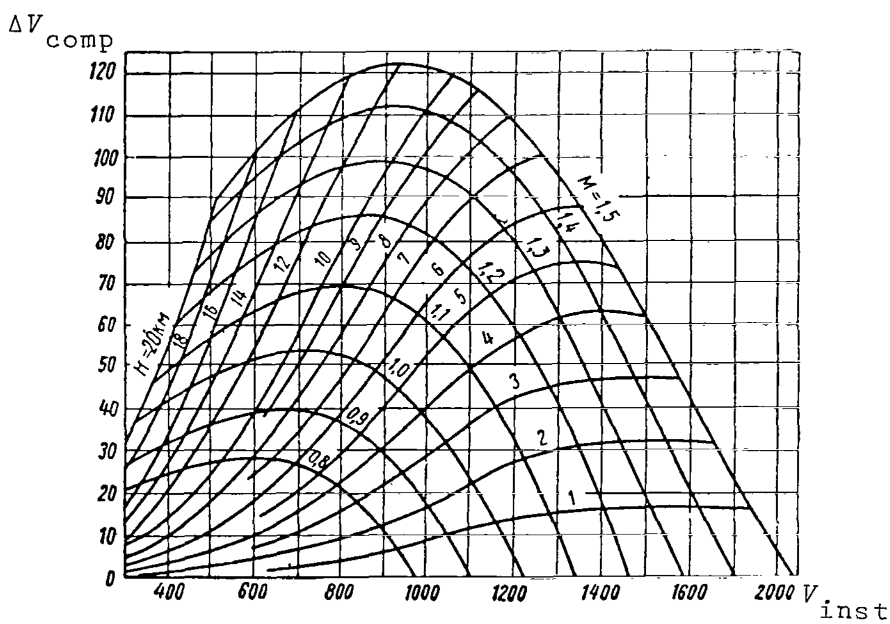


Fig. 2.33. Graph of Corrections for Air Compression.

5. Measurement of the Temperature of the Outside Air

Measurement of the temperature of the outside air during flight is necessary first of all for determining the true values of the airspeed and flight altitude.

The thermometer for measuring the outside air temperature is a remote-controlled instrument, i.e., its sensitive element is mounted outside the cabin of the aircraft and is exposed to the airflow, while the indicator is mounted on the instrument panel in the cockpit. /195

At the present time, electric thermometers are used for measuring the outside air temperature, and their operating principle is based on the changes in electrical conductivity of materials depending on their temperature.

A schematic diagram of such a thermometer is shown in Figure 2.34 and consists of an electrical bridge made of resistors.

If the arms of the bridge 1 and 1_1, as well as 2 and 2_1, have the same resistance when connected in pairs, no supply voltage will flow through bridge AB and consequently through the temperature indicator.

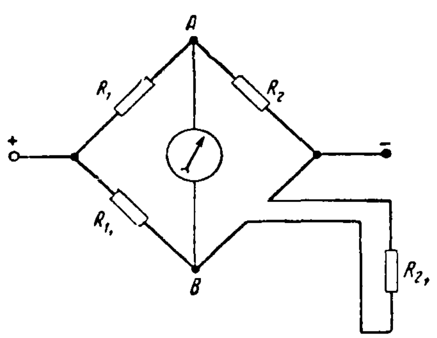


Fig. 2.34. Schematic Diagram of Electric Thermometer.

One arm of the bridge (2_1) is made of a material which has a high thermoelectric coefficient, and is mounted on the surface of the aircraft to be exposed to airflow.

Depending on the temperature of arm 2_1, its resistance changes, thus affecting the amount of current which passes through bridge AB with the temperature indicator connected to it.

Thermometers of this kind, when used at low airspeeds, indicate the temperature with an accuracy of 2-3°. However, at high airspeeds, due to drag and adiabatic compression of the airflow on the forward section of the sensor, the latter is subjected to local heating that creates methodological errors in measuring temperature.

For an exact determination of the methodological errors of this thermometer, we will require a sensor with complete braking of the airflow, as is the case in sensors used to measure the total pressure in airspeed indicators.

If we keep in mind that $\gamma = p/BT$, (2.44) can be changed to read as follows:

$$\frac{V^2}{2}+\frac{K}{K-1}\,gBT_H=\frac{K}{K-1}\,gBT_T,$$

where T_T is the temperature of the retarded flow. Therefore /196

$$T_H=T_H-\frac{K-1}{2KgB}\,V^2 \qquad (2.57)$$

If we substitute in (2.57) the values K = 1.4 and B = 29.27, we will obtain

$$\Delta T=\Delta t=\frac{V^2}{2000},$$

where V is the velocity, expressed in m/sec.

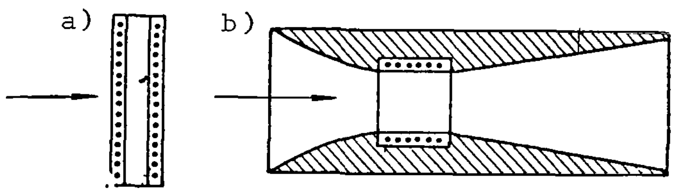


Fig. 2.35. Sensors for Electric Thermometer for Measuring Outside Air Temperature. (a) TUE; (b) TNV.

Since the conversion coefficient for changing from m/sec to km/hr is 3.6, for a speed expressed in km/hr

$$\Delta t=\frac{V^2}{2000\cdot 3.6^2}=\frac{V^2}{26\,000}. \qquad (2.57a)$$

Practically speaking, it is highly unsuitable to use thermometers for measuring outside air temperature which have complete retardation of airflow, since in this case the sensor will not be exposed to the flow and this will result in a high thermal inertia of the thermometer, i.e., rapid changes in temperature during flight, which could take place at high flight speeds, would not be detected by the thermometer.

For sensors which are exposed to the airflow, the coefficient of drag is within the limits of 0.5 to 0.85. The TUE and TNV thermometers in use at the present time have coefficients of drag which are nearly the same (approximately 0.7). The scale of corrections for the thermometer for measuring outside air temperature (TUE), located on the navigational slide rule, can be used with sufficient accuracy for the TNV thermometers as well.

The sensor of the TUE thermometer is in the shape of a rod with a winding on the surface, covered by a cylindrical housing (Fig. 2.35, a). When a flow of air passes through such a sensor, it is heated on one side. /197

The sensor of the TNV thermometer is made in the form of a de Laval nozzle. The sensitive element is located in the narrowest portion of the nozzle (Fig. 2.35, b) and the air flows symmetrically over it. Therefore, this sensor has less thermal inertia and gives more accurate readings in different flight regimes.

6. Aviation Clocks

The measurement of time plays an extremely important role in aircraft navigation, since the calculation of the path of the aircraft on the basis of the component airspeed and time is involved in almost all navigational equations. This means that an increase in the airspeed places increased demands on the accuracy of the measurement of time. It is especially important to have an exact determination of the moments of passage over control checkpoints, i.e., in this case, the exact measurement not of elapsed time but of time segments between the moments when the aircraft is passing over landmarks.

There are also factors which demand high accuracy in determining the time and the exact operation of aviation clocks. For example, the coincidence of the flight plans of individual aircraft, communication with the tower, and especially in astronomical calculations, where an error in calculating the elapsed time of 1 min could produce an error in determining the aircraft coordinates of 27 km.

The operating principle of all existing devices for measuring time is their comparison with the time required for some standard event to occur. In this case, the standard event is the period of oscillation of the balance wheel of a clock (a circular pendulum). All of the remaining mechanism of the clock acts mainly as a mechanical counter of the number of oscillations of the pendulum.

However, it exerts a considerable influence on the accuracy of operation of the clock; when the main spring of a clock is wound completely, the clock runs somewhat faster, and when the spring has run down the clock runs slower. The most important role in measuring time is played by the accuracy of adjustment of the actual period of oscillation of the pendulum.

We know that the period of oscillation of a body around its axis (torsional oscillation) is related to the deformation of the body as determined by the formula

$$T=2\pi\sqrt{\frac{j}{D}},$$

where T is the period of oscillation of the body around the axis, J is the moment of inertia of the body, and D is the modulus of torsion.

The product of the modulus of torsion times the angle through which the body rotates (ϕ) is the torsional moment:

$$M = D\phi.$$

The period of oscillation of a balance can be adjusted both /198 by changing its moment of inertia (for which purpose adjusting screws are located along its outer circumference), or by changing the modulus of torsion.

The moment of inertia of the balance wheel is changed by screwing the adjusting screws symmetrically in or out along the entire circumference, in order not to disturb the balance of the pendulum. This means that a portion of the mass is brought closer to or moved further away from the center of rotation of the balance.

The modulus of torsion is adjusted by means of a hairspring; the balance wheel is adjusted by changing the free length of the hairspring, for which purpose a movable stop, which acts as a regulator, is mounted near the point where the hairspring is fastened.

It should be mentioned that many factors affect the precision with which a clock operates, but the most important ones are temperature and magnetic effects. Therefore, a number of measures are taken to exclude these factors.

The balance wheel of an accurate clock is usually made of bimetallic material and divided along the plane of the diameter.

When the temperature falls and the flexibility of the hairspring increases (the modulus D increases), one-half of the balance expands and its ends move further away from the center of rotation, thus compensating for the temperature error in the clock.

The harmful effect of magnetic fields on the accuracy of clocks can usually be overcome by using diamagnetic parts in the balance wheel, hairspring and escapement, or else the entire clock mechanism is placed within a shielded housing made of iron alloy.

Special Requirements for Aviation Clocks

In addition to the general requirements for clock mechanisms (high accuracy, compensation for temperature and magnetic effects), aviation clocks have additional requirements placed upon them:

(a) Protection against vibration and shock, so that the clocks on an aircraft must be mounted in special shock mountings.

(b) Ensuring reliable operation under conditions of low temperature; for this purpose, aviation clocks are usually fitted with electric heaters.

(c) Reliability and accuracy of operation under various conditions. The hands, numerals, and principal scale divisions are made larger and covered with a luminous material to permit their use during night flights.

(d) The possibility of measuring simultaneously several time parameters. This means that several dials are usually driven by the mechanism.

Aviation clocks of the ACCH type (aviation clock-chronometer with heater) are made to satisfy all the conditions listed above. /199

The elapsed time is indicated on these clocks by a main dial with a central pointer. To calculate the total flight time or the flight time over individual stages, there is an additional scale in the upper part of the clock. The start of the clock hands is marked on this scale, while the time when they stop as well as the resetting to zero are accomplished by pushing a button on the left-hand side of the clock housing. This same button, when pulled out, is used to wind the main spring of the clock.

Below the "flight time" scale, there is a pilot light which is used to signal the following by means of a special shutter:

(a) Start of mechanism: red light.

(b) Stop mechanism: the light is half red and half white.

(c) Pause: white light.

To measure short time events, the clock is fitted with a sweep hand (thin central pointer) and an additional scale at the bottom of the apparatus where the minutes are counted. The sweep hand is started, stopped and held by pressing a button on the right-hand side of the housing.

In addition to the ACCH, the aviation chronometer 13 ChP is currently in use. It employs a potentiometric circuit; the version fitted with indicators is the 20 ChP. This chronometer, especially intended for purposes of astronomical orientation, is operated by remote control and consists of three main indicators:

(a) An elapsed-time indicator whose readings are always linked to the chronometer at the transmitter.

(b) Two time indicators for measuring the altitude of luminaries; their readings are also connected to the chronometer at the transmitter, but at the moment of measurement of the altitude of the luminary by means of a sextant, a stop signal is sent to one of them and the time of measurement is noted.

After the reading is made, the minute hand is set to the elapsed time according to the readings of the first dial by pushing the button. Each time the button is pressed, the hand moves forward one minute. The sweep second hand lines up with the readings of the transmitter immediately after the indicator is switched on.

These dials do not have any hour hands. The time in hours is determined by readings from a Type ACCH clock.

7. Navigational Sights

At the present time, navigational sights are used only for special purposes such as aerial photography. They are not used in passenger aircraft.

There are several types of navigational sights, which differ in their design. However, all are intended for measuring the course angles of landmarks (CAL) and their vertical angles (VA). /200

The *course angle of a landmark* is the angle between the longitudinal axis of the aircraft and the direction of the landmark. The *vertical angle* is the angle between the vertical at the point where the aircraft is located and the direction of the landmark.

The sight can be used to solve a great many navigational problems related to determination of the locus of the aircraft and the parameters of its motion.

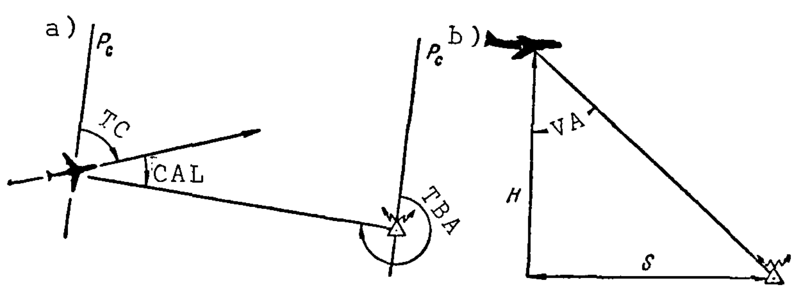


Fig. 2.36. Determining the Value (a) of Aircraft Bearing; (b) of the Distance from a Landmark to the Aircraft Vertical.

1. Determination of the locus of the aircraft in terms of the course and vertical angles of the landmark (Fig. 2.36). In this case, the true bearing from the landmark to the aircraft is (Fig. 2.36, a)

$$TBA = TC + CAL \pm 180°,$$

while the distance from the landmark to the vertical of the aircraft (Fig. 2.36, b) is

$$S = H \text{ tg VA},$$

where TBA is the true bearing from the landmark to the aircraft, TC is the true course of the aircraft, CAL is the course angle of the landmark, H is the flight altitude, and VA is the vertical angle of the landmark.

Obviously, if the aircraft course is determined by a magnetic compass, in order to solve this problem we must also add to the readings of the compass the corrections for the deviation of the compass of the magnetic declination of the locus of the aircraft.

$$\text{TC} = \text{CC} + \Delta_c + \Delta_M.$$

The correction for the deviation of the meridians between landmarks and the locus of the aircraft in this case is not taken into account, since the measurement of the vertical angles can be made satisfactorily up to 70-75°, i.e., at distances which do not exceed /201 three to four times the flight altitude.

In solving this problem, it is particularly important to know the true flight altitude above the level of the visible landmark, since errors in determining the distance will be proportional to the errors in measuring the flight altitude. Therefore, the readings of the altimeter must be subjected to corrections for the instrumental and methodological errors and the elevation of the landmark above sea level must also be taken into account if measurements are not being made in a level location.

2. Determination of the location of an aircraft in terms of the bearings from two landmarks (Fig. 2.37). In this case,

$$\text{IPS}_1 = \text{TC} + \text{CAL}_1 \pm 180°;$$

$$\text{IPS}_2 = \text{TC} + \text{CAL}_2 \pm 180°.$$

The position of the aircraft is determined by the intersection of bearings IPS_1 and IPS_2 on the map. If the direction finding is made over great distances, especially in the polar regions, the measurements of the bearings must include a correction for the displacement of the meridians.

An advantage of this method is its independence of flight altitude, and consequently, of the nature of the local relief.

However, this method requires careful measurement of the course angle of the second landmark, since the aircraft may move considerably away from the line of the first bearing during a prolonged measurement.

3. Determination of the drift angle of the aircraft according to visual points. To determine the drift angle by this means, the sight is set at a course angle of 180° and a zero vertical angle.

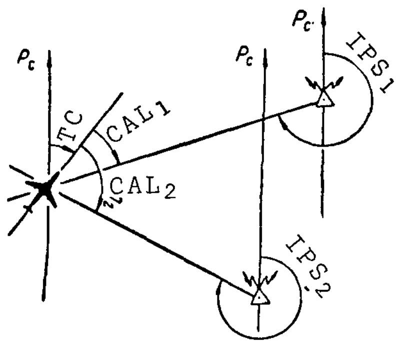


Fig. 2.37. Determining the Position Line of an Aircraft by Two of its Bearings.

With an exact maintenance of the course by the pilot, observing the directions of visual points and turning the sight to keep it parallel to the course chart, the sight is set in the direction in which the aircraft is moving. The drift angle of the aircraft is then calculated on a special scale.

This method is used for low flight altitudes, i.e., with rapidly changing visual landmarks.

4. Determination of the drift angle of an aircraft by using a backsight. The essence of this method lies in the meas- /202
urement of the course angle at which visual points recede from the aircraft. After setting the sight, as in measuring the drift angle in terms of the location of visual points (CAL = 180°, VA = 0), the pilot waits until the characteristic visual point appears in the cross hairs of the sight at the position of the bubble level in these cross hairs. Then, keeping the aircraft strictly on course, the pilot waits until the landmark leaves the cross hairs in the vertical plane at an angle of 40-50° at average altitudes or 15-20° at high altitudes. Then, by turning the sight, he matches the visual point with the course marking and calculates the drift angle.

5. Determination of the drift angle of an aircraft by sighting forward. In measuring the drift angle by sighting forward, the sight is set to the zero course angle and a visual point is selected on the course chart, which preferably lies at a vertical angle of 45 or 26.5°. In this case, with VA = 45°, the distance to the landmark will be equal to the altitude, while at VA = 26.5° it will equal half the altitude:

$$S = H \text{ tg VA}.$$

Then, keeping the aircraft strictly on course, the sight is set to zero on the scale of vertical angles and the pilot waits until the visual point crosses the transverse line on the cross hairs on the sight (traverse of the landmark). Noting the lateral deviation of the landmark in degrees, it is possible to determine the linear lateral deviation as follows:

$$S_\delta = H \text{ tg VA}.$$

The drift angle of the aircraft is determined as the ratio of its initial distance to its final distance:

$$\text{tg US} = \frac{S_2}{S_1} = \frac{\text{tgVA}_2}{\text{tgVA}_1}$$

At drift angles on the order of 10°, the tangent US can be replaced by its value, while the tangent VU_2 can be replaced by the value of the lateral deviation (LD):

$$\text{US} = \frac{\text{LD}}{\text{tgLD}_1}$$

or, with an initial value of VA_1= 45°, US = LD; with an initial VA_1=26.5°, US = 2 LD.

All three of these methods described above for determining the drift angle are used in locations which have many landmarks, i.e., where it is easy to pick out a visual landmark at the desired visual angle.

6. **Determination of the ground speed of the aircraft by means of a backsight.** To determine the ground speed by this method, the sight is set on the course angle scale to 180°, and to zero on the vertical angle scale. The bubble in the level is set at the intersection of the cross hairs.

Having selected the characteristic point as it passes through the intersection of the sight, the sweep second hand is started and the pilot waits until this point has moved to a vertical angle of 35-40°. /203

Then the course marking of the sight is made to coincide with the visual point and the sight is set to VA = 45°. The moment when the visual point again crosses the intersection of the cross hairs in the sight, the sweep second hand is stopped. In this case, the path traveled by the aircraft will be equal to the flight altitude. Consequently, the ground speed can be determined by the simple formula

$$W = \frac{H}{t}$$

where H is the flight altitude and t is the time measured by the sweep second hand.

7. **Determination of the drift angle and the ground speed of the aircraft from a landmark located to the side.** This method is used in the case when it is desired to measure the drift angle and the ground speed and the pilot has only one landmark at his disposal,

which is not located along the line of flight of the aircraft. Being careful to keep the aircraft strictly on course, he looks through the sight at the landmark and waits until its course angle is equal to 45 or 315°, depending on whether it is to the left or right of the flight path of the aircraft.

At a course angle for the landmark of 45 or 315°, the vertical angle of the landmark is measured and the sweep second hand is started.

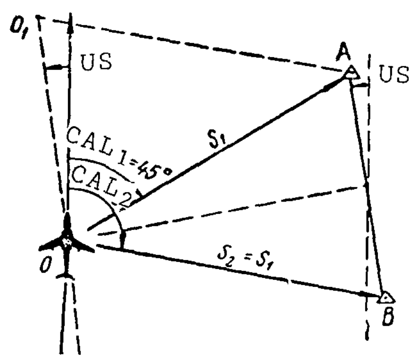


Fig. 2. 38. Determining the Drift Angle and Ground Speed by a Landmark Located to the Side.

Leaving the setting of the vertical angle in the same position, the sight is rotated to follow the motion of the landmark, noting its fixed position on the course chart. At the beginning (up to CAL = 90° + US), the distance to the landmark will decrease, but then will increase again. Consequently, the landmark will at first move away to one side from the intersection of the cross hairs and will then again begin to approach it. At the moment when the landmark is at the intersection of the cross hairs, the sweep second hand is stopped and the course angle of the landmark is calculated.

If CAL_1 = 45°, the bisectrix of the triangle OAB (Fig. 2.38) will be located at the course angle, which is equal to:

$$CAL_{bis} = \frac{45°+CAL_2}{2},$$

while the drift angle of the aircraft will be equal to CAL_{bis}-90°, so that /204

$$US = \frac{CAL_2-135°}{2}.$$

If CAL_1 = 315°,

$$CAL_{bis} = \frac{315° + CAL_2}{2}$$

$$US = CAL_{bis}-270°$$

or

$$US = \frac{CAL_2-225°}{2}.$$

At points 1 and 2 the distance from the aircraft to the landmark is equal to

$$S_1 = S_2 = H \quad VA.$$

Consequently, the distance between points 1 and 2 is determined by the formula

$$S_{1-2} = 2H \text{ tg VA } \sin\frac{\text{CAL}_1+\text{CAL}_2}{2}.$$

Clearly, the reason for the change in the course angle of the landmark from CAL_1 to CAL_2, was the shift of the aircraft from point O to point O_1, so that

$$S_{1-2} = OO_1.$$

Consequently, the ground speed is

$$W = \frac{S_{1-2}}{t}.$$

The majority of navigational problems which we have discussed, which are solved by means of mechanical or optical sights, can be solved using the radio devices which are installed nowadays aboard modern turboprop and jet aircraft, which will be described in the next chapter.

8. Automatic Navigation Instruments

In Section 2 of Chapter I, it was mentioned that in the general case, all the elements of a flight regime are not strictly fixed, with the exception of the extreme points of deviation from a given trajectory. Therefore, the crew of an aircraft must constantly deal with average values of measured navigational elements (average course, average speed, average wind, etc.).

If all the elements which have been mentioned had a constant given value, the practical problems of aircraft navigation could be solved quite simply and the question of automating the processes of aircraft navigation would be superfluous. /205

Without using automatic navigation devices, the pilot of an aircraft must systematically carry out observations using all the navigational instruments, average them for time intervals covering the observation time, and record the time of change in average values of the measured parameters. This introduces considerable tedious work into the task of aircraft navigation. However, if the pilot did not devote sufficient attention to changes in the elements of aircraft navigation, it would soon have an effect on the precision of aircraft navigation.

The simplest device used for automating the computation of the aircraft path in terms of the changing values of navigational parameters and times is the automatic navigational device, which has been devised on the basis of the general features of aircraft navigation.

At the present time, the navigation indicator Type NI-50B, is widely used. We shall now discuss its design and the method of its application.

The NI-50B navigation indicator is an automatic navigation device which calculates the path of the aircraft on the basis of signals from sensors for the course and airspeed, taking into account the measured wind speed during flight. In addition, the indicator can be used to determine the wind parameters at the flight altitude.

Calculation of the path of the aircraft with the use of the NI-50B can be performed both on the basis of orthodromic systems of coordinates for straight-line flight segments, as well as in a rectangular system of coordinates with any orientation of its axes.

Without going into the details of the design of the instrument, let us examine its schematic diagram, purpose, and operating principles of the individual parts, as well as the ways in which the system as a whole can be employed.

The navigation indicator consists of the following parts: automatic speed indicator, control unit, automatic course-setting device, wind indicator, and device for calculating the aircraft coordinates (Fig. 2.39).

The automatic speed control consists of a device which converts the pressure from the sensors of total and static pressure into electrical signals, corresponding in value to the airspeed of the aircraft, according to Formula (2.47a)

$$\frac{P_{total}-P_{st}}{P_{st}}=\left(1+\frac{V^2}{2060\,T}\right)^{3,5}-1.$$

The automatic speed control has two horizontal manometric boxes. One of them (aneroid 1) is used to measure the static pressure, while the other is used to measure the aerodynamic pressure 2 as the difference between p_{total} and p_{st}. /206

Both boxes are connected by means of linking mechanisms to potentiometers 3, which regulate the current ratio in the balancing circuit, according to the ratio of the dynamic pressure to the static pressure.

It is clear from Formula (2.47, a) that the ratio of the dynamic pressure to the static pressure is not linearly related to the airspeed of the aircraft. In order to develop electrical signals which are proportional to the airspeed, the control unit contains an automatic speed control mechanism. This mechanism consists of a magnetic signal amplifier 4, coming from the automatic speed control, activating motor 5, and a potentiometer 6 with a special profile, which levels out the nonlinearity of the signals from the automatic airspeed control. Thus, the turn angle of the axis of the potentiometer of the analyzing mechanism becomes proportional to the airspeed.

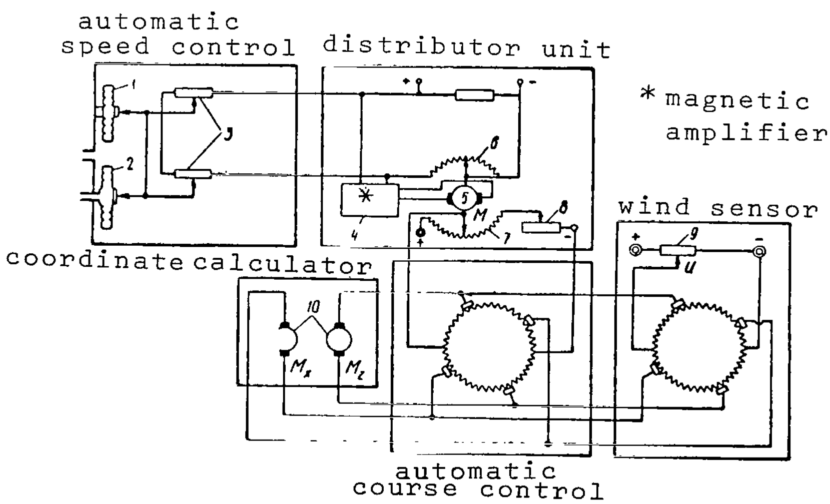


Fig. 2.39. Schematic Diagram of Navigational Indicator.

By means of a second potentiometer, connected by its axis of rotation to the activating mechanism, sends out electrical signals which are proportional to the airspeed, in the form of a DC voltage.

The *automatic course control* is intended to distribute the signals which are proportional to the airspeed, along the axes of the coordinates for calculating the path.

Let us assume that we must make a flight over a path segment with the orthodromic flight angle ψ (Fig. 2.40).

If the aircraft is now to fly with an orthodromic course γ, the airspeed must be divided into two components: /207

$$V_x = V\cos(\gamma - \psi);$$
$$V_z = V\sin(\gamma - \psi).$$

It is clear that if there is no wind at the flight altitude, these components of the airspeed must be multiplied by the flight time to give us the change in the aircraft coordinates during this time:

$$\Delta X = V_x\Delta t;\ \Delta Z = V_z\Delta t.$$

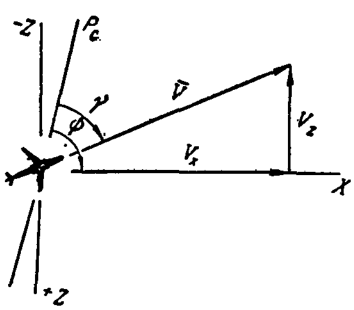


Fig. 2.40.

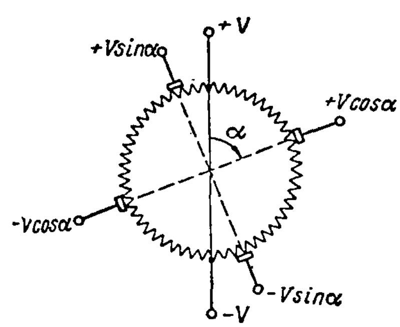


Fig. 2.41.

Fig. 2.40. Distribution of the Airspeed Vector along the Coordinate Axes.

Fig. 2.41. Sine-Cosine Distributor.

The division of the course signals by the axes of the coordinates in the automatic course control is accomplished by means of a sine-cosine potentiometer (Fig. 2.41).

The sine-cosine potentiometer consists of a circular winding with power supplied to it at two diametrically opposite points.

Two pairs of pickups slide along the coils; they are located at right angles to one another.

Obviously, if we say that the zero position of the pickups is the one in which one pair (cosine) coincides with the supply leads and the second (sine) will be located at an angle of 90° to them, then the maximum current will flow through the first pair of pickups while that through the second pair will be zero. By turning the pickups from zero to 90°, the current in the cosine pickups will drop from maximum to zero and that in the sine pickups will increase from zero to the maximum. However, the change in the current in the pickups will not take place according to the sine and cosine laws, but proportionately to the angle of rotation of the pickups.

In order for the law of change of currents to approach the sine-cosine, the winding of the potentiometer is given a profile or is /208
fitted with special regulating shunt resistors.

Rotation of the pick-up shoes of the potentiometer is involved in figuring the course of the aircraft which is arriving from a course system or other course instrument.

In order to apply the components of the airspeed to the receiving system for calculating the aircraft coordinates, the circular winding of the potentiometer is made movable and can be mounted in any position by means of a rack and pinion, located in the automatic course control, and a special scale for calculating its position.

The angle for studying the system of coordinates for calculating the path relative to the meridian from which the aircraft course is measured is called the *chart angle*. In the majority of cases, the chart angle is made equal to the orthodromic path angle of the path segment.

Hence, by applying to the winding of the sine-cosine potentiometer a voltage which is proportional to the airspeed, we obtain signals at the outputs of the potentiometer which are proportional to the component of the airspeed along the axes of the coordinates V_x and V_z.

For a precise regulation of the navigational indicator as a whole, these signals are calibrated manually by means of a potentiometer (see Fig. 2.39, Position 8), located in the control unit.

The *wind sensor* has a schematic similar to that found in the automatic course control, with the exception that the voltage which is proportional to the windspeed is analyzed directly at the sensor by means of a potentiometer (see Fig. 2.39, Position 9) and is set by manually turning knob "u" so that the setting of the pick-up shoes on the sine-cosine potentiometer agrees with the wind direction.

Thus, we have three set parameters on the wind sensor: the wind speed (u), the wind direction (δ), and the chart angle (ψ).

It is clear that the difference between angles δ and ψ gives the path angle of the wind. As a result, we obtain signals at the output of the sine-cosine potentiometer which are proportional to the component of the wind speed along the axes of the coordinates for calculating the path.

The outputs of the sine-cosine potentiometers of the automatic course control and the wind sensor are connected in series, so that we obtain signals at their common outputs which are proportional as folllows

$$V_x + u_x = V\cos(\gamma - \psi) + u\cos \text{AW}$$
$$V_z + u_z = V\sin(\gamma - \psi) + u\sin \text{AW}$$

i.e., signals which make it possible to calculate the path of the aircraft with time, considering the manual setting of the wind value for the flight altitude.

The *coordinate calculator* consists of two integrating motors /209 that work on direct current (see Fig. 2.39, Position 10), whose speed of rotation strictly corresponds to the magnitude of the signals coming from the automatic course control and the wind sensor. The revolutions of the motors are summed by two counters, whose readings are shown on a scale which is graduated in kilometers of path covered by the aircraft along the corresponding axes.

A pointer marked "N" shows the path of the aircraft along the *X*-axis, i.e., along the orthodrome, while a pointer marked "E" shows the travel along the *Z*-axis, or the lateral deviation from the desired line of flight.

The names of the pointers ("N" and "E") were given because at a chart angle equal to zero, the pointer "N" will show the path traveled by the aircraft in a northerly direction from the starting point while the pointer "E" shows travel in an easterly direction.

To set the pointers of the counter to zero (at the starting point of a route) or to the actual coordinates of the aircraft when correcting its coordinates, there is a special rack and pinion which is used to turn the "N" pointer when it is pushed inward and to turn the "E" pointer when it is pulled out.

9. Practical Methods of Aircraft Navigation Using Geotechnical Devices

Flight experience shows that in addition to a knowledge of the devices for determining each of the elements of aircraft navigation, successful completion of a flight, means that it is necessary to obtain and use the measured values, i.e., to master the devices used for aircraft navigation prior to automation.

These devices do not depend on systems of measuring flight angles and aircraft courses, since they have limited fields of application. In addition, in describing them, it is necessary to recall that the readings of navigational devices contain all necessary corrections. Therefore, in the formulas which have been found to be necessary, we have used the common designations for navigational parameters.

Under practical conditions of aircraft navigation, an important role is played by the pilots' calculating and measuring instruments. However, in many cases, instead of using these instruments, approximate calculations are performed mentally. Approximate mental extimates can be used to advantage in all cases when the problem can be solved more precisely by means of calculating instruments in order to avoid any chance gross errors.

Methods of approximate (yet sufficiently accurate for practical purposes) estimation of navigational elements in flight without the use of calculating and measuring instruments are called *pilots' visual estimates*. The rules for pilots' visual estimates will be given later on in the description of the suitable methods of aircraft navigation.

Takeoff of the Aircraft at the Starting Point of the Route /210

The *starting point of the route* (SPR) is the first control landmark along the flight path from which the aircraft will travel along the route at a given path angle ψ.

The *final point on the route* (FPR) is the last control landmark along the route, from which the maneuver to land the aircraft begins.

Regardless of the fact that the path angle of the flight is usually reckoned from the airport from which the aircraft took off up to the SPR, as well as from the FPR to the airport where it is to land, these values have significance only for general orientation in the vicinity of the airports.

In connection with the fact that the first turn of the aircraft after takeoff is made after the aircraft reaches a certain altitude (200 m, e.g.) and that many factors influence takeoff conditions (such as atmospheric pressure, wind speed and direction, flying weight of the aircraft, etc.), an exact determination of the location of the beginning and end of a turn is usually difficult. Therefore, the path angle and the distance from the first turn to the SPR has a variable nature and cannot be determined exactly.

Methods of bringing the aircraft to the initial point on the route differ somewhat from the general methods of aircraft navigation along the flight route.

The basic difference between the methods of aircraft navigation involved in bringing an aircraft to the SPR, and the aircraft navigation along the route, is that in the first case we do not have a strictly determined path angle for the flight and can reach the given point from any direction, i.e., in the given case the navigation is made in a polar system of coordinates. In the second case, we have a given line of flight, and the aircraft navigation takes place along a straight-line orthodromic system of coordinates.

In Figure 2.42, a, we see that the flight path angle from the center of an airport in the direction along the SPR and the shortest line for the aircraft's path to the SPR after takeoff and gaining altitude until the first turn are at right angles.

Since the line of flight is not constant when the aircraft reaches the SPR, the problem involves bringing the aircraft to a given point with the minimum number of changes in the course, or (in other words) along the shortest path.

Practically speaking, visual control of an aircraft to bring it to the SPR is done as follows.

If the course leading from the airport to the SPR differs from the takeoff course by less than 90°, after takeoff and gaining the desired altitude, the aircraft makes a right angle turn to the starting point of the route, so that when the wind in the vicinity of the airport causes drifting of the aircraft to the right, the landmark which is designated as the SPR must remain to the right of the aircraft course by 5-10°, depending on the direction and speed of the wind. In the case of a left-hand drift, the SPR must remain /211
at the same angle to the left of the aircraft course.

With the proper selection of the course to the SPR, i.e., when the lead angle (LA) is equal in value to the drift angle of the aircraft, the landmark will be observed at a constant angle to the axis of the aircraft, CAL = const (Fig. 2.42, b).

In this case, it is necessary to continue the flight along the previous course until the SPR is passed or (in high-speed aircraft) until there is a linear lead on the turn.

If the drift angle turns out to be less than the lead which has been taken (Fig. 2.42, c), a slipping of the landmark will be observed from the direction of the longitudinal axis of the aircraft. In this case, the aircraft must be shifted in the direction of the landmark so that its course angle turns out to be less than the initial one.

The slipping of the landmark in the direction of the longitudinal axis of the aircraft (Fig. 2.43, d) indicates that the lead which has been taken is less than the drift angle, and the aircraft must be turned away from the landmark so that its course angle is greater than the initial one.

Thus, the course to be followed by the aircraft is set visually when the SPR is located along a straight line. This problem is

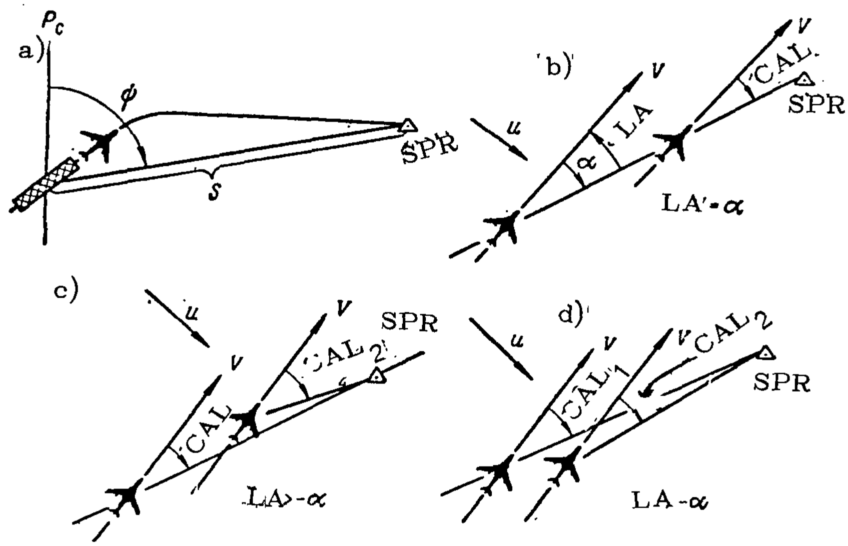


Fig. 2.42. Lining Up an Aircraft with the SPR: (a) Path Angle (ψ) and Shortest Distance (S); (b) Aircraft Course Chosen Correctly; (c) Aircraft Course must be Increased; (d) Aircraft Course Must be Decreased.

best solved when there is a navigation level on board, by using the so-called method of half corrections. This method involves the following: if the lead which has been taken turns out to be greater or less than the required one, it then changes in the required direction by half of the initial lead which was taken. If this turns out to be insufficient, it is changed again by half of the initial value until the course angle becomes stable or the sign of the correction must be changed to the opposite. /212

Reverse correction is made by one-fourth of the initial lead, and if this is insufficient or too much, a correction is made which is equal to one-eighth of the initial lead. It is not usually necessary to break down the corrections more than eight times, since the value of the correction will then be no more than 1-1.5°, which is no longer of practical importance for visual aircraft navigation.

In the absence of a sight aboard the aircraft, the course angles for the SPR are determined by visual observation; to solve this problem, the pilot requires a certain degree of experience which is gained in the course of the training of flight cruise in actual flight or in special training devices, as well as in practice flights.

Selecting the Course to be Followed for the Flight Route

The course to be followed by the aircraft along the flight route not only must be set so the aircraft passes over certain control landmarks in the proper order, but must also ensure that the flight takes place exactly according to the given line of flight.

There are three principal methods of selecting the course to be followed:

(a) When deviations occur from the line of a given path (LGP) during the flight,

(b) At a landmark along the line,

(c) In the direction of the landmark points,

The most universal and widely used method is the first one. This method involves the following: after flying over a certain control point, the calculated course to be followed along the given line of flight is determined as follows

$$\gamma = \psi - \alpha_{calc},$$

which the aircraft follows until the first characteristic point along the flight path.

If, at the moment that it is flying over this point, the aircraft turns out to be on the given line of flight, the course is then considered to be sufficiently correct.

If the aircraft has undergone some shift to the right when it passes over this point, the linear lateral deviation from the desired line of flight is determined and the required correction is found for the course of the aircraft:

$$\mathrm{tg}\,\Delta\gamma = \frac{LLD}{S_c},$$

where LLD is the linear lateral deviation and S_c is the distance covered. /213

Example: An aircraft has flown from a control landmark for a distance of 36 km and has deviated 3 km to the right of the desired path. Determine the required correction in the course (Fig. 2.43):

Solution.

$$\mathrm{tg}\,\Delta\gamma = \frac{3}{36} = \frac{1}{12};$$

$$\Delta\gamma = -5°.$$

To reach the desired line of flight, it is usually necessary first of all to make a double course correction (in our case, 10°), and then (when the aircraft has covered a distance equal to the base of the measurement, or is traveling along the line of the desired path) the lead in the course is reduced by a factor of two, leaving a correction in the course which is equal to the set angle of drift.

If the closest turning point in the route (CTR) is located at a distance which is smaller than the base of measurement, then in order to attain it, correction must be made in the course for the distance covered for the travel parallel to the line of the desired path and over the distance covered, in order to reach the desired path at the moment when the next control landmark is being passed.

Let us say that in our example the distance to the next landmark is still 30 km; the correction for the remaining distance will be equal to:

$$\operatorname{tg} \Delta\gamma_{rem} = \frac{3}{30} = \frac{1}{10}$$

$$\Delta\gamma = -6°.$$

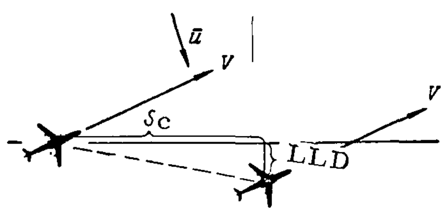


Fig. 2.43. Determination of Corrections in Course to be Followed.

Since the correction for the distance covered was equal to -5°, the total correction for the course in order to get the aircraft to the CTR must be equal to -11°.

The problem is solved similarly when the aircraft has wandered to the left of the desired path, but with the difference that the correction in the course to be followed is positive in this case.

In solving problems in determining the desired corrections in the course to be followed, we preferably use methods involving visual observation by the pilot without the use of any calculating instruments or tables. In the opposite case, while the pilot is solving the problems, the aircraft will cover a considerable distance, thus complicating the realization of the desired solutions. /214

The first method of pilot's visual estimation in this case will be the visual estimation of the lateral drift from the line of flight.

If an aircraft is traveling to the side of the above mentioned characteristic point, the distance from it by flight along the traverse is determined by the vertical angle. When the vertical angle of the point is close to 26.5°, the aircraft is located at a distance from the point which is equal to half the flight altitude; with a vertical altitude of 45°, the distance is equal to the flight altitude, while at an angle of 63.5° it is twice the flight altitude. These angles are usually determined by visual observation. Intermediate values of vertical angles and distances are determined by visual observation and interpolation. For example, if the vertical angle is roughly equal to 55°, then the distance to the point is approximately equal to 1.5 flight altitudes.

This method, with sufficient training, gives a very high accuracy for determining the location of the aircraft relative to a given point along the route, and consequently, with respect to the line of flight (on the order of 0.1 H) at vertical angles up to 65°. At very large angles (grater than 65°) from the vertical of the aircraft, the errors in distance will be greater and this method cannot be used.

The second method of visual estimation by the pilot which is used in solving this problem is the mental calculation of the required course corrections following linear lateral deviation (LLD).

For convenience in metal calculation, one radian is assumed to be 60° rather than 57.3, but this does not introduce any considerable errors (the maximum error in angles up to 20° does not exceed 1°).

This allows the required correction to be made in the course in terms of the approximate ratio of the lateral deviation to the distance covered:

LLD/S	$\Delta\gamma$, deg	LLD/S	$\Delta\gamma$, deg	LLD/S	$\Delta\gamma$, deg
1/60	1	1/12	5	1/6	10
1/40	1.5	1/10	6	1/5	12
1/30	2	1/8	7	1/4	15
1/20	3	1/7	8	1/3	20
1/15	4				

These ratios are easy to remember if we know that in order to obtain their required correction it is adequate to divide the number 60 into the distance covered, when the lateral deviation is taken per unit of measurement.

Obviously, if this method for course correction is employed and the aircraft does not reach the desired point along the line of flight, so that there is still some lateral deviation, the lateral deviation and the distance from the point at which the course was last changed can be used to correct the course. /215

Selection of the course to be followed according to a landmark along the route can be used in the case when the flight takes place along a straight-line portion of a railway or highway and means that the crew must change the course of the aircraft so that it follows this linear landmark. After changing the course by an additional turning of the aircraft, the crew returns to the desired course and travels in the desired direction once again.

The selection of the course to be followed on the basis of orientation landmarks is a variety of the latter method.

In this case, the course is selected so that the closer of two selected landmarks along the line of flight constantly (up to the moment that the aircraft flies over it) remains in a line with the further landmark. After passing by the closer landmark, the aircraft follows the desired course or choses the next landmark, located beyond the second one, and continues its flight along this line.

Change in Navigational Elements During Flight

The majority of navigational elements (course, altitude, speed) are determined in flight on the basis of indications of the corresponding instruments, with introduction of corrections for instrumental and methodological errors.

Automatic radio devices, based on the Doppler principle, make it possible to make measurements directly (during flight) of such elements as the drift angle and the ground speed.

Other methods of aircraft navigation do not permit direct measurement of the latter two elements, so that in order to determine them it is necessary to use various pilotage techniques.

In the absence of sights, the drift angle of the aircraft can be determined as follows.

Let us suppose that we are traveling along a given route and that a control landmark on this route has been passed. After 15-20 min of flying time, we select another landmark by which we test the correctness of the course which has been selected. If no lateral deviation of the aircraft occurs on this segment, it means that the aircraft course has been properly set, i.e., the drift angle is equal in value to the previous course, but has the opposite sign

$$\alpha = \psi - \gamma,$$

where α is the drift angle of the aircraft, γ is the aircraft course, and ψ is the given flight path angle.

It is not always possible, however, to correctly set the course to be followed.

If a lateral deviation of the aircraft from the line of the desired path arises in our flight segment, the course to be followed will be incorrect and the actual flight angle will be /216

$$\psi_{\Phi} = \psi_{з} + \text{arctg}\,\frac{\Delta Z}{S}$$

where ΔZ equals the deviation of the aircraft from the LGF, and S is the length of the segment over which the drift angle was measured.

The angle of deviation of the aircraft from the line of the desired flight path arctg $\Delta Z/S$ is considered to be negative if the aircraft deviates from it to the left, and positive if it deviates to the right. As in the method of selecting the course, this angle is determined by methods of visual estimation by the pilot.

In the case of improper selection of the course to be followed, the latter can be determined as the difference between the actual flight angle and the course being followed:

$$\alpha = \psi_{\Phi} - \gamma = \psi_{з} + \text{arctg}\,\frac{\Delta Z}{S} - \gamma.$$

It is much easier in flight to determine the ground speed of an aircraft: the same landmarks are used for this purpose as those used for determining the drift angle of the aircraft. To do this, it is sufficient to determine the times when the aircraft flies over the first and second landmarks, after which the ground speed is determined by the formula

$$W = \frac{S}{t},$$

where S is the distance between the landmarks and t is the flying time between the landmarks.

The division S/t is done as a rule on scales 1 and 2 of a navigational slide rule (Fig. 2.44), with the exception of those cases when the flying time is less than 60 min. For example, 6, 10, 12, 15, 20 and 30, or even 40 and 48 min are possible. In these cases, the groundspeed will be equal to $10S$, $6S$, $5S$, $4S$, $3S$, $2S$, $1.5S$ and $1.25S$, respectively, and is easily determined mentally by multiplying the distance between the landmarks by one of the numbers given above.

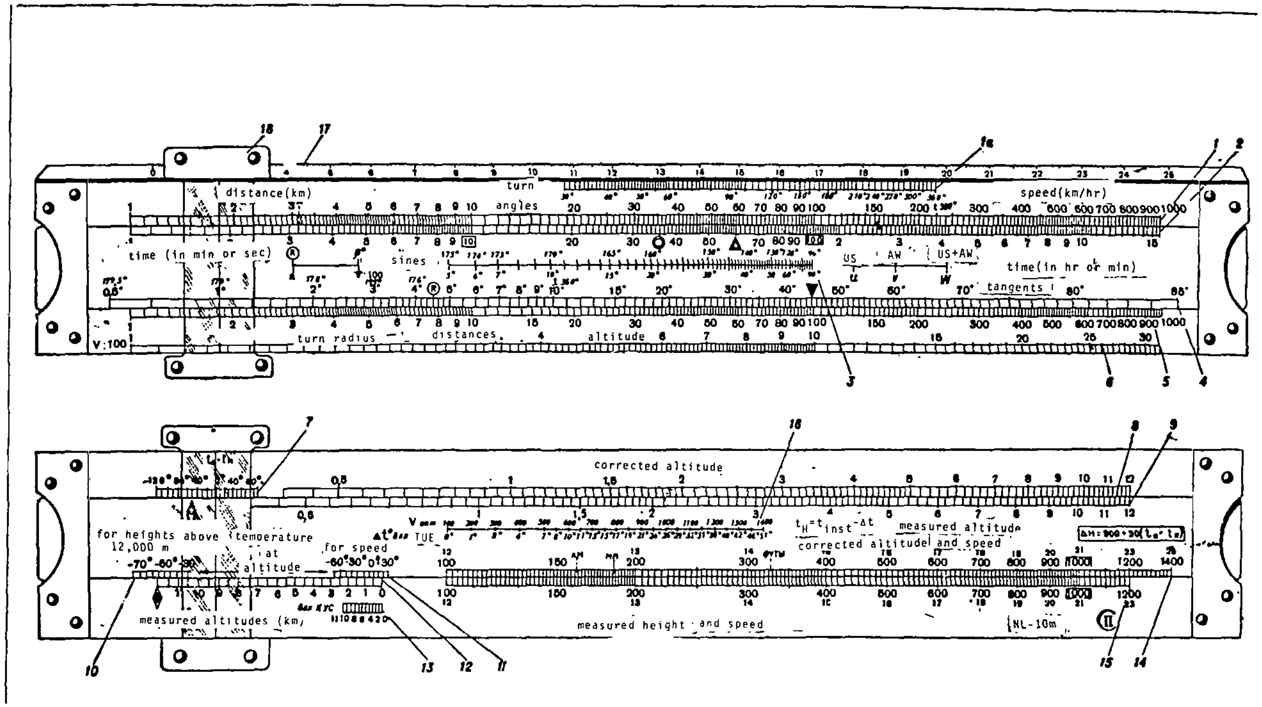


Fig. 2.44. Scales on Navigational Slide Rule NL-10M.

/217

To measure the ground speed as well as the drift angle, it is desirable to select distances between landmarks which are no less than 50-70 km apart. Over short distances, in order to avoid gross errors, it is necessary to determine and mark down very exactly the time that the aircraft passes over the control landmarks.

Measuring the Wind at Flight Altitude and Calculating Navigational Elements at Successive Stages /218

The principal factor which complicates the processes of aircraft navigation at flight altitude is the wind. With availability of exact data regarding its direction and speed, all problems of aircraft navigation can be solved by a combination of general methods of aircraft navigation independently of the visibility of terrestrial landmarks.

When the aircraft has on board only the most general devices for aircraft navigation, the problem of determining the wind at the flight altitude as well as the drift angle and the ground speed can be solved if terrestrial landmarks are visible.

The wind at flight altitude does not remain constant but is constantly changing with time and especially with distance. In order to be able to prepare the navigational data for the next stage of flight, it is necessary to determine the wind at the very end of the preceding stage and even in this case, the data on the wind which are obtained are obsolete to a certain degree and are not completely satisfactory for the needs of calculating.

Under the conditions when an aircraft is flying along an air route, there are three navigational parameters which basically determine the speed and direction of the wind at flight altitude: the airspeed (V), ground speed (W), and the drift angle for a given course.

The wind calculated on the basis of these parameters will not be reckoned from the meridian of the locus of the aircraft (LA) but from the line of flight of the aircraft.

The calculation of the path angle of the wind (AW) is carried out on the navigational slide rule by means of a key (Fig. 2.45, a).

Example: W = 360 km/hr; V = 320 km/hr; drift angle = +8°. Determine the wind angle.

Solution: (Fig. 2.45, b).

Answer: AW = 48°.

If we know the wind, it is easy to determine its speed by means of a key which is marked on the rule (Fig. 2.46, a). For our example, see Figure 2.46, b. Answer: u = 60 km/hr.

The direction of the wind relative to the meridian of the locus of the aircraft (LA) is determined by the formula

$$\delta = AW + \psi.$$

If the flight is made with magnetic flight angles, the wind direction is obtained relative to the magnetic meridian of the LA. This direction is also used to calculate the navigational elements in the next stage of the flight.

Information on the speed of the wind and its direction is transmitted from the aircraft to ground stations, also relative to the magnetic meridian of the LA, and is used for controlling the flight of the aircraft.

The angle of the wind for the next stage of the flight is /219

$$AW = \delta - \psi,$$

where δ is the wind direction and ψ is the flight path angle of the next stage of the flight.

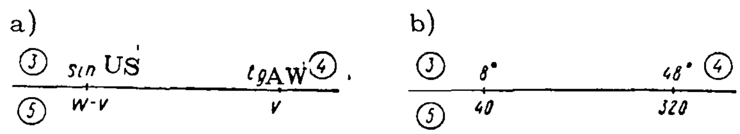


Fig. 2.45. Calculation of the Path Angle of the Wind on the Navigational Slide Rule: (a) Key for Determining the Wind Angle; (b) Determining the Wind Angle.

The values for the ground speed and drift angle of the aircraft for the next stage of the flight are calculated on the navigational slide rule by means of a key (Fig. 2.47, a).

Let us assume that the flight in the preceding stage was made with a MFA = 38°, in the next stage with an MFA = 56°, and with an airspeed of 320 km/hr. The data obtained on the wind at the preceding stage are AW = 48°, u = 60 km/hr.

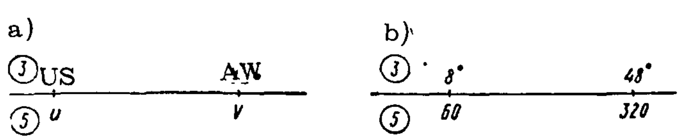


Fig. 2.46. Calculation of the Wind Speed on the Navigational Slide Rule: (a) Key for Determining the Speed; (b) Determination of the Speed.

The direction of the wind relative to the meridian of the LA is

$$\delta = 48 + 38 = 86°,$$

while the angle of the wind for the next stage of the flight is

$$AW = 86 - 56 = 30°.$$

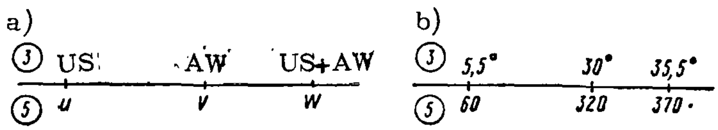


Fig. 2.47. Calculation of the Drift Angle and Ground Speed on the Navigational Alide Rule: (a) Key for Determining the Drift Angle and Ground Speed; (b) Determination of the Drift Angle and Ground Speed.

The value of the groundspeed and the drift angle for the next stage of the flight are also determined by means of the navigational slide rule (Fig. 2.47, b), i.e.,

$$W = 370 \text{ km/hr}; \quad US = +5.5°.$$

The values of the drift angle can be used to determine the calculated course to be followed in the next stage of the flight. In our case, /220

$$\gamma = 58 - 5.5 = 52.5°.$$

If the flight is made with orthodromic flight angles, then in order to calculate the navigational elements for the next stage of the flight it is unnecessary to convert the wind angle to its direction relative to the meridian of the LA. In this case, the wind angle for the next stage of the flight is determined as the difference between the wind angle of the preceding stage of the flight and the angle of turn in the route (Fig. 2.48):

$$AW_2 = AW_1 - TA.$$

In our example, $AW_1 = 48°$, $TA = 56-38 = 18°$, and $AW_2 = 48-18 = 30°$.

However, in order to transmit information regarding the wind to ground stations, it is necessary to determine the wind direction relative to the meridian of the LA.

Obviously, the true wind direction at the point LA is

$$\delta_{true} = AW + \alpha,$$

where α is the azimuth of the orthodrome at the point LA; the magnetic direction of the wind is

$$\delta_M = AW + \alpha - \Delta_M.$$

Consequently, if the calculation of the orthodromic path angles is made from the reference meridian, then

$$\delta_M = AW + (\lambda_{LA} - \lambda_{ref})\sin\phi_{av} - \Delta_M.$$

Example: $\lambda_{or}=70°$, $\lambda_{LA}=85°$, $\phi_{av}=52°$, $\Delta_M=-5°$, $\psi=38°$, $AW=48°$.

Solution: The true wind direction is

$$\delta_{true} = 48 + 38 + 15\cdot0.8 = 98°,$$

and the magnetic wind direction is

$$\delta_M = 48 + 38 + 15\cdot0.8 + 5 = 103°.$$

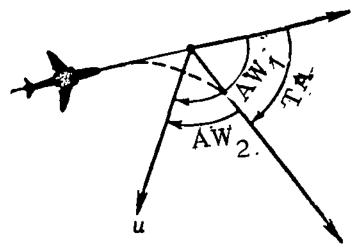


Fig. 2.48. Determination of the Wind Angle in a Successive Flight Stage.

Calculation of the Path of the Aircraft and Monitoring Aircraft Navigation in Terms of Distance and Direction

In the preceding paragraphs, we have discussed the methods of placing the aircraft on course, determining the navigational elements during flight, and calculating them for the following stages of the flight.

Under conditions of continuous visual orientation, theses methods completely ensure reliable aircraft navigation with respect to dis- /221
tance and direction, and no additional calculations are required.

However, very often it may not be possible to determine the location of the aircraft continuously relative to a given path. For example, in locations where there are no distinguishing features (steppe, desert, taiga, bodies of water, etc), visual orientation is only possible over individual sections of the route.

The conditions of meteorological visibility may not allow use of landmarks which have been selected along the route.

Therefore, it becomes necessary to use continuous calculation of the aircraft path in terms of time at certain periods, when it becomes necessary to check the aircraft path with respect to distance and direction.

Calculation of the aircraft path is always done with previously calculated parameters (the calculated course and ground speed, calculated time). At the same time, all the values and moments of change in the aircraft course are determined, which make it possible to determine the calculated position of the aircraft by plotting and thus to determine the additional errors in aircraft navigation.

Calculation of the path of the aircraft means that after the last identified landmark has been left behind, the crew aims the aircraft toward the next landmark during a certain period of time which is used to fix all the values of the actual course of the aircraft.

If the proper landmark has not been sighted when the scheduled time has elapsed, due to meteorological conditions, the calculated time for flying over this landmark is determined, and the aircraft is set to the next phase of calculated flight on the basis of the previous values for direction and velocity of the wind.

Thus, calculation of the path (flight on the basis of previously determined data) can continue until the conditions for visual orientation improve. However, it is necessary to keep in mind that the accuracy of aircraft navigation then decreases continuously due to the accummulation of errors with time, as well as in connection with the obsolescence of the data on the wind, measured prior to the last reliably sighted landmark.

When the conditions for visual orientation improve, the crew takes measures to check the path of the aircraft in terms of distance and direction.

To check the path in terms of distance, linear landmarks are usually employed, which intersect the route of the flight at an angle close to 90°.

Five to ten minutes before the calculated time for flying over these landmarks, depending on the flying time according to the previously calculated data and the speed of the aircraft, the pilot carefully begins to examine the landscape, looking for the landmark; at the moment that he flies over it, the approximate position of the aircraft is determined relative to distance and /222
time.

When flying over a control landmark, the pilot also tries to determine the lateral deviation of the aircraft from the desired path on the basis of additional features of the landmark (curves in rivers, tributaries, road junctions, populated areas, forest outlines, etc.).

Having determined the point of intersection of the landmark, the pilot projects it along the line of the desired path, fixing the position of the aircraft (in terms of distance at the moment that it flies over the landmark)and the direction.

At the present time, aircraft used for long distance flights when the ground is not visible are fitted with special radio navigational equipment. Light planes (which fly at low altitudes) are occasionally required to make long distance flights when the conditions for visual orientation are poor. Nevertheless, when such cases do occur, the flight can be made on the basis of a slight lead in the course and the direction of an acute angle of intersection with the route by a linear landmark (Fig. 2.49). In this case, the landmark must appear in the field of view somewhat earlier than the calculated time for flying past it. After this, the aircraft can be aimed at a control landmark which lines up with the linear landmark.

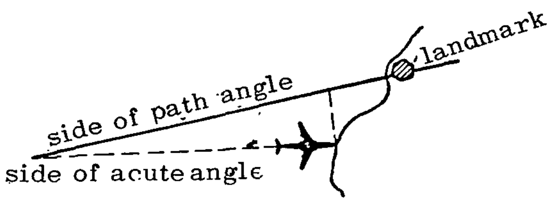


Fig. 2.49. Lead Toward the Acute Angle of the Traverse of a Landmark.

In individual cases, when the pilot does not recognize the terrain over which the aircraft must fly, after the conditions for visual observation have improved, the crew sets the aircraft on course to fly toward the next control landmark, and the pilot makes an estimate on the chart of the aircraft flight in terms of airspeed, fixed course, and flying time with these courses from the last recognized landmark.

The point obtained has a calculated wind vector during the flight time, after which the pilot compares the chart with the location in the following manner:

(a) In the region of the end of the wind vector (the most probable position of the aircraft);

(b) In the vicinity of a calm point;

(c) In terms of the wind vector direction from its beginning to end, with a continuation of the wind vector 1.5 to 2 times and turning it to the left and right at angles up to 90° from the calculated direction;

(d) Turning the wind vector (extended 1.5 to 2 times) in the remaining semicircle.

Naturally, these operations must be carried out with constant change of the calm point, depending on the direction of the movement of the aircraft. /223

If the location of the aircraft cannot be determined in this manner, other measures must be taken to find landmarks, such as the location of a characteristic linear or large-area landmark (lake, sea), and also by making inquiries from the ground, etc.

Use of Automatic Navigational Devices for Calculating the Aircraft Path and Measuring the Wind Parameters

To a considerable degree, automatic navigational devices simplify the work of the pilot in calculating the path of the aircraft and in measuring the wind parameters at flight altitude.

These devices are mounted on high-speed passenger aircraft which have complete radio navigational equipment, thus considerably increasing the effectiveness of their use.

Such devices, which are based on the general methods of aircraft navigation, can be used in straight-line systems of coordinates at any orientation of their axes.

The direction of the axes of the coordinates is selected by the pilot depending on the conditions for which the system is being used. For example, for flying along a route, it is most advantageous to combine the axis of the system *OX* with the directions of the straight-line segments of the flight, i.e., to calculate the path in an orthodromic system of coordinates in stages.

To carry out special operations in this region, e.g., at test sites for radio navigational systems for short-range operation, the axis *OX* is combined with the average meridian of the flight area (magnetic or true), depending on which system for calculating the flight angles is being used to make the flight.

In preparing to land and maneuvering in the vicinity of the airport, the axis *OX* coincides with the axis of the landing strip at the airport, etc.

In all cases when an automatic navigational device is being used, a rectangular system of coordinates should be applied to the flight chart in the given region, parallel to the axes of the system *OX* and *OZ*.

Parallel lines are drawn at 20 mm intervals, so that on charts with a scale of 1:1,000,000 this corresponds to 20 km, while on those with a scale of 1:2,000,000 it is 40 km, etc. For this purpose, a special stencil is included in the set of navigational instruments for the NI-50B indicator.

In using an automatic navigational device with orthodromic coordinates in stages, no additional devices are needed other than the general navigational divisions of the chart.

During flight, the apparatus is connected to a source of direct current, and the chart angle on the automatic course control is set in accordance with the selected system for calculating the aircraft to coordinates. The windspeed and direction are set on the wind sensor on the basis of the results of measurements during the preceding flight segment. /224

If the navigational indicator is used with an orthodromic system of coordinates in sections, the setting of the chart angle and wind is made at the end of the preceding stage of the flight before flying over a turning point in the route (TPR). On the coordinate calculator in this case, the pointer "N" is set to a value equal to the linear lead for the turn (LLT) and pointer "E" is set to zero.

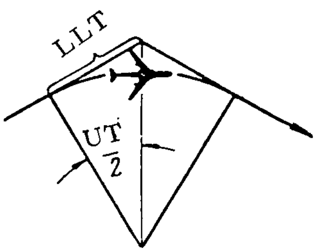


Fig. 2.50. Transition to an Orthodromic System of Coordinates in a Successive Flight Stage.

At the moment when the aircraft emerges from the turn on the new course (Fig. 2.50), the alternating current is connected to the instrument and the indicator begins to calculate the flight path.

At small turn angles in the line of flight (up to 30°), the turn trajectory of the aircraft is very close to TPR. In this case, the two pointers on the indicator should be set to zero, and the mechanism switched on when the TPR is passed as the aircraft is turning. At the beginning of the straight-line segment of flight, if possible, it is necessary to mark the established coordinates of the aircraft on the computer as the aircraft passes over a given landmark.

During flight along the straight-line segment, the pointers of indicators "N" and "E" will always show the distance covered by the aircraft from the last landmark along the line of the given path and the lateral deviation from the latter if it takes place, e.g., due to inaccurate maintenance of the course, improper studying of the data regarding the wind, or in case of dangerous meteorological phenomena.

Constant knowledge of the aircraft coordinates facilitates both visual and radial orientation. However, aircraft coordinates obtained on the basis of a computer will not always correspond precisely with the actual coordinates, since the speed and direction of the wind during flight change over the distance covered.

The navigational indicator also makes it easier to determine the wind parameters at flight altitude. This is done as follows:

At the end of a stage in the flight, the aircraft coordinates are recorded with the computer (Point B in Figure 2.51, a) and the actual location of the aircraft is determined visually or by means of radionavigational devices (Point B_1). These Points BB_1 determine the vector of the change in the wind at flight altitude for the flight time of a given stage of flight.

The problem of determining the wind vector in this case can be solved easily on a flight chart. To do this, a reverse line must be drawn from Point B and the length of the wind vector is set on the sensor ($u_p \cdot t$) during the flying time from Point A to Point B (Point O). Then the vector of OB will constitute the vector of the calculated wind (and OB_1, the actual wind) at flight altitude. /225

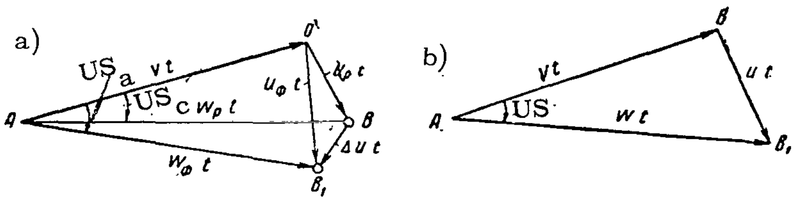


Fig. 2.51. Measurement of the Wind by Means of a Navigational Indicator: (a) Wind-Change Vector; (b) Wind Vector.

In order to obtain the value of the wind in km/hr, it is sufficient to divide the length of the vector OB_1 by the flying time between Points A and B, expressed in hours.

The problem of measuring the wind can be simplified if we consider that the wind at the sensor is zero for the flight stage, i.e., we introduce the value of AW = 0, u = 0 into the wind sensor. Then Point B will be the indication of the coordinates of the aircraft at the end of the flight stage, while Point B_1 will represent the actual coordinates (Fig. 2.51, b). Consequently, vector BB_1 will be the wind vector for the flying time in this stage.

The use of the navigational indicator in rectilinear coordinates for flights in a given region is not different in principle from using it in orthodromic coordinates and stages. However, the important advantage of the orthodromic system of coordinates is then lost, i.e., the relationship of the coordinates to the checking of the path for distance and direction. Therefore, the position of the aircraft in this case can be determined only in terms of the coordinates of the network superimposed on the chart.

The rectangular system of coordinates can be extended over a relatively small area (on the order of 400 x 400 km), since the effect of the sphericity of the Earth begins to show up in large areas.

In conjunction with this, in the case of flights by a coordinate system, it is not necessary to set a new chart angle for each change in the line of flight and to describe the coordinates of the aircraft in a new system for calculation, which to a considerable degree compensates for the loss of those advantages which we have in the orthodromic system of coordinates in stages.

Details of Aircraft Navigation Using Geotechnical Methods in Various Flight Conditions /226

The conditions for aircraft navigation using geotechnical devices are determined primarily by the presence and nature of landmarks, as well as by their contrast relative to the surrounding terrain.

The best landmarks for visual aircraft navigation are linear ones (large rivers, railways and highways, the shores of large bodies of water). Lakes, large and small populated areas, characteristic mountain peaks, etc., are also good landmarks, while grain elevators, water tanks, churches, industrial enterprises, etc., can be used for flights at low altitudes.

For aircraft navigation in an area which is poor in landmarks, we can use separate sighting points on the Earth's surface in the form of spots, individual trees, foam on the surface of the water, etc. Such points are not landmarks, since it is impossible to determine their location on a flight chart, but they can be used to measure the drift angle and the ground speed when there is a sight on board and also make it possible to increase the accuracy of aircraft navigation during flight between control landmarks.

The contrast of landmarks is determined by weather conditions, in which the visual flights must be made. With good contrast, e.g., in the presence of large populated areas, river valleys, located in forested terrain, aircraft navigation both summer and winter can be carried out with a horizontal visibility on the order of 1.5 km. At places where there is little vegetation, a visibility on the order of 10 km is required for aircraft navigation in winter.

The visibility of all landmarks, with the exception of illuminated populated areas, is considerably decreased at night, especially when the Moon is not out. Therefore, populated areas are the principal landmarks at night; their appearance at night can differ from their appearance in the day.

An important factor which determines the conditions for aircraft navigation is the stability of operation of magnetic compasses. Conditions of aircraft navigation without the use of gyroscopic compasses are unfavorable in the polar regions, as well as low altitudes in the vicinity of the magnetic anomalies.

The flight altitude also has a significant influence on the aircraft navigation conditions. In clear weather, optimum conditions for visual orientation exist at heights on the order of 1000-1500 m, since at this altitude the angular velocity at which the landmarks go by is small, all of their details can be seen clearly, and the field of view of the crew covers a very large area, which is important in comparing the charts with the landscape. /227

However, these altitudes can only be used when there is a small amount of clouds along the flight route. In cloudy weather, flights are made at lower altitudes, as low as the relief of the terrain will allow.

At low altitudes, the conditions for visual orientation are worse, since the angular velocity with which the landmarks go by increases and the area which the crew of the aircraft can scan is reduced.

An increase in the flight altitude (above 1.5 km) in clear weather have a small influence on the conditions of visual orientation, but at great heights the visual visibility of landmarks (depending on weather conditions) is much worse than at low and medium altitudes.

The selection of scales and chart projections for making a flight depend primarily on the altitude and speed of the flight. At low altitudes, it is best to use charts with a large scale of 1:500,000 or 1:1,000,000. At high altitudes and high speeds, it is best to use charts with scales of 1:2,000,000 and 1:4,000,000.

For flights along routes which are very long, charts are used which are made up of projections showing the properties of orthodromicity (the orthodrome on the chart has a shape close to a straight line), i.e., charts in the international or transverse cylindrical projection. For the polar regions, charts with tangent stereographic projection are used.

10. Calculating and Measuring Pilotage Instruments

Purpose of Calculating and Measuring Pilotage Instruments

Pilotage calculating and measuring instruments are intended for the following:

(a) Measuring distances and directions on flight charts.

(b) Calculating navigational elements both in preparing for flight and when completing it.

(c) Calculating methodological errors in the readings of navigational instruments (the readings of the airspeed, altimeter, and outside air thermometer).

(d) Calculating the elements of aircraft maneuvering.

Measurement of distances on flight charts is made by means of a special navigational slide rule. A feature which distinguishes this slide rule from conventional slide rules is the presence of several scales for measuring distances on charts with different scales. /228

Measurement of directions on flight charts is made by means of navigational protractors, made of transparent material.

The protractors are simultaneously used as triangles, which make it possible to make certain constructions on flight charts and diagrams (laying out the traverses of landmarks, parallel shift of lines, etc.).

Calculations of navigational elements, corrections to navigational devices, and elements of maneuvering are presently carried out with the aid of navigational logarithmic slide rules, the best modification of which is the navigational calculating slide rule NL-10M.

In addition, to calculate certain navigational elements, we can use special devices for setting up the speed triangle (wind-speed indicators). However, due to the improvements in navigational calculating slide rules, they have a very limited application.

Navigational Slide Rule NL-10M

The principle of the construction of navigational slide rules is the property of logarithms to make it possible to replace the processes of multiplication, division, raising to a power, extracting the square root, as well as multiplication and division of trigonometric functions, by operations involving the addition and subtraction of the logarithms of these values.

Thus, the operations described above involving numbers can be applied to the summing of the segments of a scale on the ruler, which simplifies calculation to a considerable degree.

The scales of the navigational slide rule NL-10M (see Fig. 2.44) are grouped so that one side is used for solving problems in determining navigational elements of flight as well as maneuvering elements, while the other side is used for calculating the corrections for the readings of navigational instruments.

In addition, the upper beveled edge of the ruler (Position 17) carries a scale divided into millimeters, which can be used to measure distances on the map.

The scales on the ruler 1 and 2 are intended to determine the ground speed from a known distance covered in a given time, or from a given distance at a known ground speed and time.

Therefore

$$W = \frac{S}{t} \quad \text{and} \quad S = Wt,$$

so that

$$\lg W = \lg S - \lg t \quad \text{and} \quad \lg S = \lg W + \lg t.$$

Scale 1 is the scale of logarithms of distances in kilometers /229 or flight speeds in km/hr; scale 2 is a scale of logarithms of flying time in min or sec up to the rectangular index marked 100 and beyond, in hrs or min.

The principle of solving problems by determining the airspeed over a given distance at a given time is as follows:

Let us say that an aircraft has covered a distance of 165 km in 12 min and that we must determine the ground speed in km/hr.

We set the marking on the slider to the 165 position on the distance scale; by moving the adjustable scale 2, we set division 12 on it opposite the marking on the slider. We can then read off the distance covered by the aircraft in minutes of flight opposite the number 1 at the beginning of the scale:

$$\lg W \text{ km/min} = \lg 165 - \lg 12 = \lg 13.8.$$

However, since 1 hr is 60 min, the speed in km/hr would be equal to

$$\lg W \text{ km/h} = \lg 13.8 + \lg 60 = \lg 825$$

or

$$W = 825 \text{ km/h}.$$

By combining the first and second effects, we obtain

$$\lg W = \lg 165 - \lg 12 + \lg 60,$$

i.e., in order to solve the problem, it is sufficient to set the number 12 on scale 2 opposite the number 165 on scale 1 and opposite number 60 on scale 2, which is marked with a triangular marking, and then to calculate the ground speed from scale 1 (Fig. 2.52, a).

The problem is solved analogously if the flight time is measured in sec. In our example, it will be 720 sec:

$$\lg W = \lg 165 - \lg 720 + \lg 3600 = \lg 825;$$

$$W = 825 \text{ km/h}.$$

The ground speed in this case is calculated from scale 1 opposite the number 3600 on scale 2 (the number of seconds in 1 hr), marked with a circular index.

To determine the distance at a given groundspeed and flying time ($S = Wt$), the logarithms of these numbers are added:

$$\lg S = \lg W + \lg t.$$

On the rule, the triangular or circular index on the movable scale 2 is set opposite the known ground speed on scale 1. The index marking on the slider is set opposite the given flying time on scale 2, after which the position of the indicator on scale 1 shows the distance covered in this time. /230

Example. W = 750 km/hr, t = 1 hr and 36 min. Find the distance covered.

Solution. See Figure 2.52, b.

$$\lg S = \lg 750 + \lg 1\text{ h }36\text{ min} = 1200;$$

Answer. S = 1200 km.

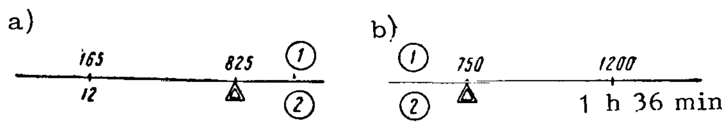


Fig. 2.52. Calculation on the NL-10M: (a) of the Ground Speed; (b) of the Distance Covered on the Basis of Ground Speed and Time.

Let us apply the keys to NL-10 for solving problems in determining the ground speed and distance covered on scales 1 and 2:

(a) To determine the ground speed for a distance covered in a known time (fig. 2.53, a or 2.54, a).

(b) To determine the distance covered from the ground speed and time (Fig. 2.53, b or 2.54, b).

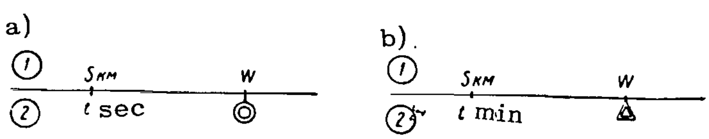


Fig. 2.53. Keys for Determining the Ground Speed on the NL-10M, on the Basis of the Distance Covered and the Time.

Movable scale 3 with the signs of the logarithms, which is the same (up to 5°) as scale 4 for the logarithms of the tangents and is also divided into scales 3 and 4, along with the fixed scale of distances or altitudes 5, which essentially repeats scale 1, are all intended for working with trigonometric functions.

The majority of problems which are solved on these scales are based on the properties of a right triangle, so that the value of the sine of 90° and tangent 45° (scales 3 and 4), whose logarithms are equal to zero, are marked on the rule by a triangular index.

If the problem is solved from a known leg, e.g., determining the error in the course on the basis of the distance covered and the linear lateral deviation (Fig. 2.55), we use scales 4 and 5 on the rule.

$$\mathrm{tg}\,\Delta\gamma = \frac{Z}{X} \text{ or } \lg \mathrm{tg}\,\Delta\gamma = \lg Z - \lg X.$$

The key to solving this problem is shown in Figure 2.56.

In the case when the hypotenuse of the triangle is known, the problems are solved by using scales 3 and 5. For example, suppose we wish to determine the location of the aircraft in ortho- /231
dromic coordinates (X_a, Z_a) on the basis of known coordinates of a landmark (X_1, Z_1), the distance and direction of which have been determined by means of a radar located on board the aircraft (Fig. 2.57).

It is clear from the figure that the orthodromic coordinates of the aircraft will be equal to

$$X_a = X_1 - R \cos \theta;$$

$$Z_a = Z_1 - R \sin \theta,$$

where R is the distance to the landmark and θ is the path bearing of the landmark (the angle between the given line of flight and the direction of the landmark).

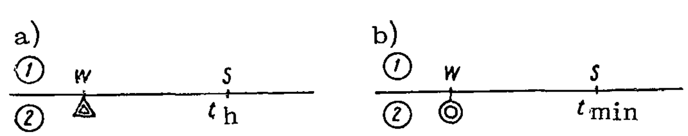


Fig. 2.54. Keys for Determining the Distance Covered on the Basis of the Ground Speed and Time, Using the NL-10M.

The difference in the coordinates of the landmark in aircraft are represented by X and Z, respectively, and are found on the logarithmic rule (Fig. 2.58, a, b).

In aircraft navigation, a number of problems are solved which are connected with the distances and directions (e.g.), the checking of a course in terms of the distance covered, determination of the position of the aircraft by using methods of visual and radar measurements, and many others. The essence of the solution of these problems is obvious from the examples given.

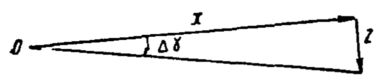


Fig. 2.55.

Fig. 2.56.

Fig. 2.55. Determination of the Course Error from the Change in the Lateral Coordinate.

Fig. 2.56. Key on the NL-10M for Determining the Aircraft Course Error.

For cases when the angles measured are greater than right angles, the sine scale 3 is numbered backwards, so that sin 180-α = sin α, for example:

$$\lg \sin 135^\circ = \lg \sin 45^\circ.$$

Scales 3, 4 and 5 can be used to solve special problems of oblique-angled triangles, e.g., the solving of speed triangles. The key for solving this kind of problem is given on the right- /232
hand side of the scale 3.

The theorem of signs, well known from trigonometry, determines the relationship between the angles and lengths of the sides of oblique-angled triangles. In the case where the speed triangle is used (Fig. 2.59), this theorem has the form:

$$\frac{\sin US}{u} = \frac{\sin AW}{V} = \frac{\sin(AW+US)}{W}. \quad (2.58)$$

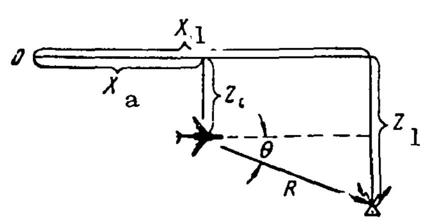


Fig. 2.57. Determination of the Orthodromic Coordinates of the Aircraft.

It is obvious that the relationship of Equation (2.58) is equivalent to the following:

$$\lg\sin US - \lg u = \lg\sin AW - \lg V =$$
$$\lg\sin(AW+US) - \lg W,$$

which is expressed by the key on the navigational rule (see Fig. 2.60, a).

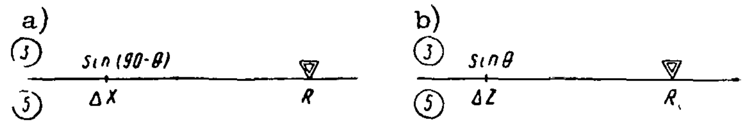


Fig. 2.58. Keys for Determining the Aircraft Coordinates on the NL-10M; (a) *X*-Coordinates; (b) *Z*-Coordinates.

Example. $MFA_g = 35°$, $V_{true} = 400$ km/hr, $\delta = 85°$, $u = 60$ km/hr. Find the drift angle of the aircraft and the groundspeed.

Solution. In our example, the wind angle is

$$AW = \quad - MFA_g = 85 - 35 = 50°.$$

Having set the slider indicator to the division representing 400 km/hr on scale 5, and also having lined up the 50° division on the logarithm sine scale 3 with the same slider indicator, we obtain the drift angle equal to 6.5°, and a ground speed of 440 km/hr (Fig. 2.60, b).

This key for solving speed triangles is suitable for determining speed and drift angle of an aircraft at known wind parameters. However, it is not suitable for determining wind parameters in measuring the drift angle and the ground speed.

This problem can be solved as follows.

Let us say that on the basis of measurements, we know the airspeed of an aircraft, the ground speed and the drift angle, /233
and we want to find the speed and direction of the wind (u) at flight altitude (see Fig. 2.59).

It is clear from the diagram that the running component of the wind at flight altitude is

$$u_x = W - V \cos US, \qquad (2.59)$$

while the lateral component is

$$u_z = V \sin US = (W - V \cos US) \operatorname{tg} AW. \qquad (2.60)$$

If we consider that the drift angle of the aircraft rarely exceeds 15°, and the cosine of the angle of drift is practically always close to unity, Formula (2.60) can be written as follows:

$$\operatorname{tg} AW = \frac{V \sin US}{W - V}.$$

However, since

$$V \sin \text{US} = (W-V) \text{ tg AW},$$

then we have the ratio

$$\frac{\sin \text{US}}{W - V} = \frac{\text{tg AW}}{V} .$$

which can be used as a key on the slide rule (Fig. 2.61, a).

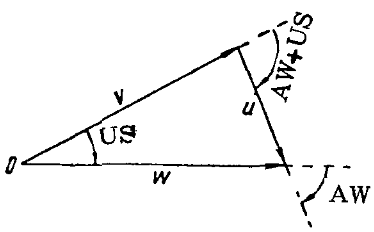


Fig. 2.59. Navigational Speed Triangle.

In practice, within the limits of the drift angles which are encountered in flight, the scales 3 and 4 on the rule coincide (the sine equals the tangent), and so that we will not have to deal with three but with two scales, we often use a key which is shown in Figure 2.61, b.

Example. V = 450 km/hr; W = 520 km/hr; US = +10°. Find the wind angle.

Solution. The difference between the ground speed and airspeed ($W-V$) is equal to 70 km/hr.

If we set this value on scale 5 opposite 10° on scale 4 (Fig. 2.61, c), we will find the wind angle to be equal to 48°. The wind speed is found with the aid of a key which is described in the sine theorem (Fig. 2.61, d).

Answer. u = 105 km/hr.

The fixed scale on the ruler 6, like scale 5, is a scale of logarithms of linear values, but the scale is twice that used on the first five scales. Therefore, when comparing any of the first five scales to the fixed scale, a number is obtained on the latter whose logarithm is equal to half the logarithm of the numbers of the first five scales.

Example. In setting the marker of the slider to the number 400 on scale 5 or 1, this marker shows half the log of 400 on the sixth scale, which corresponds to the square root of 400 or 20.

If the desired number is set on scale 6, we will obtain numbers on scales 5 and 1 whose logarithms are equal to twice the logarithm of the given number, thus corresponding to that number raised to a power of two.

The turn radius of the aircraft with a given banking angle (β), as we know, is determined from Formula (1.6). /234

$$R = \frac{V}{g \text{ tg}\beta} .$$

Therefore, the problem of determining the turn radius is solved by means of scales 4, 5 and 6:

$$\lg R = 2\lg V - \lg g - \lg \operatorname{tg}\beta.$$

Therefore, in solving this problem, it is necessary to have the logarithm of the square of the speed and to set it on scale 6. The logarithm of the tangent of the banking angle is calculated with the aid of scale 4.

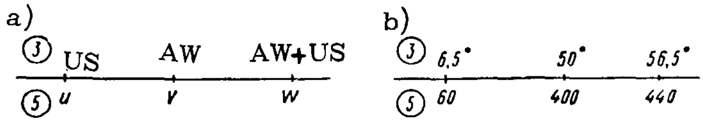


Fig. 2.60. Calculation on the NL-10M: (a) Key for Solving the Navigational Speed Triangle; (b) Solution of Navigational Speed Triangle.

If we consider that in order to determine the turning radius, the airspeed of the aircraft must be expressed in m/sec and not in km/hr, as we did on scale 6, and also that it is necessary to take into account the acceleration due to gravity g, we have a marking R on scale 4 which corresponds to the logarithm of the number

$$\frac{1}{3{,}6^2 \cdot 9{,}81} = 0{,}00787,$$

i.e., Formula (1.6) assumes the form:

$$R = \frac{0{,}00787\, V^2}{\operatorname{tg}\beta}$$

or

$$\lg R = 2\lg V + \lg 0{,}00787 - \lg \operatorname{tg}\beta,$$

which corresponds to the key for the navigational slide rule which was shown in Figure 2.62, and which is found at the beginning of the third scale of sines.

The last scale on the slide rule NL-10M is the scale 1a, which is intended to determine the turning time (t_p) of the aircraft at a given angle (UT) at a known turning radius (R) and flight speed (V). This scale is a scale of logarithms for the arc of the circumference, relative to the radius of turn of the aircraft.

Obviously, the turning time of the aircraft at the given angle /235
will be

$$t_p = \frac{2\pi R}{V} \cdot \frac{UT}{360} \tag{2.61}$$

In this formula, the value $2\pi/360$ is a constant multiplier. In order not to have to calculate it each time, scale 1a is set to the value of the logarithm of this multiplier at the left-hand side.

After dropping this multiplier, Formula (2.61) assumes the form:

$$t_p = \frac{R\,\mathrm{UT}}{V}$$

or

$$\lg t_p = \lg R + \lg \mathrm{UT} - \lg V,$$

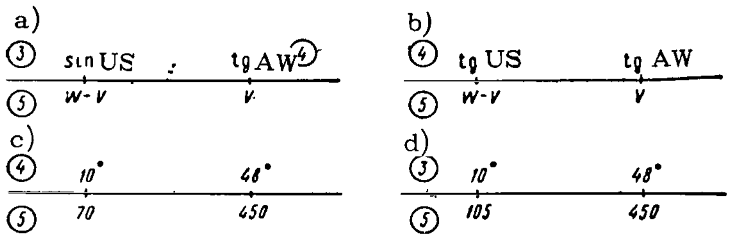


Fig. 2.61. Calculation on the NL-10M: (a,b): Keys for Determining the Wind Angle; (c,d): Determining the Angle and Speed of the Wind.

which can be expressed on the rule scales by a key shown in Fig. 2.62, b.

Example. R = 4.5 km, V = 400 km/hr, UT = 90°. Find the turning time of the aircraft.

Solution. See Fig. 2.62, c. Answer. t_p = 64 sec.

On the back of the rule are scales for making methodological corrections in the readings of navigational instruments (altimeters, airspeed indicators, outside-air thermometers).

Adjustable scale 7, with a movable diamond-shaped index and the adjacent scales (fixed scale 8 and movable scale 9) are intended for making corrections in the readings of barometric altimeters in case the actual mean air temperature does not agree with the calculated temperature obatined when adjusting the apparatus. These corrections can be made with the formula

$$\lg H_{\mathrm{corr}} = \lg \frac{T_0 + T_H}{2} + \lg \frac{H_{\mathrm{inst}}}{T_{\mathrm{av.c.}}}.$$

According to this formula, the adjustable scale 7 is a scale of logarithms $T_0+T_H/2$. For convenience in use, the logarithms $T_0+T_H/2$ on the rule are marked t_0+t. The arithmetic effects of converting temperatures from the centigrade scale to the absolute scale and their division in half are taken into consideration in the design of the scales in such a way that it is not necessary to make them each time during the flight. /236

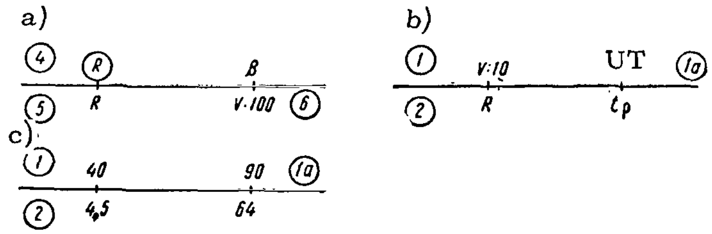


Fig. 2.62. Calculation on NL-10M: (a) Key for Determining Turn Radius. (b) Key for Determining Turn Time. (c) Determination of Turn Time.

Fixed scale 8 (corrected altitude) is simply a scale of logarithms of altitude, while the movable scale 9 (instrumental altitude) is a scale of logarithms of altitude, divided by the average calculated temperature obtained for each altitude, i.e.,

$$\lg\frac{H_{inst}}{T_{av.c.}} = \lg H_{inst} - \lg T_{av.c.}\ .$$

The key for solving problems by introducing methodological corrections to the readings of the altimeter are shown in Figure 2.63, a.

Example. The flight altitude according to the instrument is H_{inst} = 6000 m; t_H = -35°. Find the flight altitude corrected for the methodological error.

Solution. The actual temperature for a zero altitutde is determined from the temperature gradient equal to 6.5 deg/km:

$$t_0 = t_H + 6.5_{km} = -35 + 6.5 \cdot 6 = +4°,$$

so that t_0+t_H = -31°.

If we set this temperature value on the slide rule (Fig. 2.63, b), we will obtain H_{corr} = 5.74 km.

To introduce corrections in the readings of the altimeter at flight altitudes above 12 km, we use movable scale 10 with the adjacent fixed diamond-shaped index, as well as the adjacent scales: fixed scale 14 for the corrected altitude and speed, and fixed scale 15 for the instrumental altitude and speed.

Corrections to the readings of the altimeter at flight altitudes above 12 km are made by Formula (2.36).

Expressing the altitude in km, this formula can be written as follows: /237

$$\lg(H_{corr}-11) = \lg T_{H_{av}} - \lg 216.5 + \lg(H_{inst}-11). \quad (2.62)$$

Adjustable scale 10 is a scale of logarithms ($\lg T_{H_{av}}-\lg 216.5$). Scales 14 and 15 are scales of logarithms (H-11 km), so that they are simple, unique logarithmic scales on which we can carry out multiplication and division of numbers, but with additional numbers which are shifted by 11 km to calculate altitude.

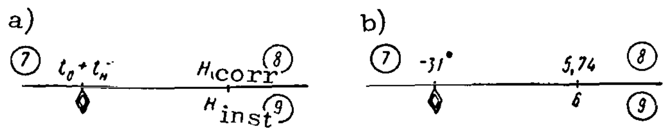


Fig. 2.63. Calculation on NL-10M: (a) Key for Introducing Methodological Correction in Altimeter Reading. (b) Determination of Correction for Measured Flight Altitude.

In accordance with Formula (2.62), the key for introducing corrections in the readings of the altimeter at flight altitudes above 12 km is shown in Figure 2.64 a.

Example. H_{inst} = 14 km; t_H = -50°. Find H_{corr}.

Solution. See Figure 2.64, b. Answer: H_{corr} = 14,400 m.

Note. Since the altitude of the tropopause at middle latitudes is not exactly at an altitude of 11 km, but can change within limits of 9-13 km, after solving the problem by means of the key shown in Figure 2.63, b, the flight altitude must be corrected for the additional correction ΔH = 900+20 (t_0+t_H) which is shown on the rule at the right-hand side below scale 14.

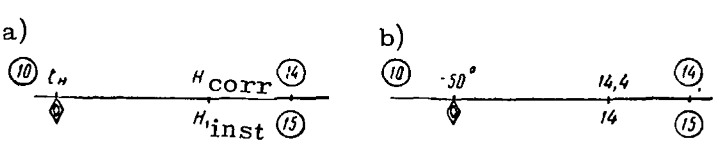


Fig. 2.64. Calculation on NL-10M: (a) Key for Introducing Correction in Flight Altitudes above 12,000 m; (b) Determination of Correction for Flight Altitude above 12,000 m.

The methodological corrections due to the failure of agreement of the actual air temperature with a calculated value are made to calibrate the speed indicator (type "US") with the aid of Formula (2.53).

In accordance with this formula, the scale 14 on the ruler for $\log V_{true}$ and scale 15 for $\log V_{inst}$ are purely logarithmic scales of linear values. Adjustable scale 11 (temperature for speed) is a scale of logarithms

$$\frac{1}{2}\lg(273^\circ + t_H),$$

while adjacent to it is fixed scale 12 (instrument altitude altitude in km) with a scale of logarithms /238

$$\frac{1}{2}\lg 288 + 2{,}628\lg(1 + 0{,}0226\,H).$$

The key for introducing corrections in the readings of the speed indicator "US" is shown in Figure 2.65, a.

Example. $t_H = -30^\circ$, $H_{inst} = 7$ km; $V_{inst} = 450$ km/hr. Find the airspeed.

Solution: See Figure 2.65, b. Answer: 638 km/hr.

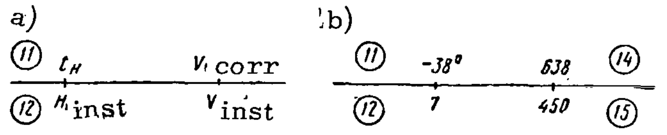


Fig. 2.65. Calculation on NL-10M: (a) Key for Introducing Correction in Readings of Type "US" Speed Indicator; (b) Determination of Correction for Reading of Type "US" Speed Indicator.

For speed indicators of type "CSI", the corrections given above are found by Formula (2.54 a).

It is clear from this formula that fixed scale 11 and movable scale 15 (for V_{inst}) and 14 (for V_{corr}) will be the same for the speed indicators of types "US" and "CSI".

Instead of fixed scale 12, we can scale 13 on speed indicators of type "CSI", which is a scale of logarithms

$$\frac{1}{2}\lg(288 - 0.0065\,H_{inst}).$$

The key for introducing corrections in the readings of these indicators is shown in Figure 2.66, a.

Example. H = 10 km, t_H = -45°, V_{CSI} = 800 km/hr. Find the corrected airspeed.

Solution: See Figure 2.66, b. Answer: 808 km/hr.

Rule scale 16 is set up according to the formula

$$\Delta t = K \frac{V^2}{26\,000}$$

and is used for introducing corrections into the readings of the thermometer for the outside air, type "TUE". This same scale can be used at subsonic airspeeds, and the error will not be greater than 1-2° for the type "TNV".

In practice, the front side of the navigational slide rule NL-10M can be used to solve a number of other problems, the keys for whose solution are directly dependent on the nature of the problem.

One example of such a problem is the determination of the deviation angle of the meridians between two points on the Earth's surface. /239

The angle of deviation of the meridians can be determined by Formula (1.82).

Obviously, this problem can be solved on a ruler by means of a key shown in Figure 2.67, a.

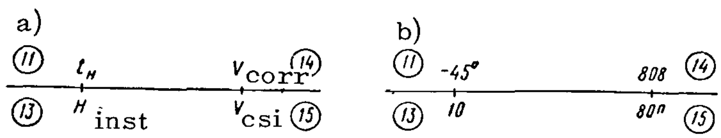


Fig. 2.66. Calculation on NL-10M: (a) Key for Introducing Correction in Reading of Type "CSI" Speed Indicator; (b) Determination of Correction for Reading of Type "CSI" Speed Indicator.

The scales on the back of the ruler can be used to solve some other problems. For example, movable scales 14 and 15 are the ones most suitable for multiplication and division of numbers.

Scale 14 is marked off with the following values: AM (American statute mile, equal to 1.63 km); NM (nautical mile, equal to 1.852 km), and foot (equals 32.8 cm). These markings are used for rapid conversion of measurements from one system to another.

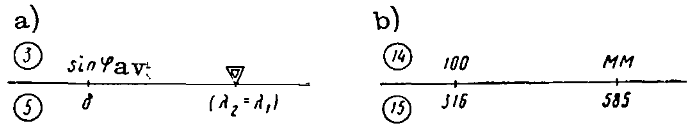


Fig. 2.67. Calculation on N1-10M: (a) Key for Determining Angle of Deviation of Meridians; (b) Conversion of the Length of the Arc of the Orthodrome into Kilometers.

Example. Convert the length of the arc of the orthodrome 5°16′ to kilometers.

Solution. 5°16′ = 316 NM (nautical miles).

Having set division 100 on scale 14 on the navigational slide rule opposite 316 on scale 15 (Fig. 2.67, b), we obtain the answer (585 km).

On scales 14 and 15, by using the settings of scales 11 and 12, we can solve problems in determining the Mach number at a known airspeed and air temperature at a flight altitude, or determine the airspeed at a given Mach number and air temperature.

Therefore, the speed of sound in air is found by the formula

$$a = 20{,}3\sqrt{273^\circ + t_H},$$

$$M = \frac{V_{\text{true}}}{a} = \frac{V_{\text{true}}}{20{,}3\sqrt{273^\circ + t_H}}$$

or /240

$$\lg M = \lg V_{\text{true}} - \lg 20{,}3 - \frac{1}{2}\lg(273^\circ + t_H). \qquad (2.63)$$

Scales 14 and 15 are scales of log V, fixed scale 11 is the scale of 1/2 log $(273^\circ + t_H)$, and fixed scale 12 is a scale of 2.628 log $(1-0.0226H)$, which is movable relative to scale 11 to the value 1/2log288.

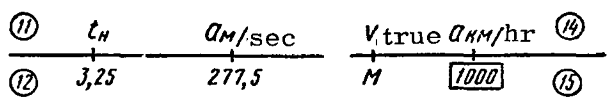


Fig. 2.68. Determination of Mach Number on NL-10M.

In order to get log20.3 from the value 2.628 log $(1-0.0226H)$, it is important to replace H by a value of 3.25 km. Therefore, if we find an altitude of 3.25 km and set it on fixed scale 12, we will obtain the key for solving the problem with a certain M number (Fig. 2.68).

Obviously, the value M = 1 corresponds to the airspeed (in km/hr) which is equal to the speed of sound.

To determine the speed of sound in m/sec, it is necessary to set the value of 0.2775 (1/36) on scale 15. If we use the rectangular index with the marking of 1000 for M = 1, then division 0.2775 will correspond to the number 277.5.

Note. In general, for converting zero altitude, corresponding to 1/2 log288, to the value of log20.3, it is necessary to shift it to the right to the value 2.51 km, and the functions of scales 14 and 15 in the key shown in Figure 2.100 will change places. Then Formula (2.63) will be valid.

However, this will call for using fixed scale 11 (temperature for speed) in reverse order. Therefore, it was found to be more convenient for solving these problems related to that Mach number to interchange the locations of the values on scales 14 and 15, and to shift the altitude to the left, to the division equivalent to 3.25 km, although theoretically Formula (2.63) is not then valid.

The fact that the numbers 2.51 (with a shift to the right) and 3.25 (with a shift to the left) are not equal is explained by the fact that zero altitude under standard conditions does not correspond to zero temperature but to +15°. Therefore, to make zero temperature match division H = 3.25, the scale must be moved by an amount such that it lines up with the marking H = -2.51 km.

CHAPTER THREE

AIRCRAFT NAVIGATION USING RADIO-ENGINEERING DEVICES

1. Principles of the Theory of Radionavigational Instruments

Geotechnical methods of aircraft navigation, although they form the basis of the complex of navigational equipment on an aircraft, do not permit a complete solution of the problems of aircraft navigation when there are no terrestrial landmarks or when the latter are invisible. /241

The principal reason for this is the variation of the wind at flight altitude, which means that the flight cannot be maintained for a significant period of time without checking the distance and direction of the path being followed.

Astronomical means, however, are not always helpful in determining the location of the aircraft, since the heavenly bodies are just as invisible as terrestrial landmarks when flying in clouds or between cloud layers. In addition, in order to determine the location of the aircraft, it is necessary to see at least two luminaries in the sky simultaneously, which is not always possible under normal flight conditions.

This has made it necessary to seek new methods of reliably carrying out aircraft navigation under any physical and geographical conditions either day or night, without dependence upon meteorological conditions, and has led to the development of radio-engineering devices for aircraft navigation.

All radio-engineering devices for aircraft navigation use the properties of the propagation of electromagnetic waves in the Earth's atmosphere to varying degrees.

We know that the phase velocity of the propagation of wave energy in dielectric media is

$$c_1 = \frac{c}{\sqrt{\mu\varepsilon}},$$

where c_1 is the rate of propagation of electromagnetic waves in the medium, c is the rate of propagation of electromagnetic waves in a vacuum, μ is the magnetic permeability, and ε is the dielectric constant. For a vacuum, $\mu = \varepsilon = 1$. /242

In addition to the phase propagation rate of electromagnetic waves, there is also a group propagation rate of electromagnetic energy.

In a vacuum, the phase and group propagation rates for electromagnetic waves are the same in all cases.

In dielectric media, especially in solids, liquids, and (to a much smaller degree) gases, the phase propagation rate depends on the frequency of the oscillatory process. This is explained by the inertia of the dielectric medium, i.e., the dielectric permeability of the medium depends on the oscillation frequency.

The dependence of the phase propagation rate upon the oscillation frequency is called dispersion. If the waves propagate in an electromagnetic medium with different frequencies, their phase rate may not be the same. In this case, the total energy of the waves will be maximum at those points in space where the phases of the waves are closest to coincidence. In addition, there will be points where the total energy of all the waves will be equal to zero, i.e., where the positive phases of the waves will be balanced by the negative ones.

The points with maximum total energy are called centers of wave energy. The rate at which the centers of wave energy move in space is the group rate of the waves.

The group rate of propagation of electromagnetic waves in space is

$$c_{gr} = \frac{c_1^2}{c_1 - \omega \frac{dc_1}{d\omega}},$$

where c_{gr} is the group rate, ω is the average spectral frequency, and c_1 is the average phase rate of the spectrum.

It is clear from the formula that with positive dispersion, the group rate of the waves exceeds the phase rate of their propagation.

Wave Polarization

Figure 3.1 is a graphic representation of a propagating electromagnetic wave in the horizontal plane as a function of the vertical open circuit.

In this case, the vector of the electrical field, and therefore the displacement currents, will coincide with the direction of the dipole of the circuit (dipole open antenna). The plane of the vector of the magnetic field coincides with the horizontal plane.

Obviously, the electromagnetic wave is a transverse wave, i.e., /243
the amplitudes of the oscillations of the electric and magnetic fields are located at right angles to the direction of wave propagation.

The direction of the plane of oscillation of the electric field is called the vector of wave polarization. In our sketch, we have electromagnetic waves with a vertical polarization vector.

In receiving electromagnetic waves, it is important to be sure that the direction of the dipole of the receiving circuit coincides with the vector of wave polarization. In this case, the oscillations of the electric field and the axis of rotation of the magnetic field coincide with the direction of the dipole, and both of these factors will bring the electromagnetic force (and consequently the conductivity currents) to the receiving antenna.

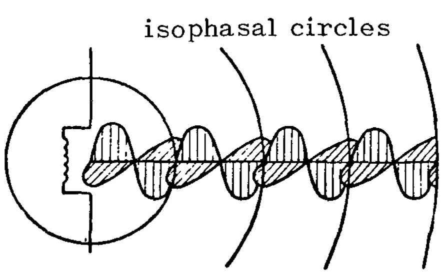


Fig. 3.1. Propagation of an Electromagnetic Wave from a Verticle Dipole.

If the waves are vertically polarized and the receiving antenna is located in a horizontal position, no emf will be produced in the dipole.

With the dipole in a horizontal position, the electromagnetic waves reaching the antenna will have a horizontal vector of polarization. In this case, the receiving antenna must be horizontal; in addition, the direction of the antenna in the horizontal plane must be perpendicular to the line to the transmitting antenna, i.e., it must coincide with the polarization vector of the waves.

The circles in Figure 3.1 join points in the horizontal plane which have identical phases for the electromagnetic waves. These circles are called isophasal.

From the viewpoint of the receiving antenna, the isophasal circles (and the isophasal spheres in the propagation area) are the directions of the wave front.

Propagation of Electromagnetic Oscillations in Homogeneous Media

In order to make use of the principles of design of various transmitting and receiving radio navigational instruments, it is necessary to become acquainted with the characteristics of the propagation of electromagnetic oscillations in inhomogeneous conducting and nonconducting media.

Electromagnetic wave processes in dielectrics constitute the conversion of the potential energy of the electrically deformed medium to the kinetic energy of displacement currents and vice versa (the kinetic energy of the field into the potential deformation of the medium). /244

For the majority of dielectric materials, polarization is not related to absorption of wave energy, because wave energy is propagated practically without losses in all directions. A decrease in the oscillation power with distance takes place due to the fact that the wave energy fills an increasingly large volume, which (as we know) is proportional to the cube of the radius of the sphere which it fills.

Significant losses in wave energy can occur in solid dielectrics with polar molecules. In this case, the polarization is not related to elastic deformation but to the motion of molecules, which causes a conversion of wave energy into heat.

In conducting media, the electromagnetic waves carry alternating conductivity currents. This means that conductors always undergo absorption of wave energy and its conversion to heat.

Thus, the propagation of wave energy in media, exhibiting both electronic and ionic conductivity, is practically possible to a slight depth which depends on the conductivity of the medium and the frequency of the oscillations. *The higher the conductivity of the medium and the greater the frequency of oscillation, the shallower the depths to which the oscillations will propagate.*

Since the propagation rate of electromagnetic waves depends on the dielectric and magnetic permeability of the medium, and the electronic or ionic conductivity of media can be assumed to be a very high (approaching infinity) dielectric permeability, the concept of optical density of media has been introduced.

The minimal optical density (equal to one) is possessed by a vacuum (where the propagation rate of the waves is equal to c). The optical density of all other dielectrics is greater than unity. In ideal conductors, the optical density is equal to infinity (the propagation rate of electromagnetic waves is equal to zero).

In portions of a medium with varying optical density, electromagnetic oscillations change the direction of their propagation. The change in direction of propagation of electromagnetic waves on the surfaces of particles of the medium with different optical density is called *refraction of rays*. In addition, under certain conditions, there is reflection of waves from the surfaces of the sections. The coefficient of reflection depends on the difference between the optical densities of the media, the frequency of the oscillations, and the angle of incidence of the wave.

When the path of a wave (propagation direction) runs from a less dense medium to a more dense one, with a certain angle of incidence to the surface, there may be no separation of the reflected wave. Such an angle is called the *angle of total internal reflection* of the denser medium. If the medium with the greater optical density is a conductor, irreversible absorption of wave energy may take place in it (conversion of wave energy into heat). /245

With the gradual change in the optical density of the medium, there is a continuous refraction (bending) of the line of propagation, called *radiorefraction*.

The optical inhomogeneity of a medium characterizes the propagation characteristics of waves of different frequencies in the Earth's atmosphere.

All harmonic oscillations in a medium are characterized by an oscillation frequency (ω) and an amplitude oscillation (E).

If we say that the amplitude oscillation is the maximum value of the intensity of the electrical field, then at any fixed point the oscillation process will satisfy the expression:

$$E = E_0 \sin(\omega t+\phi),$$

where   is the initial phase of oscillation.

The derivative of the field intensity with time will characterize the magnitude of the displacement current

$$I_{\text{dis}} = \varepsilon\frac{dE}{dt} = -\varepsilon E_0\omega\sin(\omega t+\phi).$$

while the second derivative will express the acceleration of the displacement current

$$\dot{J}_{\text{dis}} = \varepsilon E_0\omega^2 \cos(\omega t+\phi).$$

The distance between the two closest points in space which lie along the line of propagation of the wave front, in which the wave phase is identical, are called the *wavelength* (λ), which is equal to c_1/ω

Electromagnetic waves can be subdivided into four groups on the basis of their propagation characteristics in the Earth's atmosphere.

1. *Long waves,* from 30,000 to 3000 m (10-100 kHz). These waves have a surface type of propagation. Conducting media such as the Earth's surface and the upper ionized layers of the atmosphere have a deflecting effect upon them.

2. *Medium waves,* from 3000 to 200 m (100-1500 kHz) have a complex type of propagation. In the day, when the ionized layers of the atmosphere are lower, the type of propagation is superficial as in the case of long waves; at night, the medium waves have both a surface and spatial type of propagation.

3. *Short waves,* from 200 to 10 m (1500-30,000 kHz) have a spatial type of propagation. /246

4. *Ultra-short waves,* less than 10 m, have a radial type of propagation. They can be reflected from conducting layers on the Earth's surface, but only under certain conditions can they be reflected from the ionized layers of the atmosphere. Therefore, it is these waves which are used within the limits of geometric visibility of objects. The resistance to these waves on the Earth's surface is insignificant.

From the point of view of electrical conductivity and relief, the Earth's surface has a complex nature which depends on the time of year and weather conditions. The ionized layers of the atmosphere also have a varying nature.

The ionized *D* layer, which is closest to the Earth's surface, is only observed in the daytime and depends on the time of year, time of day, and geographical latitude; it may appear at heights from 60-90 km. This layer has an effect on the propagation of long and medium waves. The critical frequency of the layer is 0.4 MHz (750 m). Waves with frequencies higher than the critical are not reflected from the layer.

Above this is the *E* layer, whose ionization maximum is reached at a height of 120-130 km. This layer is the most stable one and retains its effect both day and night. The critical frequency of this layer with maximumillumination is 4.5 MHz; at night it drops to 0.9 MHz. this layer has a maximum effect on the propagation of medium and intermediate waves (the short waves at the spectral boundary with the medium waves). During the evening and morning hours, the layer changes its parameters so that the surface of maximum ionization decreases. This leads to errors in radionavigation measurements, since it reverses the vector of the wave polarization and consequently the direction of propagation of the wave front in the horizontal plane.

The third ionized layer (F) is the most unstable one both in terms of time of day as well as season of the year. Its average height is 270-300 km. During the daytime in summer, this layer divides into two parts (F_1 and F_2). In addition, the F layer shows some shifting in homogeneities, which make it difficult to predict the propagation conditions for electromagnetic waves. The F layer has an influence on the propagation of short waves.

It should be mentioned that the medium and short waves are reflected both from the ionized layers of the atmosphere as well as from the Earth's surface, so that they may undergo multiple reflection.

All of this combines to give us the complex picture of the propagation of electromagnetic waves in the Earth's atmosphere, which must be taken into account in radionavigational measurements.

The peculiarities of propagation of electromagnetic oscillations in a conducting feeder channel in receivers and transmitters include /247 the following.

Unlike constant and low-frequency alternating currents, high frequency currents propagate mainly along the surface of a conductor, since the reaction of the magnetic field within the conductor is greater than on its surface (the skin effect). This causes all high frequency conductors to be constructed with an eye toward increasing the surface, e.g., tubular and multiple-filament (stranded wire).

However, these measures are insufficient for waves in the centimeter range. It is much better to use hollow conductors for these waves, called wave guides (Fig. 3.2).

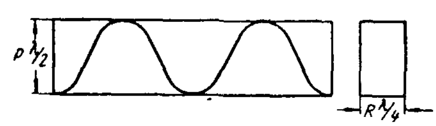


Fig. 3.2. Propagation of Electromagnetic Waves Along a Wave Guide.

In a homogeneous medium, the propagation rate of the field along the vector of the wave polarization, in the direction of the perpendicular vector of polarization, in the plane of the wave front, and in the propagation direction of the wave front are the same. Therefore, in a box-type wave guide, the distance between the surfaces of which is measured in whole numbers of half waves, the vertically polarized wave (striking the top and bottom internal walls of the box and being reflected from them) will strike the opposite wall also with a whole number of half waves. Consequently, reflection of the waves will take place in resonance with its oscillations. In this case, the box-type wave guide will act as a channel along which the waves will propagate themselves practically without any resistance.

If the distance between the walls is equal to a whole, odd number which is one-quarter of the wavelength, then (as is easily seen) the reflections from the walls of the wave guide will take place each time in opposite phase with the oscillations. In this case the wave guide will have infinite resistance, and the wave energy will not be propagated in it. In Figure 3.2, the vector of wave polarization must be turned 90° to accomplish this.

Principles of Superposition and Interference of Radio Waves

The principle of superposition is applied to wave processes, i.e., each of the wave processes acts independently of other processes which are taking place in the medium or circuits.

At the same time, the results of different processes can be summed by means of a simple superposition of oscillation vectors. If the vectors of two coherent (coinciding in frequency) processes such as the oscillations in the intensity of a field or displace- /248
ment currents, are equal in amplitude and coincide in phase, the total amplitude of the oscillations will be doubled. Under these conditions, if the oscillations are in opposite phase, the total amplitude of the oscillations will be equal to zero and no method will suffice to detect the presence of the wave processes involved.

Summing of the results of the processes in opposite phase is called *wave interference*. The case in which the result of summing of the oscillations is equal to zero is called *total interference*.

The properties of interference of radio waves are widely employed and radionavigational devices both in receivers and transmitters, especially in measuring the direction of an object.

Principle Characteristics of Radionavigational Instruments

The principle characteristics of transmitting radionavigational instruments are the following:

(1) The radiated power, characterizing the operating range of the system.

(2) Accuracy and stability of the frequency structure, as well as synchronization of special navigational signals.

As far as the antenna arrays are concerned, which incorporate certain characteristics for radiation of signals, we will discuss them under the heading of "Principles of Operation of Concrete Navigational Instruments".

Receiving navigational instruments are also combinations of tuning circuits and amplifiers which use vacuum tubes or semiconductors. In the majority of receivers, special generators (heterodynes) are used, whose construction differs for the particular difference in frequency with respect to the received signal. In special mixing tubes, the frequencies of the transmitter and heterodyne signals are combined and an intermediate frequency is produced which is equal to the difference between the frequencies given above. In such devices, further amplification of the signal is carried out with a constant, lower frequency, which makes it possible to use amplifier devices with very high coefficients of amplification, as well as to ensure a high selectivity of the receiver.

Usually, receiving radionavigational instruments fulfill two functions: (a) reception and amplification of the signals from a transmitter (b) separation and indication of measured navigational parameters.

The basic characteristics of receiver navigational instruments are the following:

(1) Sensitivity of the receiver which characterizes the possible receiving range for signals from a transmitter.

(2) Selectivity of the reception; this parameter is usually obtained by narrowing the frequency band which the receiver will pass, which usually characterizes the freedom from noise of the receiver. /249

(3) The accuracy with which the navigational parameters are selected and recorded.

Operating Principles of Radionavigational Instruments

In accordance with the laws of propagation of electromagnetic waves in space, it is possible in principle to measure the following parameters of electromagnetic waves: amplitude, phase, frequency, and transmission time of the signal.

According to the principle of technical operation, radionavigational devices are divided into amplitude, phase, frequency and time devices.

In addition, with a mutual exchange of radio signals between objects which have relative motion to one another, changes in the frequency characteristics of the signals occur which are known as the Doppler effect, which is used to build automatic airspeed indicators and devices for measuring the drift angle of an aircraft.

Measurements of the parameters listed above for electromagnetic waves from the navigational standpoint make it possible to determine the following navigational elements:

(a) The direction of the object, by means of goniometric systems;

(b) The distance to an object, by means of rangefinding systems;

(c) The difference or sum of the distances to the object: hyperbolic or eliptical systems;

(d) Speed and direction of movement of the aircraft: automatic Doppler meters for ground speed and drift angle.

For convenience of application, in many cases the navigational systems are compensated for measuring two navigational parameters simultaneously. For example, there are the goniometric-rangefinding systems, difference-rangefinding instruments, etc.

The panoramic radar located on the ground and on the aircraft are goniometric-range finding devices with a single unit of navigational equipment.

In studying methods of applying radionavigational systems, it is a good idea to classify them according to the principles by which the navigational parameters are measured. Therefore, the further subdivision of the material will be made on the basis of these principles.

Radionavigational devices can also be subdivided into automatic and non-automatic. Non-automatic devices, when they consist of systems of ground control and apparatus aboard the aircraft, are called navigational systems. Automatic devices are called *automatic navigational systems* when the operation of several types of navigational devices is combined organically on board the aircraft. For example, the automatic Doppler system for aircraft navigation, /250
which consists of a Doppler meter for the drift angle and the ground speed, course devices, and the automatic navigational instruments.

2. GONIOMETRIC AND GONIOMETRIC-RANGEFINDING SYSTEMS

The goniometric radionavigational systems are the simplest ones from the standpoint of technical requirements, and are therefore those which are most widely employed at the present time.

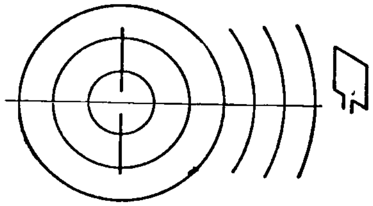


Fig. 3.3. Reception of Electromagnetic Waves by a Frame Antenna.

In the majority of these systems, the amplitude method of measurement is employed, based on the interference of electromagnetic waves. This principle serves as the basis of the operation of ground and aircraft-mounted radio direction finders, which are also called radio compasses.

Let us imagine a frame-type receiving antenna, located in a field of outwardly directed radio waves (Fig. 3.3). If the frame antenna is located relative to the transmitter so that the direction of the propagating waves will be perpendicular to the plane of the frame, the left and right vertical sides of the frame will be on the same isophasal circle. In this case, the high-frequency currents which are conducted in the sides of the frame will agree in phase and will consequently be directed toward one another. This gives complete interference of the oscillations of the currents in the frame, and there will no reception of signals from the transmitting station.

If the frame is turned around the vertical axis through 90°, so that the direction of the plane of the frame coincides with the direction of the transmitting station, the sides of the frame will be at different isophasal circles, maximally distant for the given device. Thus, the currents in the vertical sides of the frame will undergo a certain phase shift which will give maximum reception of the signals from the station. The maximum effect of the frame will be observed in the case when the distance between its sides is equal to half the wavelength. Then the currents in the vertical sides of the frame will be in opposite phase and their amplitudes will be added. However, this requirement (in the majority of cases) cannot be fulfilled, since the antenna device becomes too unwieldy; therefore, we use that part of the effect which is obtained with a phase shift through a small (frequently very small) /251
angle. In these cases, the receiving frame is supplied with many turns and a radio receiver with very high sensitivity is employed. The vector diagram of the reception directionality of the frame will have the form of a figure eight (Fig. 3.4).

The greatest accuracy in range finding is obtained with minimum reception, while at the maximum the change in amplitude of the received signal is obtained by turning the frame at a slight angle. Therefore, range finding by means of a frame is always done with minimum reception or audibility of the signal.

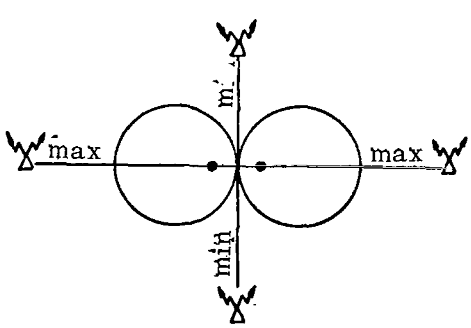


Fig. 3.4.

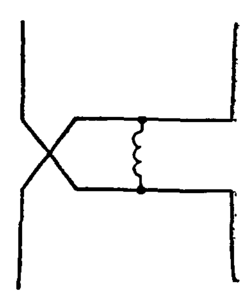


Fig. 3.5.

Fig. 3.4. Diagram of Reception of a Frame Antenna.

Fig. 3.5. Edcock-Type Antenna.

The receiving frame antenna has the shortcoming that when electromagnetic waves are being propagated through space, it picks up not only the vertically polarized wave but also the horizontal component of the polarization vector in the top and bottom sides of the frame. In addition, the frame antenna with its large dimensions and mechanical rotation is inconvenient to use. Therefore, ground-based radio rangefinding installations use special antennas which are equivalent to a frame type in the characteristics of reception for vertically polarized waves, but are free of the shortcomings mentioned above; they are called Edcock antennas (Fig. 3.5).

The picture shows one pair of Edcock dipoles with the coil of a goniometer between them. Obviously, in open dipoles, no interference will be observed when they are located on one isophasal circle. However, the difference in potential at the ends of the goniometric coil will be equal to zero, since they are connected to symmetrical points on the dipole. If the dipoles are located on different isophasal circles, then the phase shift will disturb the potential equilibrium at the ends of the coil and a high-frequency current will pass through it.

A similar pair of dipoles is mounted in the plane perpendicular to the first pair.

The high-frequency current in the goniometer coils will depend on the direction of the transmitting station relative to the crossed dipoles. /252

In a goniometric instrument, in addition to the two fixed dipole coils mounted at an angle of 90°, there is a movable searching coil, connected to the input circuit of the receiver.

If the searching coil is placed in the resultant field of the fixed coils, the reception will be maximum; when a coil is placed at an angle of 90° to the resultant field, reception will be minimal.

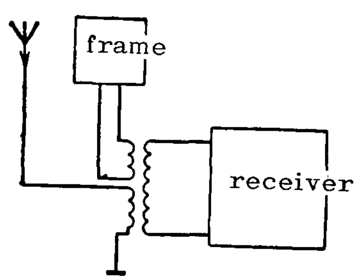


Fig. 3.6. Inclusion of an Open Antenna for Solving Ambiguity of Reception.

The horizontal wires connecting the antenna dipoles are located as close as possible to one another, so that the electromotive force conducted in them from the horizontal component vector of polarization will be in the same phase, and their total interference will appear at the inputs in the goniometer coils. Therefore, the antenna does not pick up component waves with horizontal polarization, thus considerably reducing the range finding error for waves in space.

The reception characteristics of the frame antenna (including the Edcock type) have two signs, i.e., we have two maxima and two minima of audibility, so that it can be used to determine the direction line on which the transmitting and receiving objects are located, but does not solve the problem of the sides of the mutual position of the objects (see Fig. 3.4).

To solve the ambiguity of reception with radio rangefinding instruments, an open antenna with an externally directed (circular) reception characteristic is used in addition to the frame antenna (Fig. 3.6).

The phase of the high-frequency current in the open antenna, depending on the reception direction, will coincide with the phase of one of the sides of the frame receiver and will be in opposite phase with the currents in the second side of the receiver. As a result, the current amplitudes of an open antenna will be added to one-half of the figure eight of the frame antenna and will interfere with the other half of the figure eight (Fig. 3.7, a).

In combining the characteristics of the frame and open antennas, we obtain a total characteristic which has the form of a cardioid. If we connect the open antenna and turn the frame antenna through 90° clockwise, the maximum reception shown in Figure 3.7, b will shift to the upper part of the picture while the minimum will shift to the lower part (Fig. 3.7, c). This corresponds to one side of the minimum of the frame receiver being transferred to the maximum, /253 and the second remaining minimum.

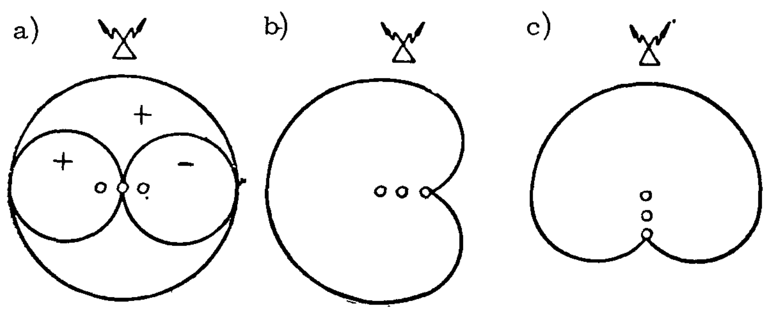


Fig. 3.7. Diagram of Directionality of a Frame Antenna Combined with an Open Antenna.

Let us suppose that we have defined a line (bearing) on which the transmitting and receiving points are located at minimum audibility. After connecting the open antenna and turning the goniometer coil through 90°, we can determine the direction of the transmitter. If the audibility of the signals increases sharply, the transmitter is located in the direction of the upper part. If it remains as before or changes slightly, the transmitter is located at the opposite side.

The principles described above for finding the direction of a transmitter are used in ground radio direction-finding installations. In this case, the transmitter is the radio on board the aircraft.

Ground radio direction-finders in principle can operate at all wavelengths. The most widely used radio rangefinders operate on short and ultra-short waves.

The position of the aircraft can be determined by means of the ground radio rangefinder in terms of the minimum audibility of the signal from the transmitter located on board. In addition, visual indicators are mounted on the ultrashort wave (USW) rangefinders, such as cathode-ray tubes.

In this case, the frame of the direction-finder or the goniometer coil is set to rotating rapidly, and the scan of the cathode-ray tube is synchronized with it. The amplitude of the scan is related in magnitude to the amplitude of the received signals in such a way that at minimum reception the maximum amplitude of the scan is observed. Then, on a scale which is marked along the periphery of the tube face, we can determine the direction of the aircraft in terms of the position of the maximum deflection of the scan.

With a relatively low density of air motion, the ground radio rangefinders are a sufficiently effective and precise method of aircraft navigation. An advantage of ground radio rangefinders is the lack of a need to mount special radio equipment on the aircraft. The radio rangefinders and receivers which are used for /254 receiving signals from ground direction-finding points mainly have other purposes, and their use for navigational purposes is not related to the increased complexity and weight of the equipment on board.

At the same time, however, the use of ground radio direction-finders has a number of serious shortcomings, which have led to a search to find new ways of radionavigational control of flight.

The most important of these shortcomings are:

(a) Lack of a visual indicator on board the aircraft to show its position, thus reducing the ease of aircraft navigation.

(b) A small capacity for the ground installations; at the same time, the radio direction-finder can only operate with one aircraft, which is clearly inadequate when there are a great many flights.

Aircraft Navigation Using Ground-Based Radio Direction-Finders

The use of ground-based radio direction-finders can be used to solve the following navigational problems:

(a) Selection of the course to be followed and flight along the straight-line segments of a route, at the beginning or end of which radio direction-finders are located.

(b) Control of the aircraft path in terms of distance.

(c) Determination of the aircraft location on the basis of bearings obtained from two ground-based radio direction-finders.

(d) Determination of the ground speed of the aircraft, as well as the drift angle, direction and speed of the wind at flight altitude.

Usually, the international "Shch"-code is used for determining the bearings from on board the aircraft. The crew of the aircraft reports its position, gives the required code for its position, and presses the telegraph key of the transmitter for a period of 20 sec.

In recent years, both state and local civil airlines have adopted USW direction finders, with visual indicators. They are oriented according to the magnetic meridian of the location of the USW direction-finder, and (depending on the flight altitude) are used in a radius of 100-200 km as a form of trace direction-finder, reporting on board the aircraft the "forward" (away) and "back" (return) magnetic bearings of the aircraft. If the crew of the aircraft requests the forward true bearing, the operator of the USW direction-finder (supervisor) calculates the magnetic declination of the location of the distance finder and reports the forward true bearing to the aircraft.

Distance finding by means of USW distance finders with a visible indicator is used in the course of communication with an aircraft, i.e., with a depressed tangent of the connected USW transmitter on board the aircraft. /255

The operator of the ground radio direction-finder, after the required measurements, gives the call letters of the aircraft, the code expression for the bearing as requested by the aircraft or used for USW communication, and gives the magnitude of the bearing in degrees.

The code expressions for the bearings in the international Shch code have the following meanings (Fig. 3.8):

ShchDR: magnetic bearing from distance-finder to the aircraft, or forward bearing.

ShchDM: magnetic bearing from the aircraft to the distance-finder (measured relative to the local meridian of the location of the distance-finder), or reverse bearing.

ShchTE: true bearing from the distance-finder to the aircraft, or the forward true bearing.

ShchGE: azimuth of the aircraft at a distance from the control distance-finding station.

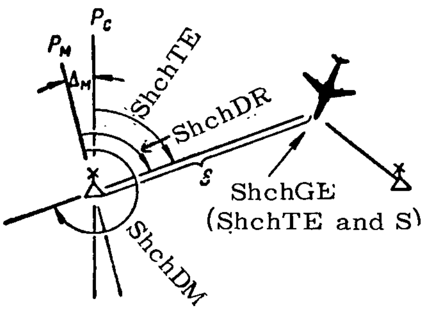


Fig. 3.8. Code Expressions for Bearings in the Shch-code.

ShchTF: location of the aircraft (coordinates or link).

Due to the small effective radius of the USW rangefinders, they are not grouped into distance-finding nets like long or medium-wave stations, but operate independently, and do not give the location of the aircraft.

Selection of the Course to be Followed and Control of Flight Direction

The selection of the course to be followed and flight along a straight-line path segment are accomplished by means of periodic inquiries and determinations of the forward or reverse bearings of the aircraft (ShchDR or ShchDM).

If the radio distance-finder is located at the starting point of a flight segment (flight from the distance-finder), then the ShchDR bearings are requested. When the aircraft is passing precisely over the ground radio distance detector and follows a constant course for a certain period of time, the first bearing of the aircraft after passing over the distance-finder can be used to determine the drift angle (Fig. 3.9). Usually in this case the ShchDR bearing will be equal to MFA_a, so that

$$US = ShchDR - MC.$$

If the ShchDR does not correspond to the given flight path angle for the path segment, then the aircraft is put on the desired line of flight after determining the drift angle and the course to be followed is set so that the total of the course and the drift angle of the aircraft will equal the given path angle. /256

It should be kept in mind that in the general case ShchDR is not equal to MFA_g, since the former is the orthodromic bearing measured at the starting point of the segment and the MFA is the loxodromic path angle measured relative to the mean magnetic meridian:

$$ShchDR - MFA = \Delta_{M_{av}} - \Delta_{M_1} - \delta_{av},$$

where Δ_{Mav} is the magnetic declination at the midpoint of the segment, Δ_{M1} is the magnetic declination at the location of the radio distance-finder, and δ_{av} is the deviation of the meridians between the initial and middle points on the path segment.

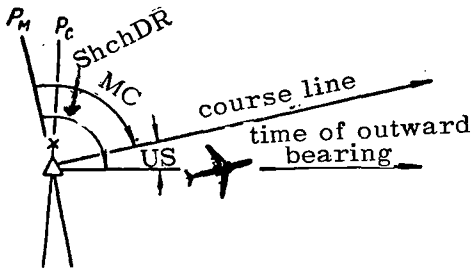


Fig. 3.9.

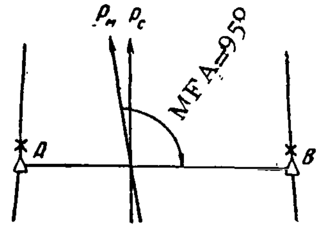


Fig. 3.10.

Fig. 3.9. Determination of the Drift Angle After Flying Over a Radio Distance-Finding Station.

Fig. 3.10. Path Segment Between Two Radio Distance-Finding Stations.

In principle, orthodromic control of the path for a loxodromic flight is inconsistent, because in practice the course to be followed in a loxodromic system of path angles is selected so that flight takes place along the orthodrome.

In order to maintain the given flight direction over the path segment with sufficient accuracy, it is necessary to note that at each bearing (ShchDR or ShchDM) the aircraft will be located on an orthodromic line of the given path and will therefore maintain this bearing.

Let us explain this by a concrete example.

We will assume that we must make a flight from a point A(λ = 105°, Δ_M = -1°) to a point B(λ = 115°, Δ_M = -7°) and return (Fig. 3.10). The magnetic flight angle of the segment is 95 or 275°, while the average latitude of the segment is 52°.

Obviously, for flight in an easterly direction from the distance-finder, located at point A

$$\text{ShchDR} = \text{MFA} + \Delta_{M_{av}} - \Delta_{M_1} - \delta_{av} = 95-4+1-4 = 88°.$$

For flight in a westerly direction, the ShchDM from this distance-finder must be equal to 268°.

For a flight in an easterly direction, the initial course of the aircraft must be set not on the basis of MFA = 95°, but from ShchDR = 88°. In the opposite case, the aircraft slowly begins to deviate from the line of the desired bearing at an angle of 7°. /257

Analogously, for the point B (ShchDR = 282°, ShchDM = 102°), the initial course must be set 7° greater than one would conclude on the basis of the MFA.

Of course, it is impossible to make flights with a constant MFA at distances at which the magnetic direction of the flight changes by 14°. This example is given only to illustrate the geometry of the process. It would be more accurate to divide this segment into four parts 150 km long with the following flight angles: MFA_1 = 90°, MFA_2 = 93°, MFA_3 = 97°, and MFA_4 = 100°. In the first two segments, we must use a distance finder which is located at Point A (ShchDR = 88°), while for the latter we must use the distance finder at Point B (ShchDM = 102°).

This division of the flight segment into parts for the case of a flight according to a ground distance finder is an approximation of the initial MFA to the ShchDR of the initial distance finder, while the latter is approaching the ShchDM of the range finder located at the terminus of the flight.

In the orthodromic system of calculating flight angles, the distance between the OFA and bearings ShchDR and ShchDM from one of the distance finders will be constant in value and will depend only on the meridian selected for calculating the path angles. In the special case when the reference meridian coincides with the meridian where the range finder is located, OFA will differ from ShchDR only in the magnitude of magnetic declination for the location of the distance finder:

$$\text{OFA} = \text{ShchDR} + \Delta_M .$$

Therefore, in an orthodromic system of calculating flight angles, the course to be followed by the aircraft changes more rarely and to a much lesser degree than in a loxodromic system, but all elements of aircraft navigation, including the speed and wind direction, are determined more accurately.

For selecting a course and maintaining the flight direction of an aircraft in terms of ground radio distance-finders, the method of half corrections is used, which consists of the following:

Let us say that at a point position of the aircraft on the line of a given path, the latter is on course with a certain anticipation of drift.

After a certain period of time, on the basis of the bearing obtained from the distance finder, it is found that the aircraft is shifting from the line of flight toward the direction of the wind vector. This indicates that the correction in the course which has been taken is insufficient. Therefore, it is necessary to return the aircraft at an angle of 10-15° to the given line of flight, and the previously employed lead in the course to be followed is doubled.

If in this case the aircraft begins to shift from the line of flight toward the side opposite the wind vector, then after the second aiming of the aircraft along the given line of flight, it is necessary to make a correction in the course which is halfway between the latter and the former. If the deviation takes place along the direction of the wind vector, the correction in the course must be increased.

In addition, if the deviation of the aircraft from the line of desired flight takes place, the difference between the latter and the former corrections is divided and added to the course with a positive or negative sign, depending on the direction of the aircraft deviation. /258

The placing of the aircraft on the desired line of flight by selecting the course with all the deviations mentioned is obligatory only in a flight from the distance-finder along a forward bearing (ShchDR). In a flight toward a radio distance-finder along a reverse bearing (ShchDM), the aircraft must follow the line of the desired path only in the case when it is going beyond the limits of the established trace. With small deviations (by distances from the radio distance-finder of up to 200 km within the limits of 1-2°), it is sufficient to select the course to be followed by the same method of half corrections relative to the last ShchDM (reverse bearing), without going to the desired line of flight each time.

The method of half corrections is the general one used for flight toward the radio distance-finder and away from it. However, in practical use, there are considerable differences between flight toward the distance-finder and away from it:

(1) In a flight from the radio distance-finder, the drift angle can be measured at the beginning of the segment, while in a flight toward the distance-finder it can be determined only after selecting the course to be followed with a stable ShchDM.

(2) In a flight from a radio distance-finder, the course to be followed by the aircraft must change in the direction opposite the change of the bearing: ShchDR increases, and the course must also decrease, and vice versa. In a flight toward a radio distance-finder, the change in the course must take place in the direction of the change in bearing: ShchDM increases, the course must be increased, and vice versa.

(3) As we have already pointed out, a flight away from a radio distance-finder in all cases must be made strictly along the given bearing, while in the flight toward a radio distance-finder (within certain limits) it is permissible to select the flight to be followed according to the last stable bearing.

Path Control in Terms of Distance and Determination of the Aircraft's Location

For the purposes of controlling the path in terms of distance, as well as determining the location of the aircraft, we can use the true bearings from the ground radio distance-finder to the aircraft (SchTE).

For checking a flight in terms of distance, we usually select the control landmarks along the flight route and determine their precalculated bearings from the radio distance-finder located to the side of the aircraft route (Fig. 3.11).

Three to five minutes before the aircraft reaches the control landmark, a series of "forward true" bearings are requested (ShchTE). When the bearing of the aircraft becomes equal to the calculated /259
one, the passage of the control landmark is noted.

By using long- and medium-wave radio direction-finders, the location of the aircraft is determined from bearings of two or three mutually related ground radio direction-finders, one of which is the command station.

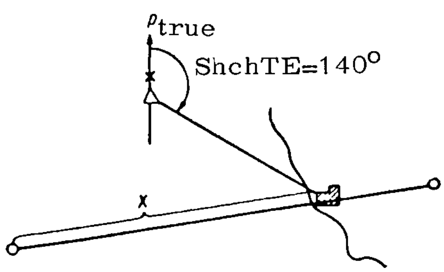


Fig. 3.11. Previously Calculated Bearing of a Landmark.

Upon request from the crew of an aircraft, with regard to the azimuth and distance from the command distance-finding station (ShchGE), the aircraft measures its distance simultaneously from two (three) distance measuring stations, while auxiliary distance finders report the measured bearing to the command distance station.

The operator of the command radio distance-finding station uses a special plotting board to determine the true bearings of the aircraft with the aid of movable rulers with their centers of rotation at the points where the radio distance-finding stations are located; having measured the distance to the aircraft (the points of intersection of the bearings), the operator transmits to the crew of the aircraft its position (the true direction and distance from the command radio distance-finding station).

If the crew of the aircraft desires to obtain data regarding the location of the aircraft in different forms (e.g., geographical coordinates or relationship to some landmark), they must ask for the ShchTF bearing from the command radio distance-finding station.

Determination of the Ground Speed, Drift Angle, and Wind

The ground speed of an aircraft can be found by using ground radio distance-finders as well as other non-automatic radionavigational devices during flight on the basis of the distance covered by the aircraft between two successive indications of its position (LA):

$$W = \frac{S}{t}.$$

The successive landmarks for the LA are the points at which the aircraft passes over previously calculated bearings along the route or locations for the aircraft marked on a map which were obtained from the command distance-finders upon request of bearings ShchGE or ShchTF.

The drift angle can be determined in three ways with the aid of ground radio distance-finders:

(1) The difference between the "forward" bearing (ShchDR) and the course of the aircraft after passing over the radio distance-finding station: /260

$$\text{US} = \text{ShchDR} - \text{MC};$$

(2) By the difference between the path angle of the flight and the course of the aircraft after selecting a stable "forward" bearing (ShchDR) or "reverse" ShchDM:

$$a = \psi - \gamma,$$

where a is the drift angle of the aircraft, ψ is the path angle of the flight, and γ is the course of the aircraft.

(3) On the basis of the path angle and the mean course of the aircraft between successive indications of the PA (A and B):

$$a = \psi_{AB} - \gamma_{\text{av}}.$$

In the magnetic loxodromic system, of estimating path angles, the best way for determining the drift angle is the first. The second and third methods give exact results only in the middle part of the path segment, i.e., when crossing the meridian, relative to which the path angle of the segment is measured. At the beginning and end of the segment, the errors are maximum.

In the orthodromic system of calculating path angles and courses, the accuracy of determining the drift angle is approximately the same for all three methods.

The speed and direction of the wind at flight altitude is determined with the aid of ground radio distance-finders in two ways:

(1) **According to the ground speed of the aircraft, the airspeed, and the drift angle.** To solve this problem, we can use a key on the navigational slide rule for determining the wind angle (Fig. 3.12, a) and for determining the wind speed (Fig. 3.12, b).

(2) **By the difference between the actual and calm coordinates of the aircraft on the flight chart.** This method means that the first location of the aircraft on the basis of the ShchGE or ShchTF bearing is marked on the flight chart. During the time that the aircraft is flying from the first location, the calm path calculation is made (according to the average course, airspeed and time), the calm position of the aircraft is determined, and also entered on the chart with simultaneous request of the second position of the aircraft in terms of the ShchGE or ShchTF. The vector between the calm point and the second position of the aircraft, determined on the basis of the ShchGE bearing, is the wind vector for the flight time over a given path segment. Let us consider the following example:

After 24 min of flight between two successive locations, the wind vector is equal to 140° in direction and 30 km in magnitude.

If we divide the modulus of the wind vector by the flight time /261
in hours (0.4), we will get the wind speed

$$u = 30:0.4 = 75 \text{ km/h}.$$

The first method of determining the wind is the one most widely employed. However, on large passenger aircraft with automatic navigational indicators on board (e.g., NI-50), by means of which automatic quiet calculation of the aircraft path can be carried out, the second method is the most suitable and precise. When this is done, it is no longer necessary to plot the wind vector on the flight chart.

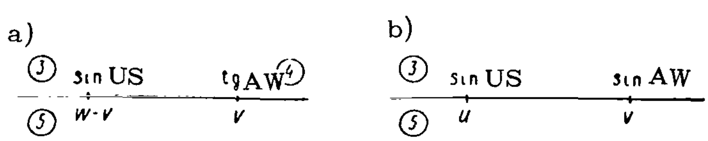


Fig. 3.12.

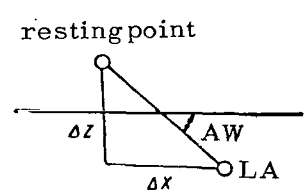


Fig. 3.13.

Fig. 3.12. Keys for Determining the (a) Wind Angle and (b) Wind Speed on the NL-10M.

Fig. 3.13. Determination of the Wind by the Difference in the Coordinates of the Calm Point and the Location of the Aircraft.

It is clear from Figure 3.13, that

$$\mathrm{tgAW} = \frac{\Delta Z}{\Delta X}; \quad ut = \frac{\Delta Z}{\sin \mathrm{AW}},$$

where

$$\Delta Z = Z_{\mathrm{LA}} - Z_{\mathrm{rp}};$$

$$\Delta X = X_{\mathrm{LA}} - X_{\mathrm{rp}}.$$

If we know the distance of the orthodromic coordinates of the location of the aircraft and the calm point, this problem is easily solved on the navigational slide rule using the following key:

For determing AW (Fig. 3.14, a), and for determining ut (Fig. 3.14, b)

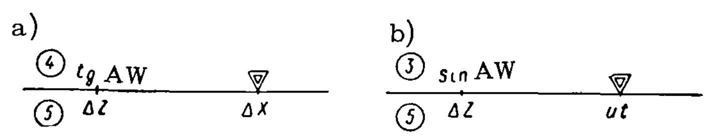


Fig. 3.14. Determination of (a) Wind Angle and (b) Wind Speed on the NL-10M.

Example: MFA = 110°; Δ_M = -7°; ΔX = 40 km; ΔZ = 20 km; t = 15 min. Find the direction and wind speed at flight altitude.

Solution: AW = 26° (Fig. 3.15, a) ut = 45 km (Fig. 3.15, b).

$$u = \frac{45\ \text{км}}{15\ \text{min}} = \frac{45\ \text{км}}{0{,}25\ \text{hr}} = 180\ \text{км/hr}$$

To determine the wind direction relative to the meridian of the aircraft's location, the calculation of the given path angle of flight should be applied to that meridian, and then the wind angle should be added. /262

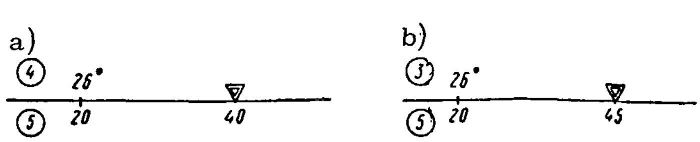


Fig. 3.15. Determination of (a) Wind Angle and (b) Wind Speed on the NL-10M.

In a flight with magnetic path angles, we will have approximately

$$\delta_M = MFA + AW$$

or in our case

$$\delta_M = 110 + 26 = 136°;$$
$$\delta = \delta_M + \Delta_M = 136 - 7 = 129° .$$

Automatic Aircraft Radio Distance-Finders (Radiocompasses)

Automatic aircraft radio direction-finders (radiocompasses) are very widely employed. Aircraft with piston engines use them as a reliable, operative, and highly precise method of aircraft navigation. Large passenger aircraft with jet engines, for a number of reasons, cannot make such effective use of radiocompasses, but they continue to use them successfully along with other more precise means of aircraft navigation.

The operating principle of radiocompasses is in no way different from the principle of operation of ground radio distance-finders. However, radiocompasses are more operative and suitable for the purposes of aircraft navigation, since they allow the crew of the aircraft to have a constant visual indication of the position of the aircraft.

The accuracy of distance-finding for ground radio stations with the aid of radiocompasses is somewhat lower than the accuracy of distance finding for aircraft with ground radio distance-finders, which can be explained by the following three reasons:

(1) Stationary radio distance-finders can have special antennas which are equivalent to frame-type antennas but are free of the effect related to the horizontal sides of the frame; on aircraft, it is not possible to install such antennas due to their unwieldiness.

(2) The bearing of an aircraft is measured with the aid of ground radio distance-finders directly from the direction of the magnetic or true meridian, passing through the radio distance-finder at a fixed setting of the antenna system relative to the vertical; /263 in distance-finding with ground radio stations by radiocompasses located on board aircraft, the error in the bearing includes the errors in measuring the aircraft course; in addition, the accuracy of distance measurement is reduced due to the longitudinal and transverse banking of the aircraft.

(3) Errors in distance-finding due to the effect on the propagation of electromagnetic waves over the relief of the surrounding medium to a certain degree is taken into account in measuring the distance of aircraft with the aid of ground radio distance-finders (by means of preliminary test flights and the recording of a curve of radio deviation).

The considerable difference in flight conditions does not permit us to solve this problem for radio compasses located on board an aircraft. On the average (with a probability of 95%), the errors in locating aircraft with ground radio distance-finders in flat country is 1-2°, and 3-5° in the mountains. The errors in measuring the distances with the aid of radiocompasses in flat areas is 3-5°, and can reach 10-15° in mountainous areas, especially at low flight altitudes.

Accordingly, the practical operating range of a ground radio distance-finder with satisfactory results of tracking is 300-400 km (except for those which work on USW, where the operating range is determined by the straight-line geometric visibility).

A satisfactory accuracy in determining the bearings of radio stations with the aid of on-board radiocompasses is obtained at distances up to 180-200 km. Nevertheless, radiocompasses have found increasingly broad application for purposes of aircraft navigation, and are more popular than the ground radio distance-finders due to their considerable autonomousness and the ease with which they can be employed.

For purposes of increasing operativeness, as well as forming a reserve and ensuring reliable operation of radiocompasses, two sets of them are used in most aircraft.

The basic control system for an on-board radio compass consists of the following:

(a) Frame antenna with mechanical device for rotating it and a mechanism for compensating radio deviation.

(b) Open antenna.

(c) Superheterodyne receiver with a device for commutation of the phase of the frame antenna and an electrical device for turning the frame antenna (tracking system).

(d) Indicator of course angles of radio stations.

(e) A shield for the remote control of the radiocompass.

The size of the antenna for the radiocompass located on board an aircraft is many times smaller than half the length of the received wave, and therefore many turns are made in the frame to increase the effectiveness. In addition, the internal space between the coils is filled with a material with a very high magnetic and dielectric permeability (ferro-dielectric). This produces a sharp decrease in the propagation rate of electromagnetic oscillations between the sides of the frame, so that /264

$$c_1 = \frac{c}{\sqrt{\mu\varepsilon}},$$

which is equivalent to a drop in the wavelength of the received signal and therefore an increase in the phase shift between the sides of the frame. To increase the magnetic and dielectric permeability of the medium, the effect of the frame will increase.

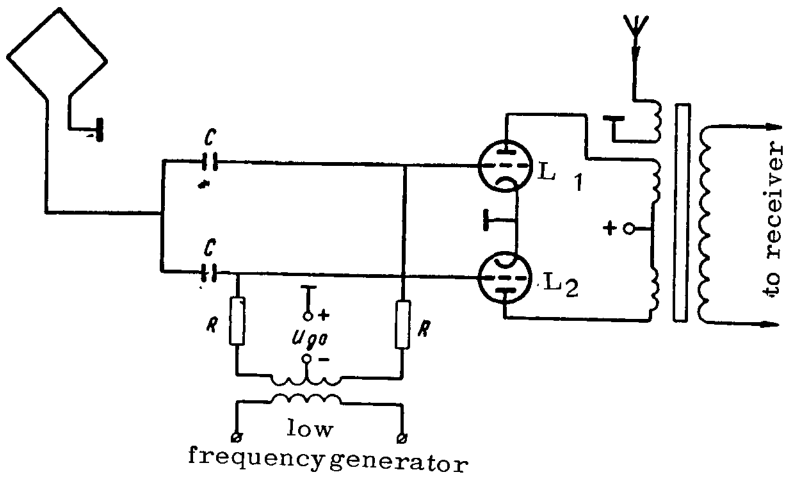


Fig. 3.16. Diagram of Amplitude Modulator at the Output of the Radiocompass Receiver.

In contrast to the amplitude ground radio direction-finders which we have discussed thus far and which belong to the "E" type (carrier-wave amplitude), radiocompasses presently use the method of amplitude modulation of the received signals (direction-finder type "M").

The essence of the method is that reception of the signals takes place simultaneously with an open and a frame antenna, with the phase of the frame antenna being constantly switched by the low-frequency generator. This means that an amplitude-modulated signal is obtained at the input of the receiver.

A simplified diagram of the amplitude modulator at the input of the receiver is shown in Figure 3.16.

The control grids of L_1 and L_2 receive a negative voltage u_{g0}, so that when the low-frequency generator is turned off, these tubes will be closed and the signals from the frame antenna will not be passed.

When the low-frequency generator is turned on, tubes L_1 and L_2 open alternately, and the signal from the frame antenna reaches /265
the input of the receiver in phases which are separated by 180°, and when these are combined with the signals from the open antenna, they undergo amplitude modulation.

Obviously, depending on the direction of the radio station (Fig. 3.17), with a fixed radiocompass frame, the amplitude modulation can be positive (Position 1), zero (Position 2), and negative (Position 3).

The tracking system at the output of the receiver is designed so that the frame of the radiocompass rotates in the direction which will produce a zero modulation of the signal.

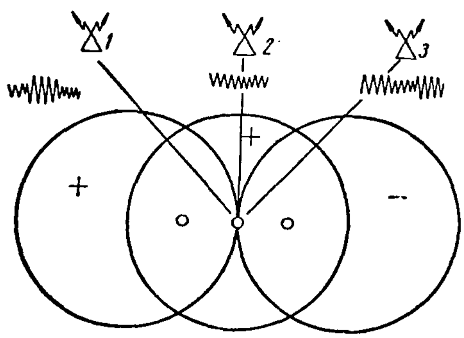


Fig. 3.17. Zero, Positive, and Negative Modulation.

A diagram of the output section of the receiver is shown in Figure 3.18.

A reference voltage on the anodes of tubes L_1 and L_2, formed previously in the positive half-periods of the rectangular pulse, is supplied to the switching circuit of the frame antenna at the input of the receiver. If the input signal is modulated by the frame signal, the average anode current of one of the tubes will be greater than that of the other. This produces a disturbance of the balance of the bridge circuit in the magnetic amplifier, made of permalloy cores, and a current passes through the rotor winding of a small motor. The stator winding of the motor is constantly supplied with a voltage which is shifted 90° in phase by capaciitor C, from a low-frequency generator which supplies the bridge circuit.

The motor will continue to rotate until the direction of the radio station is no longer perpendicular to the frame of the radiocompass, and the modulation of the signal of the open antenna by the frame becomes zero.

In the case when the frame antenna is turned toward the radio station in the opposite plane, the phase of the frame changes by 180°. In this case, in the presence of modulation, the rotation of the frame will take place not in the direction of reduction, but initially in the direction of increase of modulation, thus causing the frame to turn through 180°. In this manner, the readings from the radiocompass are all given the same sign.

A block diagram of the radiocompass is shown in Figure 3.19.

The controls for the radiocompass are mounted on a special control panel. Usually, the radiocompass has three operating regimes (besides the "off" position), so that a selector switch is mounted on the panel.

I. **Tuning.** In this regime, only the open antenna of the radio /266 compass is connected. A special vernier on the control panel is used to tune the device to the frequency of the ground radio station, either by ear or by a visual tuning indicator. When tuning by ear, reception takes place in the "telegraph" regime, i.e., the second heterodyne of the receiver is turned on to convert the intermediate frequency of the receiver to sound. In the telegraph regime, the call letters of the radio station are also heard, if the station is transmitting on a non-modulated frequency.

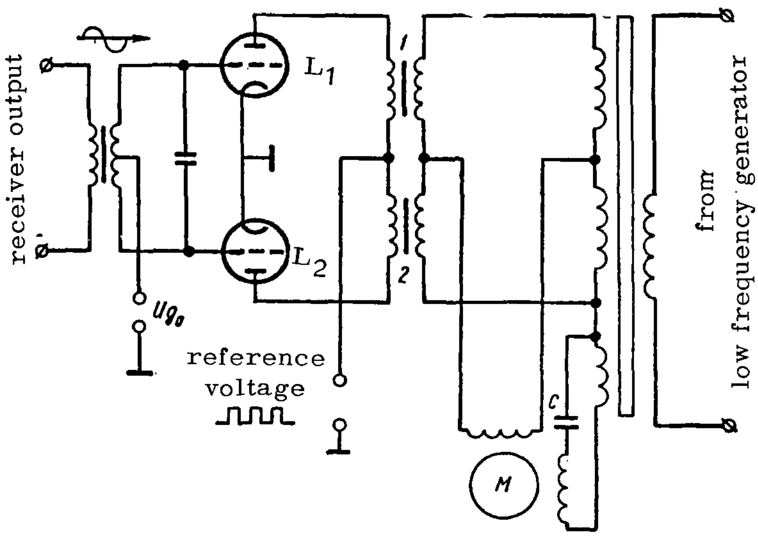


Fig. 3.18. Diagram of Output Section of Radiocompass Receiver.

II. **Compass regime.** In this regime, both the open and frame antennas of the radiocompass are connected. In this case, the tracking system of the receiver turns the frame antenna depending on the direction of the radio station and the direction of the radio station is shown on an indicator (course angle or bearing).

III. **Frame regime.** In this regime, only the frame antenna of the radiocompass is connected, and the bearing of the radio station can be determined with minimum audibility of its signals in the telegraph regime. The rotation of the frame is carried out by means of a special pushbutton switch on the control panel with the label "left-right". The reading of the bearing in this case has two signs.

Recent models of the ARK-11 automatic radiocompass do not differ /267 in their principle of operation from the operating principle described above for the ARK-5 radiocompass, but they have several design features and advantages:

(a) Complete electrical remote control.

(b) Possibility of setting the apparatus to nine previously selected channels (frequencies) in the range from 120 to 1340 kHz and switching from one receiver channel to another by means of an automatic pushbutton switch, located on the control panel. There is also a provision for smooth manual setting over the entire operating range of the radiocompass (with the tenth button depressed).

(c) Increased noise stability of the receiver.

(d) Possibility of operation in combination with a non-controlled antenna of open type with a low aerodynamic resistance and a low operating altitude (on the order of 20 cm).

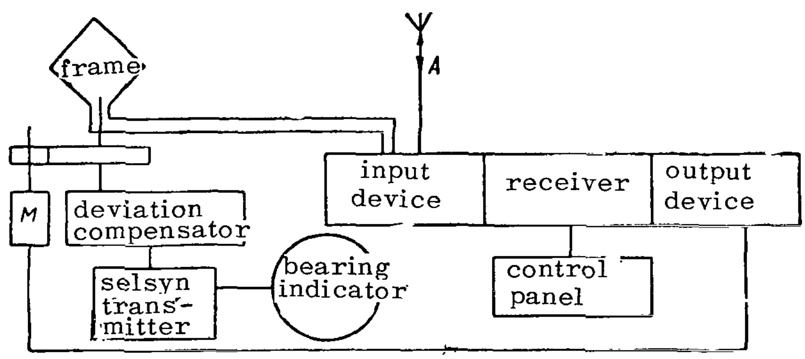


Fig. 3.19. Functional Diagram of Radiocompass.

The control panel of the ARK-11 differs in design from that of the ARK-5. In addition to the "off" position, there are four operating regimes. The first three regimes are the same as described above. The fourth regime "Compass II" is a spare and is used in the case of intense electrostatic noises when the usual distance-finding methods become unstable.

In the "Compass II" regime, instead of the open antenna, a second frame antenna is used, mounted on a common frame-antenna block, perpendicular to the basic frame and forming a unit with the basic frame.

The reference signal in this case reaches the input of the receiver not from the open but from the additional frame antenna, which is less sensitive to noise. However, the additional frame antenna which has the same properties as the main antenna, changes the phase of the reference signal by a further 180° when it is turned through 180°, so that both positions of zero reception of the main frame antenna will be positions of stable equilibrium, and consequently it is possible to have an error in determining the course angle of the radio station of 180°.

The control panel of the ARK-11 has a toggle switch for narrow and wide frequency bandpass: "wide-narrow". In the "narrow" position, the extraneous noises in the earphones are reduced and the desired radio station can be heard more clearly. /268

Other control units on the ARK-11 panel (subrange switch, knobs for coarse and fine setting, toggle switches and buttons) have the same markings as in the ARK-5.

Radiocompass Deviation

Conditions for directional reception of electromagnetic waves on an aircraft are not favorable and depend on the direction of propagation of the wave front in both the horizontal and vertical planes.

If the reception of signals from a ground radio station is being made at considerable distances which exceed 5-6 times the flight altitude, the vertical component of the vector of propagation of the wave front has less of an effect on the reception conditions. In this case, we can use a compensated curve of radio deviation, which is a function only of the course angles of the radio station.

The reason for the radio deviation is a reflection of electromagnetic waves from the surface of the aircraft or their re-reflection from individual parts of the aircraft. Since the radio compass frame is mounted in the plane of symmetry of the aircraft X-Z, the deviation at course angles zero and 180° is close to zero.

The transverse plane of the aircraft Y-Z is also close to the plane of symmetry, so that the deviation at course angles 90 and 270° is not great and passes through zero at course angles close to it.

The maximum asymmetry of the aircraft takes place relative to the directions 45, 135, 225 and 315°. Therefore, the radio deviation at these course angles reaches a maximum.

Hence, the curve of radio deviation has a quarternary appearance (Fig. 3.20) with extreme values $\Delta P = \pm$ 12 to 25° depending on the type of aircraft.

Radio deviation is compensated by a mechanical compensator located on the axis of rotation of the frame antenna. The compensator has a control strip which produces an additional revolution of the axis of the master selsyn by means of a special transmission. The required shape is given to the control strip by means of 24 compensating screws to set the readings for the radiocompass at 15° intervals on the scale from zero to 360°.

Before the first determination of radio deviation, the compensator is usually neutralized, i.e., each of the screws is unscrewed to such a position that the control strip has a shape with the correct curvature and the additional r-+ation of the axis of the master selsyn is equal to zero at all course angles. /269

To determine radio deviation, a ground radio station is selected (preferably at a distance of 50-100 km from the airport) and the true bearing is measured as accurately as possible on a large-scale chart (usually 1:500,000), and then the magnetic bearing of this radio station (MBR) is determined.

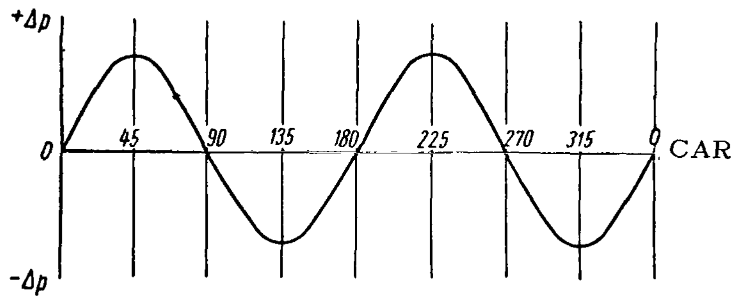


Fig. 3.20. Graph of Radio Deviation.

By means of a deviation distance finder, magnetic bearings of one or two separate landmarks (MBL) are measured from the center of the area where the radio deviation will be plotted, in the way which was described in Chapter II, with a description of their deviation of the magnetic compasses. If the area for the deviation operations at the aerodrome is constant, the MBL will be known earlier.

The aircraft is then rolled out on the runway. The deviation distance finder is installed in the aircraft in a line 0-180° exactly along its longitudinal axis, and the course angle of the landmark (CAL) is calculated to get rid of installation errors in the radiocompass. The corresponding CAR = 0:

$$CAL = MBL - MBR.$$

In the deviation distance-finder, the level line is set to the calculated CAL, and the aircraft is turned until the sight line of the distance-finder coincides with the direction of the selected landmark. In this case, the longitudinal axis of the aircraft will be lined up exactly with the radio station (CAR = 0), and the magnetic course of the aircraft will be equal to the magnetic bearing of the radio station as measured on the chart (MBR).

Turning on the radio compass and setting it to the desired radio station, the reading of the radiocompass is taken (RRC). If RRC is not equal to zero, we will have the installation error of the frame:

$$\Delta_{est} = CAR - RRC.$$

Then, without turning off the radiocompass, it is necessary to loosen the fastening screws which hold the frame to the fuselage and then (by turning the base of the frame) adjust it until the indicator points to /270

$$RRC = CAR = 0,$$

after which the frame is re-fastened to the fuselage.

The remaining installation error, if RRC is not equal to zero after the frame has been fastened down, can be compensated for immediately either by the navigator or the pilot by turning the body of the selsyn relative to the indicator scale.

After compensating for the installation error, the radio deviation is determined successively at 24 RRC's at 15° intervals. To do this, it is necessary to set the sight line of the deviation distance-finder along the longitudinal axis of the aircraft to 0-180°, loosen the dial of the deviation distance-finder and move it so that the line of sight 0-180° passes through the selected landmark, and then fasten the scale of the distance-finder once again. In this case

$$CAR = 0 \text{ and } MBR = 0.$$

The deviation distance-finder mounted in this manner makes it possible to calculate the course angles of the radio station (CAR) on the scale dial by turning the aircraft to any angle.

Consequently, if we turn the aircraft according to the indications of the radiocompass to a RRC = 15°, and then to 30, 45, 60°, etc., successively (setting the sight system of the deviation distance-finder each time to a selected landmark), we can calculate the CAR immediately from the scale on the dial.

Thus, in each reading of the radiocompass, we determine the actual course angle of the radio station and can write the radio deviation as follows:

$$\Delta_r = CAR - RRC.$$

Compensation of radio deviation is performed after it has been determined. To do this, the graph of radio deviation is plotted and the extreme values of the graph are divided into three equal parts to avoid sharp bends in the strip, after which two intermediate graphs of radio deviation are plotted.

The compensator is then removed from the axis of the frame; by turning the proper screws, compensation is made for the radio deviation in terms of the first intermediate graph, calculating the correction made in the selected portion of the radio compass by means of a special pointer on the compensator. Then the deviation is compensated by the second intermediate graph, and finally by the curve of radio deviation.

Compensation for radio deviation by all three graphs is performed in an order such that after each introduction of a positive correction there is a correction of equal magnitude but negative, i.e., with a mirror image of the course angles. Usually, the order of compensation is selected as follows: 0, 15, 345, 30, 330, 45, 315, 60, 300, 75, 285, 90, 270, 105, 255, 120, 240, 135, 225, 150, 210, 165, 195 and 180°. /271

After compensation for radio deviation, the compensator is mounted on the mechanism of the frame; the aircraft is turned and the deviation distance-finder is used to check the correctness of the operations which have been carried out. If any errors in compensation are discovered, the radio deviation is compensated once again by an additional turning of the screws corresponding to the readings of the radiocompass.

In addition to the method described above for correcting radio deviations on the ground, there are others. For example:

(a) Determination of the magnetic course of an aircraft by distance-finding at the tail (nose), as described in Chapter II, and the calculation of course angles of the radio station on the basis of it.

(b) Range finding of a radio station which is visible from the airport (e.g., a distant power radio station).

In aircraft where the frame antenna of the radio compass is mounted below the fuselage, determination of radio deviation on the ground is impractical, since the reflection of electromagnetic waves from the surface of the ground causes a distortion of the electromagnetic field. In these aircraft, the radio deviation is determined in flight.

To determine radio deviation in flight, a radio station located 200-300 km away from the flight area is selected. The flight is carried out in such a way that the aircraft crosses the line of the given bearing at each segment of the flight angle of the radio station. Usually, the order of the course angles is then selected so that the following compensation mechanism is employed in compensating for radio deviation: 0, 15, 345, 30, etc., approaching the radio station, and 105, 255, 120, 240, etc., up to 180°, going away from it.

To save time, the flight can be carried out over a 24-angle route, i.e., practically along a course which crosses the straight-line flight for 20-30 sec for each recording of the readings of the radio compass and course. However, in this case, it is necessary to determine the location of the aircraft at each point being measured and to enter it on a chart so that when the data is analyzed it will be possible to determine the bearing of the radio station from the point at which the reading was taken.

In fact, the course angle of the radio station (CAR) at the moment when the recordings are made is determined by the formula

$$CAR = TBR - TK,$$

and the radio deviation of the radio compass is determined as the difference:

$$\Delta_r = CAR - RRC.$$

Compensation for radio deviation is made after the aircraft lands in the same way as after determining it on the ground, but without checking the accuracy of the work which has been carried out, since this would require repetition of the flight.

Aircraft Navigation Using Radiocompasses on Board the Aircraft /272

Radiocompasses on board the aircraft make it possible to solve the same navigational problems as ground radio distance-finders.

(a) Path control in terms of direction and selection of the course to be followed during flight toward the radio station and away from it.

(b) Measurement of the drift angle after flying over the radio station.

(c) Checking the path for distance by measuring the distance to a radio station located to the side.

(d) Determination of the position of the aircraft by obtaining bearings from two radio stations.

(e) Determining the drift angle and the groundspeed from successive positions of the aircraft, as well as the wind parameters at flight altitude.

The solution of these problems by means of a radiocompass mounted on board the aircraft is very similar in principle of solution to the ground radio distance-finders, especially if the indicator for the course angles of the radio compass is combined with the course indicator of the aircraft and thus shows the reading for the bearing (Fig. 3.21).

The figure shows the course indicator for the navigator, combined with the bearing indicators of two radio compasses (USh-M). The course of the aircraft is measured on the inner, movable scale of this indicator (relative to a triangular mark on the outer scale), while the course angles of the radio stations are indicated on the outer fixed scale according to the position of the pointers of the radiocompasses. On the inner, movable scale, opposite the arrows, it is possible to calculate the bearings of the radio stations, while the other ends of the pointers can be used to show the bearings of the aircraft.

However, this method is practical only for use with one radiocompass, since the total correction is then shown on the scale of deviations, and is effective only for one radio station

$$\Delta = \Delta_M + \delta,$$

where δ is the difference between the meridian of the aircraft location and the meridian of the radio station, and Δ_M is the magnetic declination of the location of the aircraft.

Obviously, in the general case this correction will be different for different radio stations.

When it is necessary to obtain the true bearings of an aircraft simultaneously from two radio stations, only the magnetic declination of the position of the aircraft is set on the declination scale, and after calculating the approximate bearings of the aircraft by the opposite ends of the pointers of the radiocompasses, corrections for deviation of the meridians are made in these readings.

The necessity to make corrections for the deviation of the meridians is one of the principal shortcomings of radiocompasses. /273
This shortcoming to a certain degree can be reduced by using an orthodromic system for estimating the path angles and courses of the aircraft. In this case, the need to introduce corrections is no longer applicable, if the radio station is located on the reference meridian for computing the path angles, and in any case the correction remains constant if this condition is not observed.

The use of combined indicators considerably simplifies the operations related to the use of radio compasses mounted on board aircraft. Therefore, the methods of using them for navigational purposes must be viewed as non-recorded indicators of course angles of radio stations, assuming that in the combined indicators, the addition and subtraction of the angles according to those same rules is carried out automatically.

It is clear from Figure 3.22 that the magnetic bearing of the radio station (MBR) and the true bearing of the radio station (TBR) are added from the course of the aircraft in corresponding systems of calculation and the course angle of the radio station:

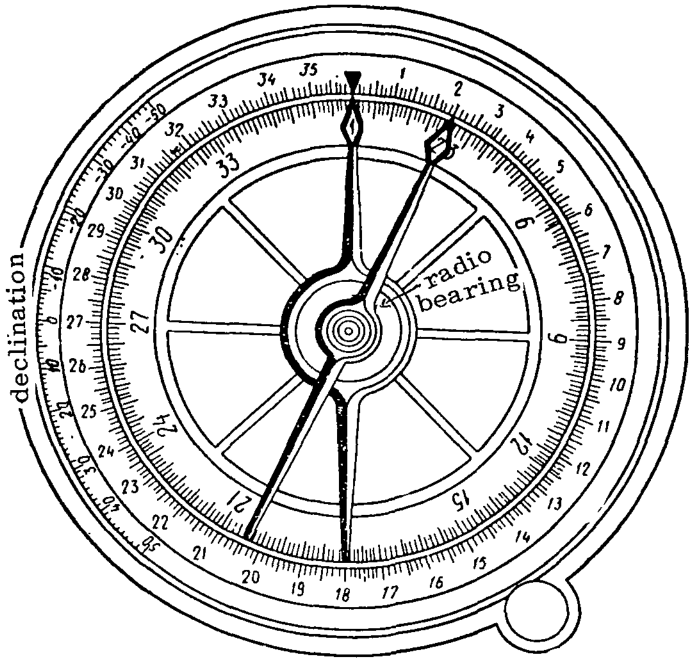


Fig. 3.21. Combined Indicator for Course and Course Angles of a Radio Station.

TBR = TC + CAR; /274

MBR = MC + CAR.

Similarly, in the orthodromic system of calculating courses

OBR = OC + CAR.

where OBR is the orthodromic bearing of the radio station and OC is the orthodromic course.

In determi-ing the location of the aircraft, the true bearings of the aircraft (TBA) are plotted on the flight chart from ground radio stations.

$$TBA = TBR + \delta \pm 180°,$$

where δ is the angle of convergence of the meridians.

In calculating the true bearing of the aircraft, 180° are added if the TBA has a numerical value less than 180° and subtracted when the value of the bearing exceeds 180°.

Consequently, in calculating courses from the truė meridian of the aircraft's location, where

$$TBA = TC + CAR + \delta \pm 180°,$$

$$\delta = (\lambda_{r.s.} - \lambda_a)\sin\phi_{av},$$

and in calculating the course from the magnetic meridian

$$TBA = MC + \Delta_M + CAR + \delta \pm 180°.$$

It is assumed that the LA is determined with the aid of magnetic compass deviation.

In the orthodromic system of calculating courses,

$$TBA = OC + CAR + \delta_{r.m.r.s.} \pm 180°,$$

where $\delta_{r.m.r.s.}$ is the angle of convergence of the meridians (reference and radio station) which is equal to $(\lambda_{r.s.} - \lambda_{r.m.})\sin\phi_{av}$.

Let us consider the means of solving problems by means of radiocompasses located on board aircraft, taking the rules mentioned above into account.

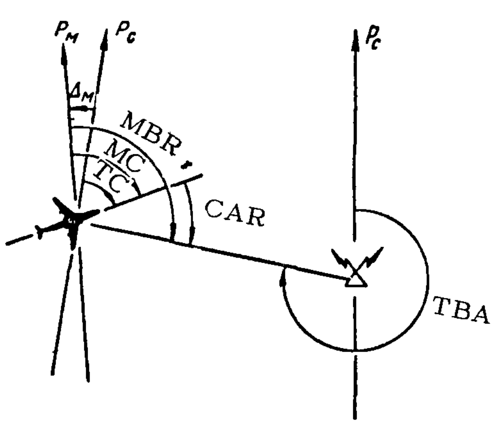


Fig. 3.22. Bearings of Radio Station and Aircraft.

Obviously, a flight along a given line of flight from a radio station or to a radio station must take place with a constant true bearing of the aircraft. In a flight from a radio station, this bearing is equal to the azimuth of the orthodrome relative to the meridian of the radio station. However, if the flight takes place in a direction toward the radio station, then the true bearing of the aircraft differs from the azimuth of the orthodrome of the meridian of the radio station by a value equal to 180°. /275

Controlling the path in terms of direction during flight toward a radio station and away from a radio station involves comparison of the actual true bearing of the aircraft with the given value.

For example, in calculating the course from the magnetic meridian of the aircraft's location, we must satisfy the equation:

$$\alpha_{init} = MC + \Delta_M + \delta + CAR;$$

$$\alpha_{ref} = MC + \Delta_M + \delta + CAR \pm 180°,$$

where α_{init} is the true bearing of the aircraft and α_{ref} is the true bearing of the radio station.

In using a combined indicator for the bearing, the total correction $\Delta = \Delta_M + \delta$ can be entered on the scale of declination. Then it is necessary to satisfy the condition that α_{init} = TBA and that α_{ref} = TBA $\pm$ 180°.

The true bearing of the aircraft is calculated from the other end of the indicator pointer, i.e., in flight toward a radio station, the direct reading of the pointer on the radiocompass must be equal to the final azimuth of the orthodrome, while in flight away from a radio station the other end of the pointer must show a reading equal to the initial azimuth of the orthodrome.

Inasmuch as the total correction (Δ) over the length of the path segment will change constantly, it is necessary to determine this correction in each measurement in order to take into consideration other indicators or entering the declinations of the combined indicator on the scale. This poses considerable difficulty in using the radiocompass in flight.

The problem is simplified considerably in an orthodromic system of calculations for the aircraft course. In this case

$$TBA = OBA + \delta_{r.m.r.s.},$$

where OBA is the orthodromic bearing of the aircraft.

The correction $\delta_{r.m.r.s.}$ is constant for all straight-line path segments; after setting it on the scale of deviations, the TBA can be read off immediately on the indicator over the entire straight-line path segment.

Note. If a radio station is located at the starting point of the route (SPR) with a reference meridian, the flight can be made directly along the orthodromic bearing of the aircraft with a zero correction on the declination scale. Correction for deviation of meridians is only valuable when the radio station is located to the side of the flight route or the meridian of the radio station does not coincide with the reference meridian.

As in the case of using a ground radio direction-finder in selecting the course to be followed along straight-line segments of a path by means of a radiocompass, it is necessary to be guided by the following rules: /276

(a) In a flight from the radio station, with a drift of the aircraft to the right, the bearing is increased and the course to be followed must be reduced; with a decrease in the bearing, the course must be increased.

(b) In the case of a flight toward the radio station, the course must be increased when the bearing increases and decreased if the aircraft bearing decreases.

In a flight along a course determined by a radio direction finder, if the course to be followed is selected on the basis of stable bearings ShchDR or ShchDM, the course is considered to have been selected by using the radiocompass on board if the course angle of the radio station remains constant.

For example, in a flight toward a radio station at a constant course of the aircraft, an increase in the course angle of the radio station corresponds to a drift of the aircraft to the left (the TBA increases). In order for the aircraft to follow a constant bearing, the course of the aircraft must be increased. When the lead in the course is equal in value to the drift angle of the aircraft, the course angle of the radio station will remain constant and equal to the drift angle both in value and in sign.

In selecting a course, the same method of half correc tion is used.

Example. It is necessary to make a flight toward a radio station with a orthodromic path angle of 82°. Let us assume that the drift of the aircraft will be to the left within limits of approximately 10°. Select a course to be followed by using the method of half correction.

In this case, after flying over the turning point in the route, it is necessary to assume an orthodromic course of 92°. The course angle the of radio station will then be equal to 350°, which is equivalent to a numerical value of -10°.

In a flight with a course of 92°, if the course angle of the radio station is increased (i.e., if we acquire the values of 351, 352, 353° in succes sion), it is necessary to place the aircraft on the line of flight and to take a lead of 15° (course equals 97°, CAR equals 345°).

Let us assume that this lead turns out to be too great; then the CAR begins to decrease, taking on values of 344, 343, and 342° Then, after a second placing of the aircraft on the path, it is necessary to take an intermediate lead in the course of 12-13° (CAR equals 348-347°). If the CAR is to be stable, it is necessary to ensure that the orthodromic bearing of the radio station is equal to the orthodromic path angle and to continue the flight with the selected course.

The course to followed in a flight away from a radio station is selected in the same manner, the only difference being that when the CAR increases, it should not be reduced but increased further.

In using other indicators, the selection of the course is also accomplished by means of stable path angles of radio stations. However, in order to control the path of the aircraft in terms of direction, it is necessary to determine on each occasion the bearing of the aircraft or the radio station by summing the course and course angles of the radio station, taking into account the deviation of the meridians of the radio station and aircraft and the magnetic declination at the point where the aircraft is located.

It should be mentioned once again that in a flight toward a radio station, the selected stable course angle of the radio station is always equal to the drift angle of the aircraft, regardless of whether the aircraft is located on the line of the desired flight or as a slight deviation from it. For example, a stable TAR = 350° corresponds to a drift angle of -10°. In flight away from the radio station, the drift angle is always equal to a stable CAR minus 180°. /277

The drift angle of the aircraft can be measured directly after flying over the radio station.

At the same time, after flying over the radio station with any constant course, the course angles of the radio station will be stable, so that

$$US = CAR - 180°.$$

However, in the majority of cases the drift angle is determined during flight away from a radio station as a difference between the magnetic (true or orthodromic) bearing of the aircraft and the magnetic (true or orthodromic) course of the aircraft:

$$US = MBA - MC,$$

or

$$US = TBA - TC,$$

or

$$US = OBA - OC,$$

where OBA is the orthodromic bearing of the aircraft and the OC is the orthodromic course of the aircraft.

In this case, we can simultaneously determine the side to which the aircraft deviates (left or right) by comparing the given path angle and the determined range of the aircraft in the system of coordinates being used; the aircraft acquires the given line of flight according to the calculated course angle of the radio station, while on the line of flight corrections are made to the course which are equal to the average angle of drift.

The monitoring of the aircraft path in terms of distance by means of the radiocompass is accomplished with previously calculated bearings of the lateral radio station.

To do this, control landmarks are marked on a map and the bearings of radio stations are determined from these landmarks. For the sake of convenience in calculation, the bearings of the radio stations are determined in the same system in which the course of the aircraft is measured. For example, for flight with magnetic path angles it is MBR and with true flight angles it is TBR, while with orthodromic angles it is OBR. Then the bearing of the radio station will consist of only two components, the aircraft course and the course angle of the radio station; on the other indicators, it can be estimated directly without any corrections from the scale of the instrument.

In approaching a control landmark, the readings for the course and the course angle of the radio station are observed. At the moment when the sum of the aircraft course and the course angle of the radio station become equal to the previously calculated bearing (in combined indicators, the bearings of radio stations become equal to the previously calculated value), the moment for flying over the landmark is determined. /278

With the aid of a radiocompass located on board, it is also possible to determine the location of the aircraft on the basis of true bearings from two radio stations. However, the accuracy of determining the aircraft location by this method, involving considerable difficulty in the process, is insufficiently high. Therefore, the method is not widely employed in aircraft navigation, being used only for determining approximate aircraft coordinates in finding lost landmarks.

The essence of the method is the following: when two radiocompasses are on board, one is set to the frequencies of two ground radio stations, located no more than 180-200 km from the aircraft. It is desirable when doing this to ensure that the bearings of these radio stations cross at an angle close to 90°.

If the indicators of the radiocompasses do not agree, the course of the aircraft, the course angles of the two radio stations, and the distance-finding time must all be described simultaneously for a given moment of time. Then the approximate true bearings of the aircraft are determined:

$$TBA_1 = MC + \Delta_M + CAR_1 \pm 180°;$$

$$TBA_2 = MC + \Delta_M + CAR_2 \pm 180°.$$

The bearings which have been obtained are plotted on the flight chart from the meridians of selected radio stations by means of a protractor and scale rule.

Having thus determined the approximate position of the aircraft, we can find its true bearing by introducing the precise value of the magnetic declination and making corrections to the deviation angles of the meridians of the radio station and the aircraft location. These corrected bearings are again plotted on the chart to give a precise position of the aircraft at the moment of direction finding.

If only one radiocompass is mounted on board the aircraft, it is necessary to consider its path when determining the location of the aircraft for the time between the moments of direction finding, and this is done as follows (Fig. 3.23).

After determining the average bearings of the aircraft, the latter are plotted on a chart, and then the flight path of the aircraft is obtained from the point of location of the first radio station for the time between the measurements of the course angles of the radio station in a direction which coincides with the course of the aircraft. A line is drawn through the point which is obtained, parallel to the first bearing up to the intersection with the line of the second bearing, defining the position of the aircraft at the moment of direction finding for the second radio station. In addition, the true bearings of the aircraft are found in the same way as in the case of two radiocompasses.

The laboriousness of the process of determining the position of the aircraft is considerably relieved if the flight is made with orthodromic courses, but the indicators of the radio compasses must match. In this case, the angles of convergence of the meridians /279
of the radio stations with the reference meridian for calculating the course are determined beforehand.

At the time of measurement, the angle of deviation of the meridians for the first and then the second radio station are entered on the scale of deviations of the indicator in succession. They are calculated from the readings of the opposite ends of the pointers of the radio compasses and are designated as TBS_1 and TBS_2. The bearings obtained are final and no corrections are required.

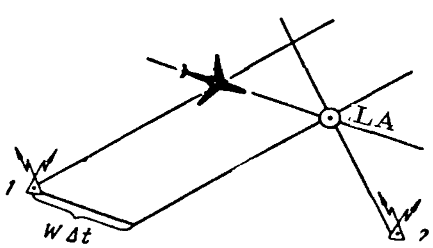


Fig. 3.23. Diagram for Locating the Position of an Aircraft from the Bearings of Two Radio Stations.

As we have already mentioned, when using radiocompasses mounted on board aircraft, the drift angle of the aircraft is determined from the stable course angles of the radio stations or measured after flying over a radio station.

The ground speed of the aircraft is determined by checking the flight in terms of distance by means of radio stations located to the side of the course or from the moments when the aircraft passes over radio stations. The latter method is not accurate, especially when flying at high altitudes, due to the error in the readings of radiocompasses when flying over radio stations.

The determination of the wind at flight altitude is accomplished on the basis of the ground speed of the aircraft, the airspeed, and the drift angle by the same methods as for ground radio distance-finders. The method of determining the wind by using the successive positions of the aircraft, obtained by distance measurement from two radio stations, usually is not used due to the inadequate precision of the determination of the aircraft location.

Special Features of Using Radiocompasses on Board Aircraft at High Altitudes and Flight Speeds

High altitudes and flight speeds cause deterioration of the conditions for using radiocompasses aboard aircraft for purposes of aircraft navigation.

The use of radiocompasses and the observation of all rules for retaining accuracy of distance finding is a laborious process, so that the increase in flight speed, calling for operativeness of navigational calculations, creates difficulties in using radiocompasses on board the aircraft.

This shortcoming can be largely overcome by using combinations /280
of bearing indicators, especially in the orthodromic system of calculating aircraft courses.

In addition, another shortcoming of aircraft radiocompasses which operate on medium and short waves, due to the increased speed of flight, is the effect of electrostatic noise on their operation.

At high airspeeds, especially in clouds and in precipitation, a considerable electrification of the aircraft surfaces occurs. Static electricity, emitted at pointed portions of the aircraft (including open antennas) creates noise and radio interference in the frequency range at which radiocompasses operate. Despite the measures which are taken to prevent the charges from flowing by using special discharge devices, as well as shielding the open antennas of the radiocompasses, this shortcoming can be overcome only partially and manifests itself in very difficult flight conditions.

High flight altitudes have an effect mainly on the accuracy of operation of radio compasses and especially on the accuracy of determining the moment when the aircraft flies over a radio station.

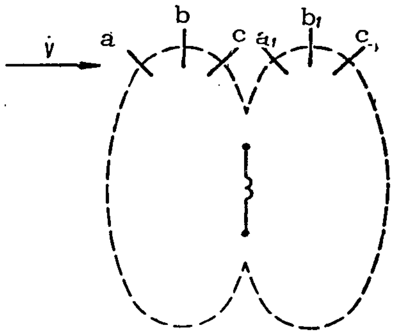


Fig. 3.24. Operation of an Open Antenna When Flying Past a Radio Station.

The decrease in the accuracy of operation takes place due to the change in the nature of radio deviation at different angles of deviation of the propagation vector of radio waves. The latter changes within wide limits when the aircraft approaches the location of a radio station.

A diagram of the appearance of errors in determining the moment when the aircraft flies over a ground radio station is shown in Figure 3.24 where there is a picture of the electrical field radiated by an open vertical antenna on a ground radio station.

At large distances from the radio station, the electromagnetic wave is vertically polarized. However, there is a space near the radio station and above it where the polarization shifts to the horizontal,then back to the vertical but in opposite phase.

Let us assume that an open antenna of the radiocompass is tilted backward (position a, Fig. 3.24) and the aircraft is approaching the radio station at a high flight altitude in the direction of vector $\overline{V}$.

Obviously, at position a the antenna will have zero reception. The reception of the antenna will then increase, but in a phase which is opposite to the reception up to the point a. This leads /281
to a rotation of the radiocompass frame by 180° until the aircraft passes a radio station. Then, after passing the station, the phases of both the frame and open antennas change almost simultaneously (at the point a_1).

Thus, the change in the readings of the radio compass by 180° takes place until the moment when the aircraft passes over the radio station (at point a) and only the oscillation of the needle will be observed from then on.

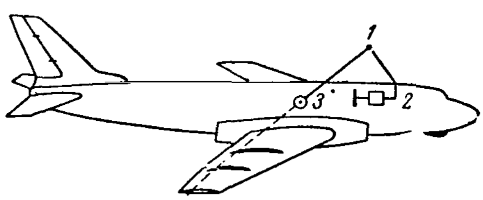


Fig. 3.25. Equivalent of an Open Antenna on Board an Aircraft.

When the antenna is tilted forward (position c), the oscillations of the radiocompass needle will begin at point c, while the passage by the radio station with rotation of the needle through 180° will be noted at point c_1, i.e., there will be a delay in marking the passage.

For a strictly vertical antenna (position b), the movement of the needle through 180° can take place prematurely. Then the pointer can make a reverse turn and again show the passage by the radio station at point b_1.

It should be mentioned that an equivalent to the open antenna of the radiocompass in terms of its inclination in the vertical plane is the resultant combining the upper or lower points of the antenna with the electrical center of the aircraft, constituting its grounding or counterweight (Fig. 3.25).

In this figure, point 1 is the top of the open antenna, point 2 is the receiver and point 3 is the electrical center of the aircraft.

Obviously, straight line 1-3 is the equivalent of the inclination of the open antenna, which is forward in this case. Thus, the setting of the open antenna above the fuselage in its forward section causes a delay in the reading of the moment when the radio station is passed.

Mounting of the antenna in the same position with respect to the center but below the fuselage leads to a preliminary reading of the moment when the aircraft passes the radio station. The opposite picture is observed when the antenna is mounted behind the electrical center of the aircraft.

The best place to mount the antenna is above or below the electrical center of the aircraft, but in this case an advance or delay in the readings is observed; in some cases, these deviations depend on the height and speed of flight, but there can also be a double reading involving both an advance and delayed indication.

This system for the creation of errors in measuring a flight only approximately reflects the reasons for these errors. In practice, they will depend both on the angle of pitch on the aircraft and on the accuracy with which the passage of the aircraft over the radio station is determined.

For example, if the aircraft is passing a radio station to the side, then obviously there will not be an indication of passage with movement of the needle through 180°, but a deterioration in the passage over the radio station, i.e., errors in determining the passage of the traverse of the radio station will be practically non-existent. /282

Usually, the exact passage of an aircraft over a radio station occurs only in special and exceptional conditions. Therefore, in practice there is always a consideration of the effect of passage with the effect of error, which does not make it possible to consider the magnitude of the delay advance in marking the passage.

Depending on the type of aircraft and the flight conditions, these errors can occur within limits equal to 1-3 flight altitudes, excluding the case of exact determination. However, exact determination can occur at distances which exceed the flight altitude of the aircraft, beyond the limits of a zone with horizontal polarization, i.e., at very considerable deviations of the aircraft from the given line of flight.

Details of Using Radiocompasses in Making Maneuvers in the Vicinity of the Airport at Which a Landing is to be Made

The maneuver of approaching for a landing usually begins at a relatively low flight altitude (1200-4000 m) with a gradual reduction of the airspeed. Therefore, the effects related to height and flight speed in this case are considerably reduced.

Unlike a flight along the route, where the main operation in aircraft navigation involving the use of radiocompasses is measuring of bearings, the work is shifted when maneuvering the aircraft mainly to measuring course angles of the radio stations at individual points along the maneuver. This is profitable in this respect: regardless of the orientation of the landing strip, and consequently of the course of the aircraft at different stages of the maneuver, the system for using radio compasses is based on two or three standards which are used at all airports.

Of course, if there is a tendency to drift in the aircraft course, the corresponding corrections must be made in the readings of the course angle of the radio station which are equal in magnitude and sign to the lead which has been taken.

The effectiveness of using radiocompasses in the vicinity of airports where landings are to made is increased also by the fact that the flight is made at short distances from ground radio stations, which gives relatively small linear errors in determining the position of the aircraft in view of the errors already committed in measuring the course angles of the radio station. In addition, it is no longer necessary to calculate the magnetic declination and the deviation angles of the meridians. The accuracy of aircraft navigation in the vicinity of the aerodrome, using radiocompasses, is considered to be quite satisfactory in all stages of the maneuver with the following exceptions:

(a) Determination of the starting point of the maneuver by /283 flying past the power radio station, if the maneuver is beginning at a high altitude.

(b) On a landing strip, where it is necessary to have very high accuracy of flight along a given trajectory for bringing the aircraft in for a landing.

Ultra-Shortwave Goniometric and Goniometric-Range Finding Systems

As we have mentioned, radiocompasses have significant advantages over ground radio distance-finders with respect to uninterrupted visual information on board the aircraft regarding its position. This means that they have been very widely employed and are installed in practically all types of aircraft as a rule in a double set. In addition, there are a number of important shortcomings for radiocompasses mounted on board aircraft, which reduce the accuracy and feasibility of aircraft navigation.

In addition to the errors caused by the effect of the local relief, which affect all systems for short-range navigation, radiocompasses have the following shortcomings:

(a) Unfavorable conditions for directional reception of electromagnetic waves on board the aircraft (radio deviation of an unstable nature);

(b) An increase in the errors in distance finding due to inaccurate measurements of the aircraft course;

(c) The necessity to consider the deviation of the meridians and magnetic declinations when using magnetic compasses to determine bearings;

(d) The effect of static noise in the range of received radio frequencies at high airspeeds;

(e) The effect of flight altitude on the accuracy of measuring the range and determining the moment of flying over the radio stations.

In addition to these shortcomings, radiocompasses are subject to a general disadvantage of goniometric systems: the need to plot bearings on the chart from two ground points to determine the location of the aircraft.

Therefore it seems natural to try to build devices for short-range radionavigation which would have the advantages of the radiocompasses mounted on aircraft but would not have the shortcomings from which they suffer.

Such devices are the goniometric and goniometric-rangefinding systems which operate on ultra-shortwaves.

A common feature of these systems is the directional radiation of electromagnetic waves by ground instruments and their directional reception on board the aircraft. This feature, together with the range of waves employed, gives three very important advantages for navigational systems:

(1) It frees the system from radio deviations on board;

(2) The bearing of the aircraft becomes independent of the aircraft course, magnetic declination, and deviation of meridians; /284

(3) It sharply increases the freedom of the system from static and atmospheric interference.

There are several types of goniometric directional radio beacons and receiving devices to carry aboard aircraft, which operate on ultra-short waves.

The usual principle of operation for these systems is the rotation of the directional characteristic of the radiation of the transmitting antenna of a ground installation with its reception aboard the aircraft by an open (omnidirectional) antenna for a transmitter of reference signals, related to the passage of the characteristic of the antenna through the starting point for measuring bearings, e.g., through the northern direction of the magnetic or true meridian of the beacon.

Figure 3.26 shows the schematic diagram of a radio beacon with a rotating directional antenna. The generator of low frequencies produces a frequency which is synchronized with the rotation of the directional antenna. On the axis of rotation of the antenna is a special disk, which generates the reference signal related to the position of the rotating antenna. The reference signal passes through the modulator and transmitter to reach the open antenna of the radio beacon.

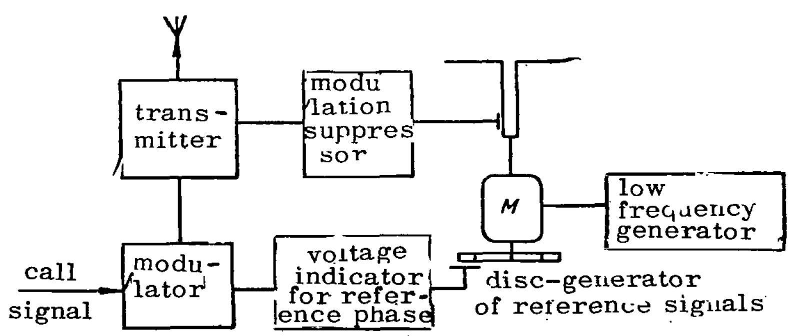


Fig. 3.26. Schematic Diagram of a Radio Beacon With Rotating Directional Antenna.

From the transmitter, the signal reaches the rotating antenna through a modulation suppressor, so that the amplitude of the signal radiated by the antenna in any given direction depends only on the position of the antenna relative to this direction. Consequently, the signal from a directional antenna is modulated by a low frequency whose phase relative to the relative signal is shifted through an angle equal to the azimuth of the aircraft.

Thus, two signals reach the aircraft on the carrier frequency in addition to the call-letter signals:

(1) Reference signal for beginning the reading. /285

(2) The signal from the directional antenna, whose amplitude maximum coincides with the moment when this antenna crosses the line to the aircraft.

The receiver on the aircraft has three channels (Fig. 3.27):

(1) Channel for picking up the call letters from the beacon (earphones);

(2) Reference-channel;

(3) Azimuthal voltage channel.

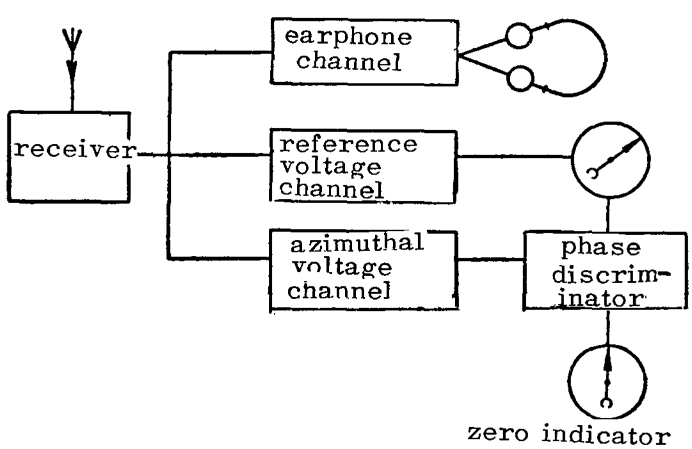


Fig. 3.27. Apparatus for Goniometric System Aboard an Aircraft.

The indicator mechanism is usually based on measurement of the phase ratio of the reference and azimuthal signals with low frequency by the compensation method, i.e., the phase of the reference signal is changed by an automatic phase shifter so that it coincides with the phase of the azimuthal signal. In this case, the signal on the phase discriminator will be equal to zero while the bearing indicator on the aircraft will act as the pointer of the phase shifter.

For flight along a given bearing, the phase shifter is set to a given position, so that the signal on the phase discriminator (and therefore on the zero indicator device) will be equal to zero only in the case when the aircraft is located exactly along the line of the desired bearing.

The goniometric system for short-range navigation, operating on ultrasonic waves, in the case when the characteristic of the rotating directional antenna has a sharply pronounced maximum (which usually is achieved by using reflex reflectors), can be built on the principle of time ratios rather than phase ratios.

In this case, the reference signal, when the rotating directional antenna passes through zero reading, has a pulsed character, /286 and the equipment on board must include a generator of a reference frequency as well as special delay devices to determine the bearing of the aircraft in time between the moments when the reference and azimuthal signals are received.

The geometry of navigational applications of USW beacons with directional radiation is exactly the same as the use of ground radio distancefinders. All the problems of aircraft navigation such as selection of the course to be followed, monitoring of the path for distance and direction, determination of the location of the aircraft from two beacons, measurement of the drift angle and ground speed, measurement of the wind at flight altitude, etc., are solved in exactly the same way as for ground radio distance-finders. In flight away from a radio beacon, the rules for flight along the ShchDR bearing are observed, while in flight toward a radio beacon it is the rules for bearing ShchDM which are followed.

If the flight is made using a zero-indicator instrument, the method of selecting the course in flight from the beacon and toward the beacon with a corresponding switch in the mode of operation of the receiver leads us to only one type: the pointer of the zero indicator shows the direction of deviation of the aircraft from the LGF.

The main difference between using USW beacons and ground radio distance-finders is only that in order to determine the location of the aircraft from two bearings using a radio direction-finder, the plotting of the bearings is done by the operator of the command distance finding station, while in the case of USW beacons it is done by the crew of the aircraft.

Nevertheless, goniometric USW beacons have a much wider range of application than ground radio-distance-finders and aircraft radiocompasses, thanks to the constant indication of bearings on board the aircraft.

The instrumental accuracy of goniometric USW navigational systems is higher than for ground radio distance-finders. The practical accuracy under average conditions of application is also somewhat higher or equal to the accuracy of distance-finders. However, in using radio distance-finders, it is possible to consider to a certain extent the influence of the local relief on the radius of application, which cannot be done for USW beacons. In this respect, the USW beacons have less favorable operating conditions than ground radio distance-measuring stations.

The operating range of a USW system S is limited by the limits of direct geometric visibility from the ground beacon to the aircraft with an insignificant increase caused by radio refraction.

It is determined by the approximate formula

$$S = 122\sqrt{H}.$$

However, if there are some obstacles along the path of the propagation of the radio waves (e.g., mountain peaks), they will appear insurmountable for USW.

From the standpoint of navigational applications, it is very advantageous to combine the operation of a goniometric USW system with range finders. /287

Rangefinding USW navigational systems are usually of impulse type (Fig. 3.28).

The aircraft transmitter sends out impulses of ultrashort waves, which reach the receiver aboard the aircraft at the same time as a reference signal.

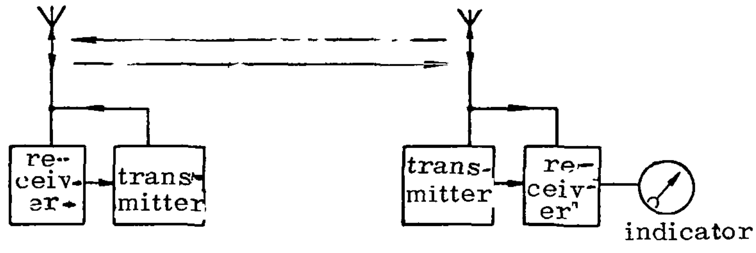


Fig. 3.28. Diagram of Long-Range Navigational System.

The ground receiver receives pulses of wave energy emitted by the aircraft, amplifies them and sends them out again through a transmitter into the ether, to be received by the aircraft.

The range indicator aboard the aircraft has a generator of standard frequencies, a frequency-divider circuit, and a delay line for the reference pulse to measure the time required for the signal to pass from the aircraft to the ground beacon and back to the aircraft.

While the signal is traveling, the duration of the delay in the reference pulse prior to its combination with the received signal determines the distance to the ground beacon, which is usually used as a visual indicator of the azimuth of the aircraft (the direct-reading instrument for distance and azimuth, DRIDA).

The combination of azimuth and distance readings makes it very easy to solve the problems of aircraft navigation, especially if the beacon is mounted at the starting or end point of a straight-line flight segment. In the latter case, the crew of the aircraft has a constant supply of direct data regarding the position of the aircraft relative to the line of flight in terms of direction and distance.

When the ground beacon is located to the side of the path to be covered by the aircraft, the problem of determining the aircraft coordinates is solved analytically, or very simple calculating devices are used to convert the polar system of coordinates for the position of the aircraft into the orthodromic system.

One type of such device is the computer which is installed for zero indication of the position of the aircraft on the line of the path to be traveled during flight in a given direction.

Let us assume that we have a straight-line path segment from point A to point B (Fig. 3.29). /288

If we are given the path angle of the segment (ψ), measured relative to the meridian for calculating the bearings (magnetic or true meridian of the location of the ground beacon), and we know the azimuth of the end point of the segment (A_{fin}) as well as the difference from it to the beacon (R_{fin}), the shortest distance from the beacon along the line of flight (R_s) (disregarding the sphericity of the Earth), can be determined by the formula

$$R_s = R_{fin} \sin (\psi - A_{fin})$$

or for any point lying on the line of flight

$$R_s = R_i \sin (\psi - A_i).$$

In other words, the given line of flight is the geometric locus of points for which

$$R_i \sin(\psi - A_i) = \text{const} = R_{fin} \sin(\psi - A_{fin}).$$

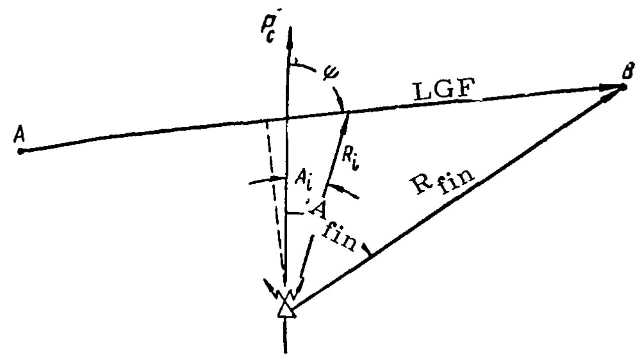


Fig. 3.29. Diagram Showing Operation of Computer for Zero Indication of Path Line.

Thus, having set the path angle of the segment (ψ) on the calculator, the distance from the beacon to the end point of the segment (R_{fin}) and the azimuth of the end point (A_{fin}), we can find the navigational parameter R_s which corresponds to the position of the aircraft exactly on the line of flight.

If it turns out in the course of the flight that R_s is greater than the given value, then in the example shown in Figure 3.29 the aircraft deviates from the LGF to the left, and the pointer of the zero indicator also moves to the left. With a deviation from the LGF to the right, the arrow of the zero indicator will move to the right.

When a flight is made in a direction which is opposite to that shown in Figure 3.40, the value sin (ψA_i) and consequently R_s, will have a negative sign, so that when the aircraft moves to the left of the line of flight the pointer of the zero indicator will also move to the left regardless of the fact that the absolute distance R_s in this case decreases, and vice versa.

The calculating device for zero indication of the given line of flight is very simple and makes it possible to solve only one problem, i.e., to select the course to be followed by the aircraft for a flight along a given line of flight, in a manner similar to that for a flight from a radio beacon or along the ShchDR bearings, using the method of half correction. /289

It is better to solve the problem or to use computors to solve it using the computation of numerical values of orthodromic coordinates of the aircraft, working on the basis of the indications from the DRIDA:

$$X_a = X_м + R\cos(A - \psi_м);$$
$$Z_a = Z_м + R\sin(A - \psi_м).$$

In this case, the angle of shift of the aircraft relative to the line of flight is determined very simply as the ratio of the change in the coordinate Z to the distance covered between the points of two measurements (X_{cov}):

$$\mathrm{tg}\,\Delta\psi = \frac{\Delta Z}{X_{cov}}.$$

Example. The distance covered between measurement points is equal to 60 km. The coordinate Z varies from zero to +4 km. Find the required correction in the course for traveling parallel to the line of flight.

Solution

$$\Delta\psi = \mathrm{arctg}\frac{4}{60} = 4°.$$

We will assume that it is necessary to travel along this line of flight for another 60 km so that the correction in the course must be -8°, but at the moment when the coordinate Z becomes zero (and if this takes place as we have calculated at 60 km) the course will have to increased by 4°.

If the juncture with this course takes place earlier or later, then it is necessary once again to determine the angle of shift of the aircraft ($\Delta\psi$) and to move the aircraft to the right by this angle. For example, if we have an initial shift from the desired path of 4 km, the aircraft will reach the line of flight in 80 km, so that

$$\Delta\psi = \mathrm{arctg}\frac{4}{80} = 3°.$$

In the orthodromic system, the problem of checking the path for distance and determining the ground speed is solved simply

$$W = \frac{X_2 - X_1}{t}.$$

For lack of a calculator, the problems in finding the angle of shift of the aircraft and checking the path for distance and direction, can be solved analytically by means of a navigational /290 slide rule. In addition, these same problems can be solved by plotting on the chart the indications of the azimuth and distance of the aircraft as obtained from the beacon.

On a flight chart which has an indication of the given line of flight, two points based on the bearings and distances from a ground beacon are plotted every 15-20 min. On the basis of the positions of these points relative to the line of flight, we can determine their orthodromic coordinates *X* and *Z*. It is then easy to solve the problems in determining the angle of shift of the aircraft ($\Delta\psi$), and also the drift angle and the required angle for turning the aircraft, the ground speed as well as the wind parameters at flight altitude.

Details of Using Goniometric-Range Finding Systems at Different Flight Altitudes

A special feature of ultrashort waves is their ability to be reflected from the interfaces of media with different optical densities, and especially from conducting media in a more sharply pronounced form than is the case for waves of shorter frequencies.

In addition, at short wavelengths, the interference which arises with combination of oscillations shows up more rarely than in the case of long waves, since the small difference in the path of the coherent waves in the case of short wavelengths gives a considerable shift in their phase.

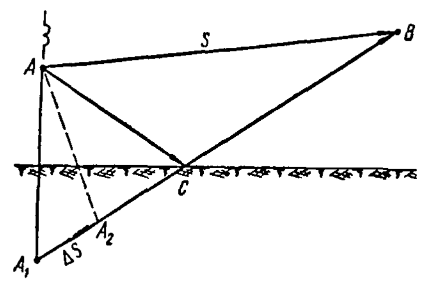


Fig. 3.30. Diagram Formation of Lobes of Maximum Radiation.

Let us say that the antenna of a ground transmitter of a goniometric or range-finding system is mounted at a certain altitude above the surface of the ground (point A in Fig. 3.30).

The electromagnetic wave at the receiver point B will be propagated along two paths:

(a) Along the straight line AB,

(b) Along the broken line ACB with reflection at point C off the Earth's surface.

It is clear in the diagram that straight line A_1B is equal to the broken line ACB, since the angle of incidence of the wave is equal to the angle of reflection.

Let us draw line AA_2 in such a way that triangle ABA_2 is an isosceles triangle.

Obviously, line A_1A_2 will represent the path difference of the rays in the straight and reflected waves.

The reflection of radio waves involves a phase shift in the wave which depends on the optical properties of the reflecting medium. A purely mirror reflection changes the phase of a wave by 180°. With a small differene in optical densities of the media, /291 when the propagation of the reflected wave takes place along a curve with a dip in the reflecting medium, the phase shift can take place differently. Let us say that upon reflection, the phase of a wave remains fixed. Then the resultant of the direct and reflected signals at the receiving point B will have a maximum when the path difference of the beams has a value which is an even whole multiple of the half wave:

$$\Delta S = 2\varkappa \frac{\lambda}{2}; \quad \varkappa = 0, 2, 4, \ldots 2n$$

and a minimum if κ is an odd multiple of the half-wave:

$$\varkappa = 1, 3, 5 \ldots (2n-1).$$

Thus, there will be an interference pattern for the propagation of radio waves in the vertical plane with maxima and minima of directionality of the radiation characteristic (Fig. 3.31).

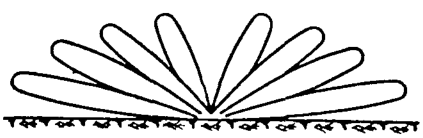


Fig. 3.31. Multilobe Radiation Characteristic of Electromagnetic Waves.

A change in the phase of the wave with reflection from the Earth's surface causes corresponding changes in the distribution of the maxima and minima of the characteristic of directionality, but the total structure of the interference pattern will be similar to that shown in the diagram.

The interference pattern of shading in the directions of radi- of radio waves by objects on the Earth's surface, as well as the altitude at which the antenna is mounted above the Earth's surface, introduce considerable corrections in the possible range of reception of ultrashort waves.

The operating range of a system, expressed by the approximate formula $S = 122\sqrt{H}$, is maximum at a sufficient power of the transmitter and sensitivity of the receiver, if the aircraft is located in the lobe of the maximum of directionality. However, at certain heights and distances, there can be "dips" in audiblity, when the aircraft passes through regions of radio shadow or interference minima. In addition, special features using USW goniometric-range finding devices at high flight altitudes are related to their rangefinding sections.

Rangefinding instruments can be used to measure not only the /292
horizontal but also the sloping distance from the aircraft to its radio beacon (Fig. 3.32). Therefore,

$$S_h = S_H \cos\theta$$

or

$$S_h = \sqrt{S_H^2 - H^2}$$

In the special case when the aircraft is passing above the radio beacon

$$S_h = 0; \quad S_H = H.$$

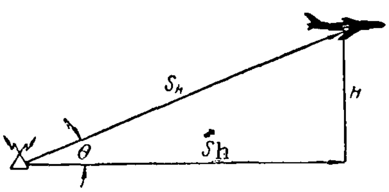


Fig. 3.32. Sloping and Horizontal Distance to Radio Beacon.

Let us suppose that an aircraft is flying along a given route with R_s = 10 km, at an altitude which is also equal to 10 km with the use of a type $R_i \sin(\psi - A_i)$ = const calculating device. With $\psi - A = 90°$, distance R must be equal to 10 km, i.e., the aircraft must deviate from the given course and pass over the radio beacon. The height errors in goniometric-rangefinding devices have some important shortcomings in their use in the shortrange applications and especially in maneuverings in the vicinity of an airport.

Consideration of altitude errors is very important due to the rapidity with which the aircraft passes over the beacon, when the errors in measuring the distance change so rapidly that it becomes impossible to enter corrections without using special calculating devices.

Therefore, the use of goniometric-range finding instruments for navigational measurements usually limits the distance from the beacon to 3-4 flight altitudes, i.e., it defines an effective zone around the beacon with this radius.

For example, at a flight altitude of 12 km, the radius of the inoperative zone thus defined must be equal to approximately 50 km.

Fan-Shaped Goniometric Radio Beacons

The possibilities of aircraft radio compasses are increased considerably by using fan-shaped goniometric beacons (Fig. 3.33).

The picture shows the schematic diagram of a radio beacon. The two outermost antennas are set to some wavelength and the power for them is in opposite phase. The total characteristic of the three antennas gives the multilobe picture of radiation as seen in Figure 3.34. The number of lobes depends on the ratio of the length of the base line between the end antennas to the wavelength, and their direction depends on the ratio of the phases in the outer and inner antennas of the radio beacon. /293

With a change in the phase of the middle antenna by 180°, the positions of the lobes shift to their mirror images (the solid and dotted lobes in Fig. 3.34), while the points where the dotted and solid lobes intersect become axes of equal signals.

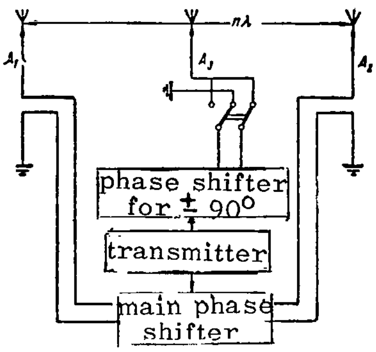


Fig. 3.33.

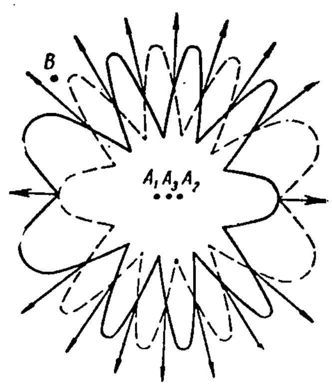


Fig. 3.34.

Fig. 3.33. Fan-Shaped Radio Beacon.

Fig. 3.34. Radiation Characteristic of a Fan-Shaped Radio Beacon.

During the periods between commutations, if we transmit short and long signals in the forms of dots and dashes in an overlapping pattern, signals of only one type will be heard within the edges of the solid lobes (e.g., long signals), while within the limits of the dotted lobes, only short signals will be heard. In zones of equal signals (near the axes of intersection of the lobes), one will hear a continuous tone. If we then smoothly change the phase ratio in the end antennas, the lobes will begin to rotate, e.g., to the right, and the phase ratios will change in the reverse direction: each of the solid lobes will change places with the dotted lobe to the right of it, and each dotted lobe will change place with the solid lobe to the right of it.

Let us assume that an aircraft is located at Point B (see Fig. 3.34), i.e., within the limits of a dotted lobe, near the right-hand limit of the solid lobe, with each operating cycle of the beacon beginning after a pause in radiation.

In this case, at the beginning of a cycle and after the pause, several fading dots will be heard, then a continuous signal, and finally a long series of dashes.

If the aircraft is located in the middle of the lobe, the series of dots will be equal in length to the series of dashes. At a point/294 which is close to the right-hand limit of the dotted lobe, the series of dots will be longer than the series of dashes.

A similar picture for the audibility would be obtained when the aircraft is located within the limits of the solid lobe with the sole difference being that at the beginning of the cycle the dashes would be heard, and the dots would be heard only after the continuous tone.

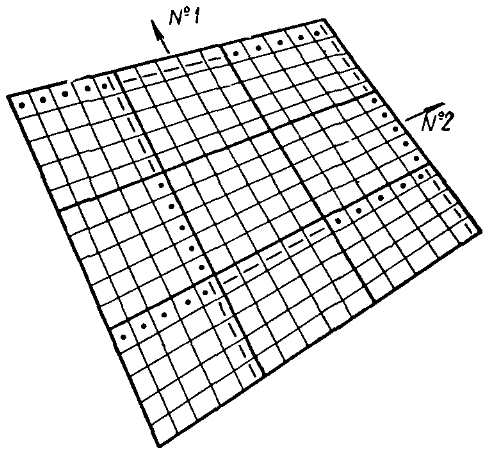


Fig. 3.35. Grid of Position Lines for an Aircraft on the Basis of Fan-Shaped Radio Beacons.

Thus, to obtain the bearing of an aircraft, it is sufficient to plot the orthodromic lines on a chart according to the location of the axes of equal signals in order to obtain the bearing of the aircraft, and also to divide the angles between them by orthodromes as well as thin lines in a ratio which is a multiple of the number of signals in the cycle.

With a sector width between the axes of equal signals of 15° and 60 signals per cycle, each signal will correspond to 15 min of angle. If the sector between the axes of equal signals is then divided into five parts, the thin orthodromic lines will diverge at angles of 3°. This narrow sector will contain 12 signals of the same type (Fig. 3.35).

For example, if an aircraft is located at the sector of points on the first thin line to the right of the axis of equal signals, then 12 dots will be heard which will fade into a continuous tone, after which there will be 48 dashes. On the second line there will be 24 dots and 36 dashes, 36 dots and 24 dashes on the third, etc. At the limit of the sector (the axis of equal signals), a total of 60 dots and 60 dashes will be heard.

Note. Practically speaking, if we consider that part of the signals (dots and dashes) are mixed with the continuous tone, the number of audible signals will be less than 60, so that after counting them the number of audible signals should be taken subtracted from 60, then divided in half and added to the number of signals of both types that were heard.

If the aircraft is located between the thin orthodromic lines plotted on the chart, then the line of the bearing of the aircraft can easily be found by interpolation of the distance between the plotted lines. /295

Fan-shaped beacons make it possible to determine very accurately the position lines of an aircraft. To do this, with the aid of a radiocompass or by generally calculating the path of the aircraft, it is necessary to determine the approximate position of the aircraft with an error which is no greater than the width of one sector. Then, having listened to the operating cycle of the beacon with the radiocompass, or with the coherent receiver, we can determine the position of the aircraft in the sector.

A similar method is used to determine the second line of position of the aircraft, using the second fan-shaped beacon, whose family of position lines intersects the lines of the first beacon. In order not to take into account the shift of the aircraft during the time between the taking of bearings from the two beacons, it is desirable to listen to the operating cycles of the two beacons simultaneously using two members of the crew who are using two radiocompasses or one radiocompass and the coherent radio receiver.

The accuracy of distance finding with the aid of fan-shaped beacons during the daytime is no worse than 0.1-0.3°. Under the most unfavorable conditions for distance measurement (in twilight when working with the space wave, or at the boundary for the use of surface waves), the errors can reach 3 and sometimes 5°. In a further zone of distance measurement, and also the short-range zone, with operation on a surface wave, the errors do not exceed 0.5-1°.

The operating range of a fan-shaped beacon during the daytime reaches 1350 km on dry land and 1750 km above the sea. At night above dry land, this figure is 740 km and above the sea, 950 km.

An important advantage of fan-shaped beacons is the independence of distance measurement from the aircraft course, magnetic declination, deviation of meridians, and radio deviations aboard the aircraft, as well as the simplicity of obtaining position lines of the aircraft during flight. One shortcoming that should be mentioned is the lack of a continuous indication of a navigational parameter, such as we have in the conventional distance measurement with ground radio stations.

Unlike the radio beacons with non-directional and omnidirectional operation, which are mounted as a rule at the turning points of air routes, flight along the bearing line of a fan-shaped beacon is only a very rare case. Therefore, the principal method of aircraft navigation using fan-shaped beacons is determining all navigational elements including the wind parameters at flight altitude by successive measurements of the LA.

This method is the most suitable one for fan-shaped beacons because the location of the aircraft can be determined in this manner with a sufficiently high accuracy.

It is often desirable to carry out aircraft navigation during /296 flight using fan-shaped beacons with conventional distance-finding from radio stations. For example, in a flight toward a radio station or away from a radio station, it is desirable to use bearings from fan-shaped beacons for checking the path for distance and determining the ground speed.

3. DIFFERENCE-RANGEFINDING (HYPERBOLIC) NAVIGATIONAL SYSTEMS

The best navigational devices from the standpoint of geometry are the goniometric-range finding systems, since the lines of position of an aircraft in these systems always cross at right angles. However, the technical requirements of such systems, which satisfy the requirements of accuracy in aircraft navigation and operate outisde the limits of direct geometric visibility, are connected with very high technical difficulties.

The azimuth lines of position are divergent because as the range of operation of a system increases, increasingly high requirements are imposed on the measurement accuracy, while beyond the limits of direct geometric visibility it is very difficult to retain directionality of transmission or reception due to the effect of local relief and especially the ionized layers of the atmosphere.

The situation is somewhat better as far as the circular position lines are concerned. Circular lines do not diverge, so that the requirement for accuracy in determining them remains constant at all distances.

In addition, the linear error in determining the position of the aircraft in a goniometric system is proportional to the sines of the angles of the propagation errors:

$$\Delta Z = S \sin \Delta A.$$

In range finding systems, these errors are proportional to the cosines of the angles of the propagation errors:

$$\Delta S = S(1 - \cos \Delta A).$$

At small angles, on the order of 6°, the cosine is practically equal to unity. Therefore, the errors in determining the distance are usually many times less than the errors in the azimuthal shift (Fig. 3.36).

We can see from the figure that the linear error in determining the direction $CC_1 = S\sin\Delta A$, and the linear error in distance is $\Delta S = ABC - AC$ ◘ $S(1-\cos\Delta A)$.

However, the technical achievement in measuring distance over long distances is much more complex than that in measurement of the azimuth.

As we saw in the case of USW systems, distance is determined by retranslation of signals from on board the aircraft by a ground beacon and their reception back on board the aircraft. This method, which is relatively easily accomplished at short distances, turns out to be unsatisfactory over long distances for use on medium and long waves. /297

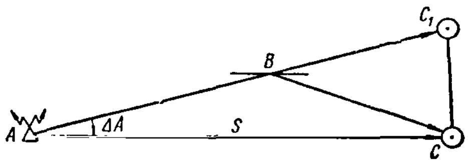


Fig. 3.36. Errors in Measuring Bearing and Range with Reflection of Electromagnetic Waves from Obstacles: A: Location of Ground Radio Beacons; B: Point of Mirror Reflection of Radio Waves; C: Location of the Aircraft (Actual); C_1: Measured Position of the Aircraft; ΔA: Angular Error in Propagation.

The best method of measuring long distances at the present time is the maintenance of a calibration frequency on board the aircraft. The generator for the calibration frequency is set at the frequency of a ground transmitter and retains a given frequency for long periods of time by means of special stabilizing elements.

By means of these special timing devices, the calibration frequency can be converted to a lower frequency which is synchronous with the signals of the ground beacon. If we take the signals from the ground stations and compare them in time with the signals from the generator of the calibration frequency, we can determine the distance to these radio stations.

However, this method has not been widely employed due to the complexity involved in keeping a highly stable reference frequency on board the aircraft, although it offers considerable promise in future.

It is simpler to solve the problem of determining the position line of the aircraft on the basis of the distance between the distances to two ground radio stations. In this case, there is no necessity for a strict synchronization of the operation of the ground installations with those on board. Only the transmission of signals from the ground stations must be synchronized. The aircraft generator for the calibration frequency in this particular case acts only as a central measuring gauge to determine the time intervals between the moments of reception of the signals from the two radio stations.

Synchronization of the operation of the apparatus at ground radio stations can be achieved incomparably more easily than synchronization of a ground apparatus with one aboard an aircraft, since the distance between ground stations remains constant, thus allowing us to use a synchronizing device for two or three stations together. In addition, ground installations are not limited by size and weight restrictions, not to mention the apparatus on board.

The methods of measuring the difference in distance to ground radio stations can involve either time (pulse) systems or phase systems. Each of these methods has its own advantages and disadvantages. /298

An advantage of the phase methods is the higher instrumental accuracy of the measurements, but in this case the result of measuring is obtained ambiguously, i.e., there may be several isophasal paths simultaneously with different distances to the ground radio stations, which differ in magnitude and are multiples of the length of the measured wave. On each of these paths, the result of measurement is the same and must be used as a measure for determining the pathway along which the aircraft is traveling.

The pulse methods of measuring distance have somewhat less instrumental accuracy, but their results are more definite.

Of course, it should be mentioned that for long-range navigational systems, the instrumental accuracy of measurement which can be attained at the present time both by the pulse and phase methods is sufficiently high so that their errors are many times less than other systematic errors which are related to conditions of propagation of electromagnetic energy. Since the errors in operation of the systems under propagation conditions of radio waves are practically the same for both pulsed and phase systems, the advantage of phase methods of measurement may be restricted only to short distances from ground radio stations (in the short-range zone of effectiveness).

Operating Principles of Differential Rangefinding Systems

Differential rangefinding systems of aircraft navigation consist of two pairs of synchronously operating ground radio stations and a receiving-indicating apparatus aboard an aircraft. For purpose of reducing the amount of ground equipment for the system, one of the transmitting radio stations (the master) is made common for two pairs so that the system can include three ground radio stations.

The operation of the two slave stations is synchronized with the master station by synchronizing signals sent out by the master station.

Let us begin by examining the geometry of the operation of one pair of ground radio stations (Fig. 3.37).

Two ground radio stations are located at points F_1 and F_2. The line connecting points F_1 and F_2 will be considered as the focal line of the base, while the points F_1 and F_2 are the foci of the system.

Let us assume that at point M there is an aircraft which is receiving signals from radio stations F_1 and F_2. At the beginning, the aircraft will receive a signal from the first radio station and then from the second. The difference in the distances from the aircraft to these radio stations is determined by the difference in time between the arrival of the signals in the pulse system or by the difference in modulation of the phases in the waves received from the two radio stations in the phase system. /299

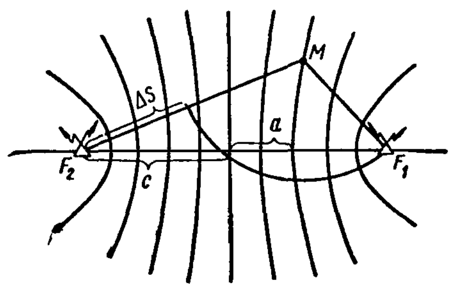


Fig. 3.37. Hyperbolic System of Position Lines.

We know that the line which is the geometrical locus of points, the difference in whose distance to two given points is a constant value, is called a *hyperbola*. The given points, to which the distances are measured, are called the *foci of the hyperbola*. Consequently, knowing the difference of the distances to the two radio stations, we can always plot the hyperbolic line of the position where the aircraft is located.

The hyperbolic line with a difference in distances equal to zero becomes a straight line perpendicular to the focal axis and dividing the distance between the foci of the system in half (see Fig. 3.37). This line is called the *imaginary axis of the hyperbola*.

The distance along the focal axis of the family of hyperbolas from the foci to the imaginary axis is called the parameter c.

It is obvious that the difference in distances from the foci of the hyperbola to any point along its branches is equal to twice the distance along the focal axis from the imaginary axis to the peak of the hyperbola. This distance is called parameter a. Accordingly, the difference in distances from any point to the foci of the hyperbola is always equal to $2a$.

The maximum density of hyperbolic lines of position is found along the focal axis between the foci of the system, where the distance between the peaks of the hyperbola is equal to the difference in parameters a.

The magnitude of the value $2a$ is measured by navigational parameters of the system so that the accuracy in determining the lines of position of the aircraft depends on the accuracy with which this parameter is measured. Consequently, an error in determining the position line of the aircraft on the focal axis is equal to the error in measuring parameter $2a$, divided in half.

As we see from Figure 3.37, the family of hyperbolas is divided by a family of position lines. At distances from the center of the system which exceed $2c$, the hyperbolas practically become straight lines, whose direction coincides with the direction of the radii extended from the center of the system. Thus, the hyperbolic system is converted into a goniometric one.

However, the density of the lines of position in this case will not be equal along the circumference, as is the case in purely goniometric systems. At a given distance from the center of the system, the maximum density of position lines will be found at the imaginary axis of the hyperbola, gradually decreasing along the circumference as they approach the focal axis. The density of position lines at a distance greater than c on the focal axis becomes so small that the system becomes unsuitable for determining the location of the aircraft. /300

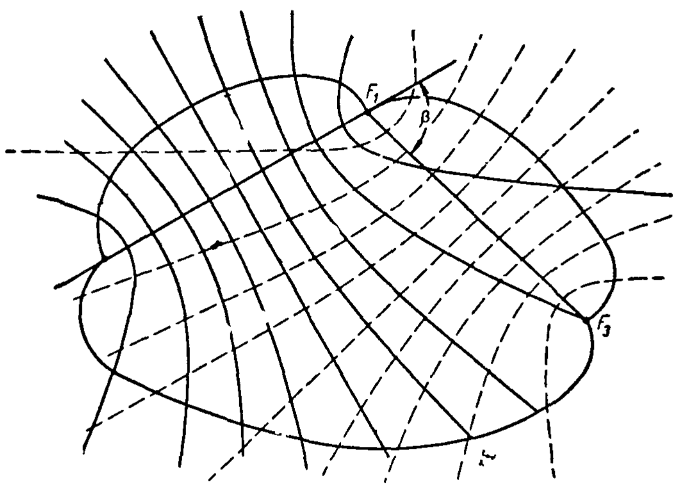


Fig. 3.38. Effective Area of Hyperbolic Navigational System.

The master station of the second hyperbolic pair can be located along the extension of the focal axis of the first pair. In this case, the angle of fracture of the base (β) is equal to zero.

If the master station of the second pair is not located on the focal axis of the first pair, there is a definite fracture of the base (Fig. 3.38).

The angle of fracture of the base creates a more favorable condition for intersection of the position line in that region of application of the system toward which it is directed, since the angle of intersection of the hyperbolas in this case approaches a right angle and therefore the accuracy in determining the locus of the aircraft is increased when two position lines intersect.

However, this involves a decrease in the quality of the conditions for navigational exploitation, as well as a narrowing of the range of application for the system from the opposite side of the focal axis. In addition, the analytical solution of problems related to navigational application of the system is complicated to a significant degree by the fracture of the base, e.g., in converting hyperbolic coordinates to geographical orthodromic ones (see Chapter I, Section 7).

The complex of equipment for a hyperbolic navigational system aboard an aircraft usually consists of the following: a non-directional receiving antenna, a matching block for the antenna with a receiving device, a receiver, and an indicator. /301

The matching block serves to produce parameters of the receiving antennas when signals are received from ground radio stations.

Signals received by the antenna are transmitted to the indicator for measurement of the navigational parameter. The indicator has a generator for a calibration frequency, which produces standard signals for purposes of measurement, and a number of frequency dividers which are required for forming electronic markings on the reading scales, as well as repetition frequencies for the scan on a cathode ray tube, synchronized with the transmissions of signals from ground radio stations.

The signals which are received pass to the scan of the cathode ray tube, where the operator controls their size according to the amplification of the receiver. The synchronization of the scan on the screen is then regulated with the frequency of the received pulses so that the latter remain fixed on the screen.

The operator then mixes a reference (selecting) signal from the generator with the signal from the master station, which is achieved by the intermittent introduction of small distortions in the generator for the calibration frequency, so that the pulses of the signals begin to move across the screen. The motion of the pulses stops when the signal of the master station coincides with the reference signal of the generator (usually a rectangular base at the beginning of the scan).

To measure the time difference between the arrival of the signals and the signal from the slave station, a selecting pulse is given which is related to the delay in scanning of the reference signal, after which the indicator is switched to the reference regime and the reading is taken on the electronic scale.

In some types of receiver indicators, the recording of the reading is made on a dial with two or three scales (for different scanning rates), for example, beginning with thousands of microseconds, then hundreds and finally tens, with interpolation up to units of microseconds. This provides increased accuracy of readings due to the many-fold increase in the scale of the indicator.

In systems with automatic tracking of the signals from ground stations, the time intervals between the moments of arrival of the signals are calculated on mechanical counting dials, whose rotation is related to the delay mechanisms for the selecting pulse. The reference signal from the generator is then reinforced, together with the signal from the master station, by an automatic frequency adjustment of the calibration generator.

Thus, there is an automatic tracking of the signals from the radio station and a constant numerical indication of the output navigational parameter of the system, and the difference in distances from the aircraft to the ground radio stations is expessed /302 in microseconds of radio wave propagation.

In phase systems, by means of distributing elements in the calibration generator, its phase is matched with the phase of the signals from the master radio station, after which a phasometer is used to measure the phase difference between the calibration generator and the slave radio station, and the position line of the aircraft is determined from this difference.

As we have already pointed out, if the difference in distances to the radio stations includes several periods of the modulating frequency of the ground stations, the determination will be ambiguous.

The solution of the ambiguity of this estimate can be accomplished by several methods.

(1) An initial setting of the coordinates of the aircraft with automatic tracking of the radio station signals. In this case, using known coordinates of the aircraft (e.g., on the basis of the visual determination of the aircraft location), the indicator is set by hand to show the isophasal line on which the aircraft is located. If constant tracking of the radio station signals is then carried out, completely reliable readings of the position line will be obtained.

A shortcoming of this system is the necessity to relate the aircraft to the local terrain on the basis of the initial reading of the hyperbolic coordinates. In addition, during flight, there may be readings of other isophasal lines, due to interference, which can be determined and corrected only by a repeated relation of the aircraft to the local terrain by means of other methods.

(2) By modulation of the carrier frequency of the ground radio stations at very low frequencies (with long modulating waves, considerably increasing the possible difference in distances from the aircraft to the radio stations). In this case, at a low frequency phase, the rough position of the isophasal line of a carrier frequency or the frequency of the second modulation with a small, long period can be determined.

(3) By using several carrier frequencies for the ground radio stations, the isophasal line can be considered to be determined if it is simultaneously on the isophasal lines for all frequencies at which the measure ment is carried out (usually three frequencies, since two will be inadequate in some cases). On adjacent isophasal lines, for each frequency used, the isophasal lines of other frequencies will not coincide with the readings of the phasometer.

Navigational Applications of Differential-Rangefinding Systems

Differential-rangefinding navigational systems, like fan-type beacons, are intended primarily for determining the locus of the aircraft on two position lines. Therefore, the principal method of aircraft navigation using these systems is the determination of the navigational elements on the basis of a series of determinations of the LA. /303

By recording and plotting on a chart a series of points for the locus of the aircraft, recording the time at which they were passed, and using a scale ruler and protractor to measure the distances between them on the chart, as well as the distance from the first recording of the LA to the second, it is easy to determine the speed and flight angle of the aircraft.

$$\psi = \alpha_{1,2};$$

$$W = \frac{S_{1,2}}{t},$$

where $\alpha_{1.2}$ is the azimuth of the second recording of the LA from the first and $S_{1.2}$ is the distance between the recordings of the LA.

The drift angle of the aircraft is determined as the distance between the actual flight path angle and the average course of the aircraft over the segment between two successive recordings of the LA:

$$a = \psi - \gamma_{av}.$$

With a known groundspeed and drift angle, taking the airspeed into account as well as the course to be followed, the wind parameters at flight altitude can be determined with the aid of a navigational slide rule.

In special conditions, when the flight direction coincides with one of the branches of the hyperbolic flight lines, the flight can be made along the latter. To do this, it is sufficient to maintain a constant reading for the calculator of hyperbolic coordinates of one pair. The family of position lines for the second pair in this case is used to monitor the path for distance.

Monitoring the path for distance by means of the readings of one of the counters can be used in the case when the aircraft navigation in terms of direction is carried out using two devices, e.g., the USW bearing of a goniometric system or a fan-type beacon.

To increase the feasibility of using differential-rangefinding systems, the hyperbolic coordinates can be converted to orthodromic or geographical ones (see Chapter I, Section 7).

In some hyperbolic systems of aircraft navigation, e.g., that of Decca and Dectra (England), simplified methods of automatic plotting of the aircraft course on a special chart use the movement of a pen in mutually perpendicular directions. For this purpose, special charts are made on which the hyperbolic lines of the first and second family are laid out at right angles. Naturally this results in distortion of the contours of the terrain on the chart, /304 as well as the scale and geographic grid, and the line of flight of the aircraft is also bent.

Such a method of recording has a number of shortcomings (e.g., in relation to the calculation of orthodromic coordinates for the air craft), but it is very easy to achieve from the technical standpoint and its shortcomings are considerably reduced if the path of the aircraft has markings for distance.

Methods of Improving Differential Rangefinding Navigational Systems

The design of hyperbolic systems contains elements whose improvement leads to a conversion of the system to a hyperbolic-rangefinding or hyperbolic-elliptical system.

Such elements include the standard frequency generators aboard the aircraft. When these generators operate in a highly stable regime, the reference signals from these generators can be kept so precise that it becomes possible to measure distances to one of the ground radio stations. To do this, it is sufficient to combine the phases of the frequencies of the generator aboard the aircraft and the ground radio station with an initial distance setting (e.g., the takeoff point of the aircraft). Further changes in distance can be determined by the deviation of the phases of these frequencies or by the deviation of the pulse signals, if the system is operating in a pulse regime.

Measurement of distance in connection with one pair of hyperbolic position lines makes it possible to considerably improve the accuracy with which the locus of the aircraft is determined over long distances, and one pair of ground radio stations will suffice for measurements. However, the conditions for measurement between the foci of the system near the focal axis will remain unfavorable (Fig. 3.39).

It is more advantageous in this case to use the hyperbolic network of position lines (see Chapter I, Section 7).

However, since we know the difference between the distances to the two radio stations as well as the distance to one of them, it is easy to determine the sum of the distances to these radio stations, e.g., if

$$S_2 > S_1 \text{ and } \Delta S = S_2 - S_1,$$

so that

$$S_2 = S_1 + \Delta S$$

and

$$S_1 + S_2 = 2S_1 + \Delta S.$$

Similarly, for the case when $S_2 < S_1$,

$$S_1 + S_2 = 2S_1 - \Delta S.$$

Therefore, in order to obtain the number of the hyperbola, it is sufficient to use the difference in distances, while to obtain the number of the ellipse, we must double the distance to one of the radio stations and add the difference in distances with the /305 corresponding sign.

One great advantage of the hyperbolic-elliptical network is the orthogonality of the intersection of the position lines at any point in the field which is involved. On individual sheets of the chart, the hyperbolic-elliptical network has the appearance of a nearly rectangular grid with noticeable curvature of the position lines only in the vicinity of the foci of the system.

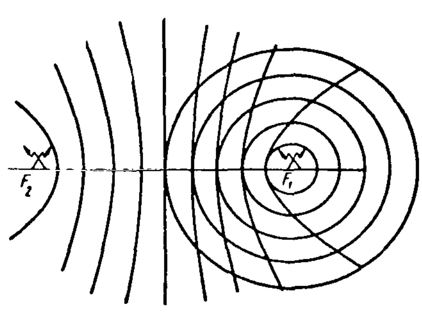


Fig. 3.39. Combination of Hyperbolic and Rangefinging Navigational Systems.

When using a hyperbolic-rangefinding (and especially a hyperbolic-elliptical system, a system of position lines) the accuracy with which the coordinates of the aircraft are determined over long distances is increased several fold, so that the practical range of application of the system is also considerably increased, with the use of only one pair of ground radio stations.

It should be mentioned, however, that a serious obstacle to the development of systems of long-range navigation for use on high-speed aircraft is the low noise sta bility of operation of these systems, since only very long waves can be used for navigation over long distances.

4. AUTONOMOUS RADIO-NAVIGATIONAL INSTRUMENTS

In recent years, there has been a considerable increase in the use of radio navigational instruments which are housed completely aboard the aircraft and operate without the need for ground facilities. Such instruments are called *autonomous radio-navigational instruments* or, if their operation is combined with some other navigational equipment aboard the aircraft, *autonomous navigational systems*. These include aircraft navigational radar, Doppler systems for aircraft navigation, and radio altimeters.

All autonomous radio-navigational instruments operate on ultra-short waves, since they have a very high (practically complete) freedom from interference during operation (not counting artificial interference).

Doppler meters for measuring the ground speed and drift angle of the aircraft measure the motion parameters of the aircraft directly relative to the Earth's surface, which clearly differentiates them from all existing forms of navigational equipment, especially with regard to problems of automation of aircraft navigation and pilot- /306
age of aircraft.

Aircraft Navigational Radar

Aircraft navigational radar is a very flexible and effective method of aircraft navigation during flight over land or sea close to coastal regions.

In terms of the geometry of their use, aircraft radar devices can be included among the goniometric-rangefinding systems. However, in comparison to the goniometric-rangefinding navigational systems, they have a number of tactical advantages:

(1) The high saturation of ground landmarks makes it possible to select the most suitable ones for measurement in navigation.

(2) The lack of errors in determining the bearings of landmarks from the radio deviation of both the aircraft itself and the local relief, something which affects all non-autonomous navigational systems.

(3) The possibility of visualizing ground landmarks with the purposes of determining ground speed and drift angle to a better degree than with optical methods.

(4) The possibility of identifying dangerous meteorological conditions in flight (thunderstorms, powerful cumulus and cumulonimbus clouds).

(5) The high accuracy and ease of the measurements using only one operational frequency.

At the same time, the navigational use of aircraft radar has several shortcomings:

(a) The bearing of the aircraft can be used only as a basis for measuring the aircraft course, thus lowering the accuracy of distance findings.

(b) A certain amount of experience is needed for correct recognition of ground landmarks and the possibility of errors in determining a landmark, since they are not labelled.

The operating principle of radar is based on the ability of electromagnetic waves at high frequencies to be reflected from objects located along their propagation path (from the interface between media with different optical densities).

To obtain a panoramic image of the terrain, a rotating or scanning antenna is used to cover a certain sector, so that its position must be synchronized with the position of the scanning beam on the screen of a cathode ray tube. In addition to synchronizing the direction of the antenna with the scanning direction of the beam, it is also necessary to ensure that the beginning of the scan is synchronized with the moment when the USW pulses are omitted from the antenna transmitter .

The radar transmitter emits high-frequency pulses whose propagation direction depends on the position of the rotating antenna /307
at the moment of emission. The scanning of the indicator beam begins simultaneously with the emission of the pulse, moving from the center of the screen toward the periphery; the direction in which the beam moves coincides with the movement of the antenna.

Since the propagation rate of electromagnetic waves is very high, and the rotational speed of the antenna and the scan rate are slow, the pulse of wave energy can cover the distance to the irradiated object and return in a period of time which is sufficiently short so that the antenna has not yet moved through any noticeable angle. Therefore, the direction of the antenna at the moment of receiving the signal coincides with its direction at the moment of emission. The received reflected signal is amplified in the receiver and passes to an indicator, where it controls the brightness of the scanning beam.

Thus, the radar screen shows the following:

(a) The direction of the object on the basis of the antenna position at the moment of emission and reception of the signal.

(b) The distance to the object on the basis of the time required for the signal to travel between the moment when it is emitted to the moment when it is received.

(c) The nature of the object, on the basis of the brightness of the scanning beam at the point where the reflected wave is received.

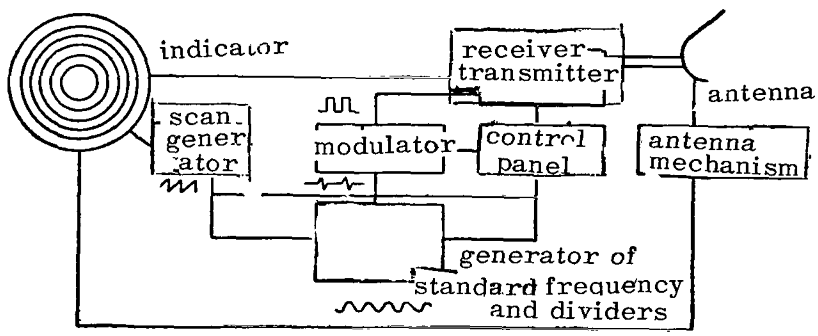


Fig. 3.40. Diagram of Aircraft Radar.

The radar screen has a long afterglow so that when the antenna has made a complete revolution, the screen still shows a trace of all the irradiated objects on the Earth's surface which are located in the field scanned by the radar.

The main section of the radar, controlling the operation of the entire system, is the standard-frequency generator with frequency dividers for forming distance markings and a signal-transmission frequency synchronized with the saw-tooth scanning image on the screen (Fig. 3.40).

The signals from the standard-frequency generator reach the modulator, where they are converted to high-voltage rectangular oscillations of a special length. The high-voltage pulses from the modulator pass to the transmitter magnetron, where high-frequency groups are generated according to the pulse length. /308

The high frequency reaches the antenna through a wave guide and is radiated into space.

At the same time, in synchronization with the pulses of high voltage which are sent to the transmitter, the scanning generator forms a saw-tooth voltage which controls the scanning beam on the screen. The scanning rate depends on the steepness of the slope for the saw-tooth waves. At a low scanning rate, a fine image scale is obtained as well as long-distance detection of objects. When the scanning rate is increased, the scale of the image decreases proportionately with the distance covered by the radius of the screen. The control of the scanning rate is achieved with the aid of a switch on the control panel.

The duration of the high-frequency pulses controls the resolving power of the radar. For example, a pulse duration of 2 microseconds corresponds to a propagation distance of the wave (back and forth) of 300 m. Hence, if two reflecting objects are located at a distance of less than 300 m along the propagation direction of the wave, they will appear as one object. Therefore, when working at course scales, the modulator is set for the formation of shorter pulses with an increase in the frequency of their transmission.

The resolving power of the radar in terms of azimuths is a function of the sharpness of the directionality of the antenna beam.

The azimuths of ground landmarks can be determined immediately by the position of the antenna (and therefore by the scanning line on the screen), and the antenna mechanism is fitted with a selsyn mechanism for tilting the indicator. The azimuth reading is made on a scale located along the periphery of the screen.

To measure the distance to a landmark, pulses from the frequency divider are sent to the receiver (and therefore to the scanning beam). These pulses increase the brightness of the beam at certain distances from the center of the screen, forming circular distance markings.

When using the radar on different scales the distance markings are shifted to different distance intervals. For example, with a scale of 10 km for the radius of the screen, the markings are usually 2 km apart; when using scales from 10 to 100 km, the markings are 10 km apart; at a scale of 200 km, they are 20 or 40 km apart.

Now let us follow the path of the high-frequency pulses from the transmitter to the object and back again, and see how they control the brightness of the scanning beam.

The high-frequency pulse passes through the wave guide to the /309
radiating horn of the antenna, after which it is shaped into the required directional diagram for radiation by means of a reflector. Usually, the directionality of the antenna in the horizontal plane is made as sharp as possible. To do this, it is necessary for the phase of the beam when emerging from the antenna to remain constant over its entire perpendicular cross section (Fig. 3.41), i.e., the reflection in this plane must have a shape such that the wave path from the horn radiator to the surface of the reflector and along its chord of emergence is uniform.

The characteristic of directionality of the radiation in the vertical plane must be such that the illumination of the terrain from the vertical of the aircraft is as uniform as possible over the entire effective radius of the radar. To do this, it is necessary to have the maximum amount of wave energy transmitted at small angles to the plane of the horizon, i.e., over the maximum range, and to have the smallest amount of energy radiated along the vertical of the aircraft. Such a *characteristic is called the cosecant-square*, i.e., the reflectors in the vertical plane are given a shape such that the amount of energy radiated into space is roughly proportional to the square of the cosecant of the angle of the plane of the horizon to the propagation direction.

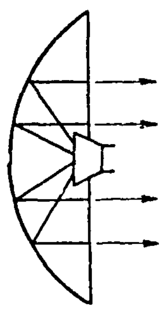


Fig. 3.41. Radar Antenna for Use Aboard Aircraft.

In some types of radar, an acicular characteristic of directionality is employed, i.e., one which is sharpest in both the horizontal and vertical planes, combining it with the cosesant-square in the vertical plane, e.g., by a scanning cycle. This is achieved by using specially shaped reflectors with a telescoping deflector or by sending energy to the antenna by different wave guides for the acicular and cosecant-square antenna characteristics of the radar.

Antennas with cosecant-square characteristics are used for circular-scan radar, mounted below the fuselage of the aircraft. Antennas with combined radiation are used for sector-scan radars and are mounted in the nose of the fuselage to cover only the area ahead of the aircraft. In this case, the radar screen is made with the center displaced so that the maximum area of the screen can be used.

Usually, the tilting of the antenna in the vertical plane (and therefore the characteristics of directionality of the radiation) is adjusted manually by means of a special electrical device and a switch on the control panel of the radar.

The transmitting antenna of the radar acts simultaneously as /310
a receiving antenna, since the directional characteristics of the antenna are reversed, i.e., used both for emitting and receiving the wave energy.

In order for the pulses of wave energy emitted from the transmitter not to return immediately to the wave guide of the receiver, special arresters are used which block the wave energy from entering the receiver at the moment when the transmitter is operating.

The transmission frequency of the pulses of wave energy from the transmitter is set so that the time intervals between them are not shorter than those required for propagation of electromagnetic waves to the most distant object at a given operating range for the radar and for its return to the aircraft. When using the radar at large-scale settings, the decrease in the pulse duration is accompanied by an increase in the transmission frequency, thus preserving the average power of the transmitter. Hence, the reception of the reflected signals takes place in the time intervals between the pulses of wave energy emitted by the transmitter.

The radar receivers have special vacuum devices (klystrons for generating high frequency) which play the same role as heterodynes in conventional receivers.

The signals received by the antenna are mixed with the frequency of the klystron; an intermediate frequency is produced which then goes on (after detection and amplification)to control the brightness of the scanning beam.

In addition to the special features of the radar which we have discussed above, the receiver has additional circuits and control units. In particular, to allow the frequency of the klystron to be changed, there is an automatic frequency adjuster (AFA), etc.

For improved contrast of the image on the screen, in addition to the devices for adjusting the overall amplification of the receiver, the operator of the radar can use a separate signal amplifier which operates at high and low levels. This makes it possible to distinguish shaded or illuminated objects on the Earth's surface as desired. For example, to examine populated areas, the high-level signals are increased and the low-level signals are reduced (by decreasing the brightness of the background of the screen). To pick out rivers and lakes, the low-level signals are increased, thus improving the visibility of shaded objects against a brighter general background.

It should be mentioned that for the formation of high-frequency pulses by the transmitter, very high voltages must be produced in the modulator; this means that at high altitudes (i.e., at low atmospheric pressure), there may be flashovers in the wiring of these units. Therefore, these units (including the wave guides of the transmitter) are hermetically sealed and the required pressure is /311
maintained in them by a special pump or by systems for pressurizing the aircraft cabin.

Indicators of Aircraft Navigational Radars

The aircraft radar is an autonomous goniometric-range-finding and sighting device, so that its indicator must be made so that all required navigational measurements can be performed satisfactorily with it.

Circular indicators are the ones which are of greatest interest from the navigational standpoint (Fig. 3.42).

The center of this indicator, marking the position of the aircraft against the panorama of the field of vision, coincides with the center of the screen. Around the edge of this screen is a scale of bearings, which can be rotated manually; in the upper part of it is a course marking which shows the position of the longitudinal axis of the aircraft. The scale of bearings is set to its own divisions by means of a "course" rack and pinion device, for setting the course of the aircraft by the course markings, according to the readings of the course instruments.

The sighting lines of the indicator are marked on the protective glass of the screen, which can be rotated by means of "sight" rack and pinion. For convenience in sighting, three movable points for longitudinal sighting lines are provided, and one transverse sighting line is provided for indicating traverses when flying over landmarks.

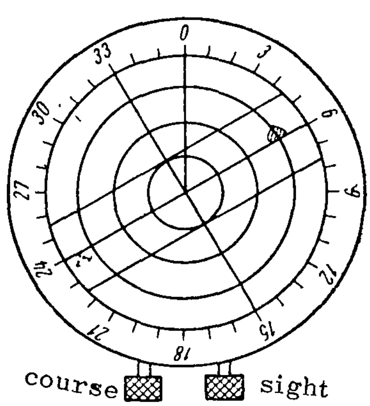


Fig. 3.42. Indicator for Radar with Circular Screen.

When the radar is operating, circular distance markings appear on the screen, and the deflection of the luminous course lines of the aircraft may also be included.

In the lower part of the indicator unit, in addition to the "course" and "sight" adjustments, there are other controls: "scale illumination", "beam scan focus", "beam brightness adjustment", and some types of indicators also have a "vertical and horizontal centering of scan".

Thus, the circular screen of the radar can be used to measure bearings precisely or determine the course angle of a landmark, its distance, as well as the provisional line of motion of the landmark for purposes of determining the drift angle /312
and the ground speed on the basis of the traverse of the flight over the landmark.

Sector-type radar screens have somewhat fewer possibilities (Fig. 3.43).

Since the center of rotation (the position of the aircraft on the screen) is shifted from the center of the screen on this type of indicator, there is no possibility for using a rotating scale of bearings or sighting lines, thus complicating the determination of bearings and the motion of landmarks on the screen.

Instead, the screen is fitted with a system of divergent lines for the course angle of the landmark (CAL). The determination of the bearings in this case is made by adding the course angle of the landmark to the course of the aircraft by the formula:

$$TBL = TC + CAL;$$

$$TBA = TC + CAL \pm 180^\circ + \delta.$$

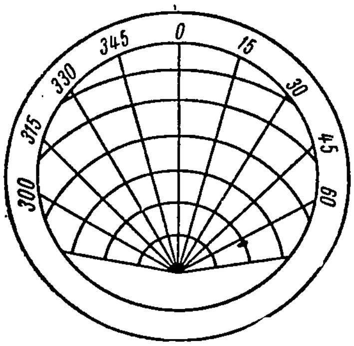


Fig. 3.43. Indicator for Sector-Type Radar.

It is very difficult and not always possible on these indicators to determine the moment of flying over the traverses of landmarks.

Instead of visualizing the movement of landmarks, the solution of navigational problems on these indicators is more often accomplished by a succession of measurements of the LA on a chart. An exception to this is constituted by landmarks which move across the screen in the immediate vicinity of the course marking, and can be used to determine the drift angle by the parallelism of their shifting, using the lines of the course angles and the ground speed when passing over the distance markings on the screen.

Nature of the Visibility of Landmarks on the Screen of an Aircraft Radar

For purposes of aircraft navigation using aircraft radar, the following landmarks can be used:

1. **Large populated areas and industrial enterprises.** The visibility and outlines of these landmarks depend on the number and location of metal structures and coverings in the object. Populated areas and industrial enterprises appear as bright spots on the screen, as a rule, with sharply bounded outlines. This means that the outlines of the landmarks coincide closely with their outlines on a chart or as they are seen by visual observation, as groups of structures with non-metallic coverings show up much less clearly /313 and are visible from shorter distances than metal structures and coverings.

Populated areas show up most clearly with maximum amplification of the high-level signals and a minimum amplification of the low-level signals.

2. **Rivers and lakes.** During the summer, these landmarks are visible as dark areas and spots whose outlines match those of the landmarks against the a lighter background of the surrounding terrain. In the winter, when these bodies of water are covered by a smooth layer of ice, only the river valleys are seen, especially against forested areas. Ice packs on rivers can be seen in the form of bright spots against a darker background of snow covered banks. Rivers and lakes can be distinguished by amplifying the low-level signals to increase the brightness of the entire background

of the screen. Then the dark objects will be observed as still darker areas against the light background.

3. **Mountains.** These landmarks appear on the radar screen in a form which is very close to their natural one, i.e., as they appear to visual observation. Mountains can be distinguished by a suitable selection of signal amplification at both high and low levels.

4. **Forested areas.** Landmarks of this type can only be seen clearly in winter, against a general background of snow-covered surface, by amplifying the low-level signals; in summer, against a background of vegetation and cultivated areas, forests are seen very dimly and cannot be used as landmarks.

5. **Highway and railway bridges.** These landmarks show up especially well against the background of large rivers. The railways themselves show up clearly only when there are embankments or steel structures for supporting catenaries for electrified railways.

In summer, the development of powerful cumulus and cumulonimbus clouds shows up very clearly on radar screens. Areas which are dangerous for flight (with a large-droplet structure, and therefore with intense turbulence and high intensity of electrical fields) appear on the screen in the form of bright spots with diffuse edges.

These storms can be distinguished very well with maximum amplification of high-level signals and minimum amplification of low-level signals. Amplification of low-level signals reduces the contrast of the images of these dangerous storms, but areas of radar shadows begin to appear, which are very clear on the screen and are characteristic signs of storm clouds.

In observing terrestrial landmarks and clouds in which there is thunderstorm activity, it is necessary (besides adjusting the amplification level of the receiver) to choose the proper inclination of the radar antenna. As a rule, landmarks which are located close to the aircraft are observed with an increased inclination of the antenna downward, while those further away (and storm clouds) /314
are viewed with a slight inclination downward or with the antenna aimed upward, depending on the flight altitude and the viewing range.

The inclination of the antenna can be selected to provide the optimum clarity of the images of the landmarks on the screen.

Use of Aircraft Radar for Purposes of Aircraft Navigation and Avoidance of Dangerous Meteorological Phenomena

Aircraft radar can be used to solve all problems of aircraft navigation, beginning with the recognition of landmarks over which the aircraft is flying and ending with measurement of all basic elements of aircraft navigation.

For recognition of terrestrial landmarks, it is desirable to use operating scales of the radar which coincide with the scales of flight charts.

With an indicator screen radius of 55 mm, an image scale of 1:1,000,000 produces a range of 55 km on the screen. This operating scale for the radar is most suitable when using maps with a scale of 10 km to 1 cm.

Hence, when using charts with a scale of 1:2,000,000, one must use a radar scale of 100 km; 110 km is possible, if the design of the radar permits.

The sharpest distinction of radar landmarks is obtained by using the proper selection of contrast in the image by using various amplifications of the signals at high and low levels, adjusting the inclination of the antenna to the proper angle, and setting the beam brightness on the screen.

The location of the aircraft can be determined very accurately in terms of the bearing and direction from a point landmark. Point landmarks in this case can be the centers of populated areas, characteristic features of the shores of rivers and lakes, individual mountain peaks, etc.

When using circular scan radar, with a rotating scale of bearings, the locus of the aircraft is determined by the same method used for goniometric-rangefinding navigational USW systems, i.e., plotting the bearing and distance from a landmark to the aircraft on a chart. However, it is necessary to take into consideration the correction for the deviation of the meridians, if the difference in the latitudes of the landmark and locus of the aircraft is significant.

In using sector-type radars, the bearing of the aircraft is obtained by adding the aircraft course and the course angle of the landmark, as is done when using aircraft radio compasses with non-integrated indicators.

As in the case when USW rangefinding systems are used, the measurement of distances with an aircraft radar means that the radar /315 measures not the horizontal but the oblique distance (OD) of the landmark. Therefore, when measuring distances to landmarks, which are less than five times the flight altitude (H), the measurement must include a correction ΔR, which always has a negative sign:

$$\Delta R = -(\sqrt{\mathrm{OD}^2 - H^2} - R);$$
$$R = \mathrm{OD} - \Delta R,$$

where OD is the oblique distance, H is the flight altitude, and R is the horizontal distance.

If the oblique distance to the landmark is equal to the flight altitude (the correction for the flight altitude becomes equal to the oblique distance), the horizontal distance will be equal to zero. This is also reflected in the panorama of the image, when a dark spot appears in the middle of the indicator screen with a sharp limit for the beginning of image formation. The beginning of image formation is separated from the center of the screen by a distance which is equal to the flight altitude on the scanning scale. *This spot is called the altimetral* and is used for measuring the true altitude of flight above the local relief.

TABLE 3.1.

Oblique distance, км	Flight altitude, км											
	1	2	3	4	5	6	7	8	9	10	11	12
	corrections, км											
5	0,0	0,5	1,0	2,0	5,0	—	—	—	—	—	—	—
10	0,0	0,0	0,5	1,0	1,5	2,0	3,0	4,0	6,0	10,0	—	—
15	0,0	0,0	0,0	0,5	1,0	1,5	2,0	2,5	3,0	4,0	5,0	6,0
20	0,0	0,0	0,0	0,0	0,5	1,0	1,5	2,0	2,5	3,0	3,5	4,0
25	0,0	0,0	0,0	0,0	0,0	0,0	0,5	1,0	1,5	2,0	2,5	3,0
30	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,5	1,0	1,5	2,0	2,5
35	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,5	1,0	1,5	2,0
40	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,5	1,0	1,5
45	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,5	1,0
50	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0

For making corrections in the measured distances for the flight altitude, we can use Table 3.1.

The location of the aircraft can be determined by means of aircraft radar and directly in stages of orthodromic coordinates. To do this, the scale of bearings on the indicator must be set not to the course of the aircraft, but to the lead angle (LA) on the /316
course of the aircraft relative to a given orthodromic path angle of flight or drift angle.

The sighting device can then be used to determine the path bearing of the landmark (PBL). For example, with LA = $\gamma-\psi$ = -10°, the bearing scale must be set to 350° opposite the course marking; with a course angle of 40°, the path bearing of the landmark (PBL) will be equal to 30°; with a negative drift angle, and therefore a positive lead angle, such as 10°, e.g., the bearing scale must be set to 10° opposite the course marking.

Knowing the path bearing and the distance to a landmark (R), we can very simply determine the orthodromic coordinates of the aircraft:

$$X = X_1 - R\cos\text{PBL} = X_1 - R\sin(90°-\text{PBL});$$
$$Z = Z_1 - R\sin\text{PBL}.$$

These formulas are different from (1.71) and (1.71a) only in the sign of the second terms on the right-hand side. This is explained by the fact that when we are using goniometric-rangefinding systems, the direction is reckoned from a ground beacon to the aircraft, while in this case it is reckoned from the aircraft to a ground landmark.

Example. The radar landmark has orthodromic coordinates $X_1 = 250$ km; $Z_1 = 80$ km and is observed with a path bearing of 40° as a distance of 125 km. Find the coordinates of the aircraft X and Z.

Solution:

$$X = 250 - 125 \cdot \sin 50° = 250 - 96 = 156 \text{ км}.$$
$$Z = 80 - 125 \cdot \cos 40° = 80 - 82 = -2 \text{ км}.$$

Thus we have found that the aircraft is located at a distance of 156 km from the last PBL, 2 km to the left of the LGF, without resorting to a plotting of the bearings on the flight chart.

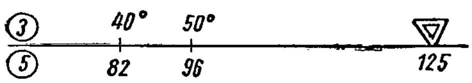


Fig. 3.44. Determination of Orthodromic Coordinates of an Aircraft on the NL-10M.

In solving this problem, it is very convenient to use the calculating navigational slide rule.

To do this, the triangular index on scale 4 is set to the distance of the landmark along scale 5. The slider indicator is then set on scale 3 to the marking which corresponds to 90° -PBL and PBL, while the values R sin (90° -PBL) and R sin PBL are set on scale 5 (Fig. 3.44).

After this, there remains only the calculation of these values from the coordinates of the landmark and the determination of the aircraft coordinates.

The problem is considerably simplified when the path bearing of the landmark is equal to 90° (flight over the traverse of the landmark). Then

$$X = X_1; \quad Z = Z_1 - R.$$

It should be mentioned that the determination of the aircraft /317
coordinates when flying over the traverse of a landmark is advantageous, since in this case the errors in measuring the path bearing of the landmark have absolutely no effect on the accuracy of determination of the lateral deviation of the aircraft from the

line of flight. This is very useful for monitoring the path in terms of direction and correcting the course of the aircraft by using autonomous Doppler measurements of the ground speed and drift angle.

This method of determining the orthodromic coordinates of an aircraft is also suitable for use with sector-type radars. In this case, the path bearing of the landmark is determined by the formula

$$PBL = CAL + LA.$$

This problem can then be solved in the same way as for circular-screen radars. However, on sector-type screens as a rule, it is not possible to determine the markings of the traverse of flights over landmarks. Therefore, for an accurate control of the path, taking into account the reduced accuracy of direction finding, due to the lack of sighting lines, it is necessary to image the landmarks at course angles which are as large as possible.

The ground speed of the aircraft and drift angle can be determined most easily with the aid of aircraft radar by using successive measurements of the LA, (locus of the aircraft), especially in orthodromic coordinates, when it is not necessary to plot the locus of the aircraft on a chart. The essence of this method has been described above. However, the method used for measuring the ground speed on the basis of successive measurements of the locus of the aircraft, is insufficiently practical for measurements of the drift angle of an aircraft. The fact is that for an accurate measurement to be made along a given path, it is necessary to determine the drift angle quite frequently and rapidly, so that the method of successive measurements of the locus of the aircraft requires a large base for measurements.

In some cases, it is advisable to use other methods for determining the ground speed (e.g., if visual points lie in the field of vision of the radar which do not allow the position of the aircraft to be determined) since they do not appear on the chart. However, they are suitable for determining the drift angle and the ground speed by visual methods.

There are several methods of determining the drift angle and the ground speed by visual means. Let us discuss several of them which are most often employed:

1. Measurement of the drift angle of an aircraft on the basis of the secondary Doppler effect. The directionality of the characteristic of radiation from an aircraft radar in the horizontal plane is made as narrow as possible. The narrower the beam for the propagation of electromagnetic waves, the better the resolving power of the radar in a tangential direction (perpendicular to the radius of the scan). However, in order to produce a very narrow charac- /318
teristic of radiation, we must use an antenna reflector on the radar which has very large dimensions. Therefore, the practical width of the beam is 2-3°.

The widening of the characteristic of directionality within these limits is undesirable in principle for surveying the terrain, but can be used very advantageously for measuring the drift angle by the so-called secondary Doppler effect. The essence of this method is the following.

Let us say that we have stopped the rotation of the radar antenna at a certain angle to the direction of the aircraft's motion (Fig. 3.45).

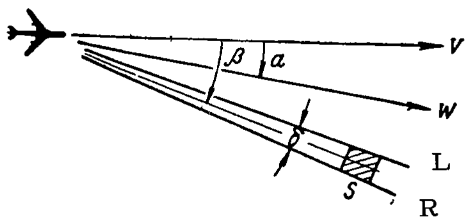


Fig. 3.45. Creation of the Secondary Doppler Effect.

In the picture, we can see the reflection of the electromagnetic waves from the elementary area S which we have selected.

The high frequency reflected from the Earth's surface, when received aboard the aircraft, will not be equal to the frequency radiated by the radar, but will have a certain positive or negative frequency shift which is called the Doppler effect.

Let us also note that the Doppler effect is proportional to the cosine of the angle between the direction of the aircraft's motion and the direction of the wave propagation (i.e., $\beta-a$). Angle β here represents the course angle of the antenna position of the radar, while the angle a represents the drift angle of the aircraft.

For the sake of simplicity, let us consider the Doppler effect only for two extreme limits of the beam with a common characteristic of radiation directionality, the left-hand beam is marked L and the right-hand beam R in our diagram.

The solution of the characteristic will be represented by the angle δ, so that

$$f_D L \sim \cos\left(\beta - a - \frac{\delta}{2}\right);$$

$$f_D R \sim \cos\left(\beta - a + \frac{\delta'}{2}\right),$$

where f_D is the Doppler frequency.

Thus, we see that the Doppler effect on the left-hand edge of the beam is greater than on the right-hand side, so that the frequency received by the antenna from the left-hand side of the beam will be somewhat higher than that from the right.

The frequencies of the left (L) and right (R) boundaries of the beam will be combined in the receiver and produce an intermediate frequency as follows /319

$$f_r = f_DL - f_DR,$$

which will amount to amplitude modulation of the received signal.

Now let us say that the direction of the antenna coincides with the direction in which the aircraft is moving, i.e., $\beta=\alpha$. Then the Doppler frequencies of the left and right sides of the beam will be uniform in value and proportional according to the cosines $\delta/2$:

$$f_DL \sim +\cos\frac{\delta}{2};\ f_DR \sim -\cos\frac{\delta}{2},$$

and the amplitude modulation from the edges of the beam will be absent.

In actuality, there will be a very low-frequency amplitude modulation owing to the difference in the Doppler frequencies of the edges of the beam relative to the effect of the center of the beam (the bisectrix of the radiation characteristic), but due to the very small difference between the cosines of the angles, the beat frequency will be very low (expressed in Hertz), while the visual effect of the beat is maximum.

With circular rotation of the antenna, the beating of the frequencies is not noticeable to the eye, since each of the luminous points is rapidly crossed by the scanning beam and appears on this screen as an individual point with subsequent afterglow.

A slight impression remains of the secondary Doppler effect in a fixed antenna, when its direction differs considerably from the direction in which the aircraft is moving, since the flickering of the points in this case takes place at high frequencies and is blurred by the afterglow on the screen.

If the direction of the antenna slowly approaches the direction in which the aircraft is moving, the luminous points all begin to flash at a reduced frequency and increased amplitude. A slow but bright flashing of the luminous points on the screen indicates a coincidence of the direction of the antenna with the direction in which the aircraft is moving. The drift angle of the aircraft is determined by the position of the scanning lines on the screen with maximum secondary Doppler effect.

Measurement is performed best of all with a large-scale operation of the radar (20 km for the screen radius), using a scanning delay of 20 km. It is then necessary to make a corresponding amplification in the receiver for the common amplification channel, in both the high and low signal levels, as well as the corresponding inclination of the antenna.

One advantage of the method of determining the drift angle of the aircraft according to the secondary Doppler effect is its high accuracy. With a little experience in selecting the receiver /320
amplification and the angle for tilting the antenna, measurements can be made literally within several seconds.

Several types of sector-type radars have a special operating regime and an additional indicator for measuring the drift angle according to the secondary Doppler effect.

2. Measurement of the drift angle and ground speed by sighting points near the course. If a clearly visible point is located near the line of flight of the aircraft on the radar screen, the ground speed and the drift angle of the aircraft can be measured by the movement of this point.

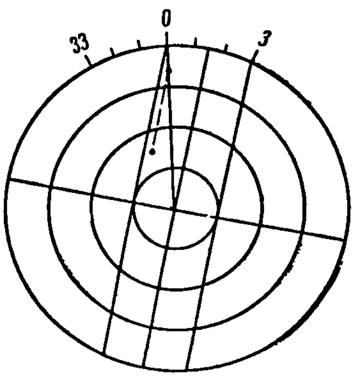


Fig. 3.46. Change in the Drift Angle and Ground Speed of a Point Near the Course Indicator on a Radar Screen.

To avoid gross errors in measurement due to altitude errors, the sighting of the points must be made at distances from 60 to 30 km. At the moment when the point being observed crosses the 60 km marking, the timer is switched on. The bearing scale of the radar is then set with the zero division opposite the course indication, and the sight is moved so that the landmark is located along its parallel lines (Fig. 3.46). When the object crosses the 30 km marking, the timer is switched off and the flight time of the base is calculated.

The drift angle is calculated directly from the bearing scale, with negative drift angles being calculated as added to 360°. To determine the ground speed, the correction for flight altitude for a distance of 30 km is added to the length of the base, taking into account the correction for a distance of 60 km as equal to zero. Thus, at flight altitudes of 8-10 km, the length of the base turns out to be equal to:

At a height of 8 km, 30.5 km; at a height of 9 km, 31 km; at a height of 10 km, 31.5 km.

The ground speed is determined by means of a navigational slide rule (Fig. 3.47, a).

Let us say that at a flight altitude of 10 km the time required to fly along the base between the 60 and 30 km markings is 2 min and 15 sec (Fig. 3.47, b). The ground speed in this case is 840 km/hr.

This method can be used with sufficient accuracy for measuring the drift angle of the aircraft. The accuracy of determination of the ground speed is obtained with a low and therefore very small /321 measurement base. Thus, e.g., at an airspeed of 900 km/hr, the error in measuring the flight time on the base(which amounts to 4 sec) produces an error in measuring the ground speed of about 30 km/hr. In addition, at large drift angles, when the vector of the motion of the target point does not agree with the radius of the screen, errors arise in determining the measurement base from the distance markings on the screen.

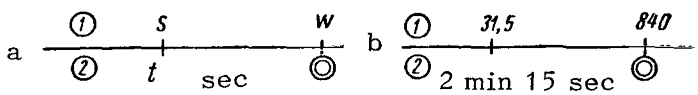


Fig. 3.47.

Fig. 3.47. Determination of Ground Speed of a Point Near the Course Indicator on a Radar Screen.

Fig. 3.48. Determination of Drift Angle and Ground Speed by Means of a Right Triangle.

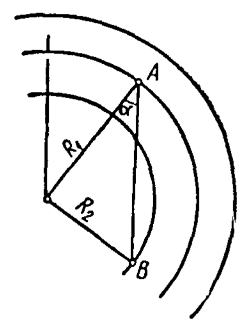


Fig. 3.48.

3. **Determination of the drift angle of the aircraft and the ground speed by means of a right triangle.** This method is more convenient and precise in comparison to the sighting of the motion of a landmark near the course. In addition, the use of the right triangle method makes it possible to select more freely the landmarks on the radar screen in order to track them.

The bearing scale of the radar is set to zero opposite the course marking, after which the course angle of the landmark is measured with the sight, its distance on the circular markings is observed, and the timer is switched on. Leaving the sighting instrument in a fixed position, the operator observes the motion of the landmark across the screen. At the moment when it crosses the perpendicular line on the sight (Fig. 3.48), the timer is switched off, the distance to the landmark is determined by the circular markings, and the flight time along the base is calculated.

Corrections are then made in the first and second measurements of the oblique distance for the flight altitude; angle α between the position of the sighting line and the direction of the movement of the landmark is then determined as follows:

$$\operatorname{tg} \alpha = \frac{R_2}{R_1},$$

and the length of the measurement base is:

$$S = \frac{R_1}{\cos\alpha} = \frac{R_1}{\sin(90-\alpha)}.$$

This problem is easily solved on a navigational slide rule (Fig. 3.49). /322

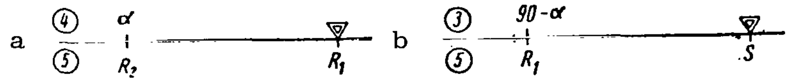


Fig. 3.49. Keys for Determining the (a) Acute Angle of the Triangle and (b) Measurement Base on the NL-10M.

The drift angle is determined as the difference between the first course angle of the landmark and the angle α (Fig. 3.49, a), while the ground speed is determined as the length of the base relative to the time required to cover the distance (Fig. 3.49, b).

Example. At a flight altitude of 10 km, the course angle of a landmark was initially equal to 8° at OD_1 = 60 km. The oblique distance at the moment when the landmark crosses the transverse line in the sight was 23 km. The flight time along the base was 5 min and 35 sec. Find the drift angle of the aircraft and the ground speed.

Solution. The correction for the flight altitude for the first distance will be considered as equal to zero. The correction for the second distance (OD = 23 km, *H* = 10 km) is equal to 3 km, so that the horizontal distance is HD_2 = 20 km.

On the navigational slide rule, we find the angle α = 18.5° (Fig. 3.50, a) and the length of the measurement base is *S* = 60 km (Fig. 3.50, b).

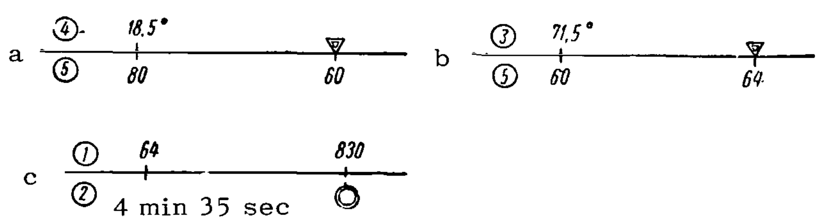


Fig. 3.50. Determination of (a) the Acute Angle of a Triangle, (b) the Base and (c) the Ground Speed on the NL-10M.

Therefore

$$US = 8 - 18.5° = -10.5°.$$

The ground speed (W) is therefore equal to 830 km/hr (Fig. 3.50, c).

4. Determination of the ground speed and drift angle of an aircraft by double distance finding using a sighting point with equal oblique distances.

This method is the most precise of the methods which we have discussed which use sighting of landmarks. However, it calls for the maximum time for measurement and calculation.

When a highly visible point shows up in the forward part of the screen, the crew waits until it reaches one of the circular distance markings (Fig. 3.51). At the moment when this point crosses the distance marking, the timer is switched on and the course angle of this point is measured. The crew then waits until this point moves across the screen and crosses the same circular distance marking at the rear of the screen. At the moment when it crosses it, the timer is switched off and the course angle of the point is measured once again.

Since in this case $HD_1 = HD_2$, the line of motion of the point /323
(from A to B) is perpendicular to the bisectrix between CAL_1 and CAL_2, i.e., if the point moves to the right of the course line of the aircraft, the drift line of the aircraft is determined by the formula

$$US = \frac{CAL_1+CAL_2}{2} - 90°,$$

and if the point moves to the left of the course line

$$US = \frac{CAL_1+CAL_2}{2} + 90° - 360°,$$

or

$$US = \frac{CAL_1+CAL_2}{2} - 270°.$$

To determine the ground speed, a correction for flight altitude is made in the oblique distance at points A and B and the length of the measurement base is determined by the formula

$$S = 2R\sin\frac{CAL_2-CAL_1}{2}.$$

If $HD_1 = HD_2$ exceeds five times the flight altitude, the correction for altitude is considered to be zero.

Example. H = 10 km, $HD_1 = HD_2$ = 60 km; CAL_1 = 32°; CAL_2 = 152°; the flight time along the base is 8 min and 15 sec. Find the drift angle and the ground speed.

Solution.

$$US = \frac{32° + 152°}{2} - 90° = +2°;$$

$$S = 2 \cdot 60 \sin \frac{152 - 32}{2} = 120 \sin 60°;$$

By using a navigational slide rule, we can solve the latter equation and find the ground speed (Fig. 3.52).

$$S = 105 \text{ км};$$
$$W = 765 \text{ km/hr}.$$

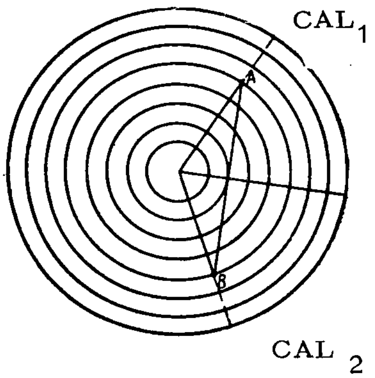


Fig. 3.51.

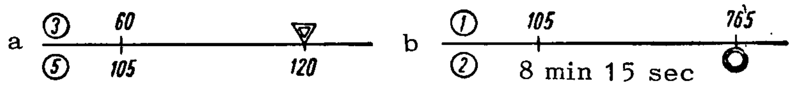


Fig. 3.52.

Fig. 3.52. Determination of (a) the Measurement Base and (b) the Ground Speed on the NL-10M.

Fig. 3.51. Determination of the Ground Speed and Drift Angle by Double Distance Finding of a Landmark at Equal Oblique Distances.

We should mention that in solving problems in determining the drift angle of an aircraft by the four methods enumerated above, the bearing scale of the radar may be set to the aircraft course rather than zero, e.g., according to the orthodrome. Then the course angles in all the formulas will be replaced by the bearings of the landmarks, and the result of the solution will not be the drift angle but the actual flight angle of the aircraft. /324

Autonomous Doppler Meters for Drift Angle and Ground Speed

Autonomous meters for ground speed and drift angle of an aircraft, based on the Doppler effect, offer broad perspectives for automation of the processes of aircraft navigation and pilotage of aircraft.

Fig. 3.53. Diagram of Formation of Doppler Frequency With a Moving Object.

Continuous measurement of the motion parameters of an aircraft makes it possible to use simple integrating devices for a precise automatic calculation of the aircraft path in time. In addition, a constant knowledge of these parameters makes it possible to regulate them in such a way that the aircraft follows a given flight trajectory with a minimum number of deviations.

All other radio devices for aircraft navigation make it possible to determine only the locus of the aircraft. The motion parameters of an aircraft can be determined only discretely for individual path segments, using the navigational devices described above.

As we pointed out at the beginning of Chapter One, the flight regimes of an aircraft are almost never stable, with the exception of the end points of curves along separate parameters. A strictly stable flight regime for all parameters simultaneously is never encountered. Therefore, automatic or semi-automatic calculation of the path on the basis of motion parameters measured over individual segments is a very approximate and unreliable method.

The operating principle of Doppler meters is the following.

Let us say that we have a moving source of electromagnetic oscillations at a high frequency A and a fixed object B which reflects these oscillations (Fig. 3.53).

If the source A remains fixed relative to object B, then after a period of time which is required for the electromagnetic waves to travel from point A to point B, electromagnetic oscillations will be set up in the latter at the same frequency as those emitted by the source. /325

When the source of oscillations moves toward point B, each successive cycle of oscillations is emitted somewhat closer to this point; its propagation time to reach point B is somewhat less than in the preceding cycle, so that the moments at which the oscillations arrive at point B can be compared.

Let us call the wavelength of the source λ, and the propagation rate of electromagnetic waves c. With a fixed source, the frequency of the oscillations (f) both at the point of emission and at the point of reflection of the waves will be equal to

$$f = \frac{c}{\lambda} .$$

With a movable source, the number of oscillations reaching point B per unit time is increased by the number of wavelengths contained in the distance covered by the aircraft in that same unit time, i.e.,

$$f = \frac{c+W}{\lambda} = \frac{c}{\lambda} + \frac{W}{\lambda} .$$

The increase in the frequency W/λ, produced by the motion of the source, is called the *Doppler frequency* (f_D).

Similarly, the oscillation frequency at the point of reflection will decrease if the source recedes from the reflection point for the electromagnetic waves.

Doppler meters work on the same principle of signal transmission as aircraft radars, i.e., frequencies are received that have been emitted by aircraft sources after their reflection from the Earth's surface. Therefore, a double Doppler frequency is received which arises along the path of electromagnetic waves, from the aircraft to the reflecting surface and along the reverse route from the reflecting surface back to the approaching or receding aircraft.

There are three ways of separating the Doppler frequency in receiving signals aboard an aircraft:

(1) The internal coherence of the signals, when the received frequency is combined within the receiver with a frequency radiated by the source, as a result of which there is a beating of the double Doppler frequency;

(2) External coherence, when the receiving antenna picks up signals which have been reflected from the ground as well as signals radiated by the transmitting antenna through the external medium;

(3) Autocoherence of the signals; in this case, the frequencies of signals reflected from the Earth's surface in the forward and backward radiation of the receiving-transmitting antenna are combined in the receiver without the frequency radiated by the antenna. Since the oscillation frequency is increased by 2 f_D relative to the preceding beam, and decreased by the same value for the following beam, the beat frequency will be equal to four times the Doppler frequency. /326

If we agree to call the Doppler frequency the beat frequency separated in the receiver as a result of superposition of the signals, then for the cases of internal and external coherence we will have

$$f_D = \frac{2W}{\lambda},$$

and for the case of autocoherence we will have:

$$f_D = \frac{4W}{\lambda}.$$

In principle, Doppler meters with internal and external coherence can be made with a single-beam antenna, but with autocoherence a minimum of two beams is required. In practice, as we will see later on, it is convenient to use antennas with three or four beams. Recently, the most widely employed type is the Doppler meter with four-beam antennas.

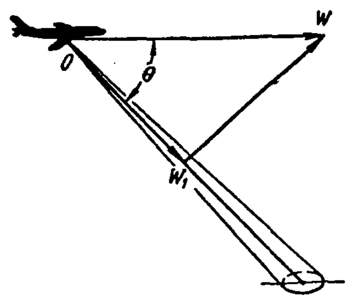


Fig. 3.54. Projection of the Ground-Speed Vector on the Direction of the Radiation of Electromagnetic Waves.

Since the characteristics of directionality of the antennas of Doppler meters in the general case do not coincide with the vector of the ground speed of the aircraft, it is necessary to consider the actual Doppler frequencies separated in the receivers.

Usually, the slope of the antenna beams of the meter is selected so that the areas of their reflection from the Earth's surface are not too far from the aircraft, i.e., the power of the transmitter is used most effectively. The slope angle of the beam relative to the horizontal plane is called the angle θ (Fig. 3.54).

Obviously, when the beam is inclined relative to the plane of the horizon, the Doppler frequency will be proportional not to the modulus of the ground speed vector, but to its projection in the direction of the antenna beam. For example, for a meter with internal coherence,

$$f_D = \frac{2W}{\lambda} \cos \theta .$$

On the other hand, the ground speed vector of the aircraft can be divided into two vector components: /327

$$\overline{OW} = O\overline{W}_1 + W_1 W.$$

The vector W_1W is directed perpendicular to the antenna beam, and therefore the Doppler effect is not produced. The vector

$$\overline{OW}_1 = \overline{OW} \cos .$$

is effective.

In addition to the fact that the antenna beam is set at a certain angle to the vertical plane, the antenna beam is usually directed at a certain angle to the longitudinal axis of the aircraft in the horizontal plane. For example, with a four-beam antenna, the longitudinal axis of the aircraft is the bisectrix of the angles between the directions of the forward and rear beams of the antenna (Fig. 3.55). The angle between the longitudinal axis of the aircraft and the direction of the antenna beam in the horizontal plane is called the angle β.

Hence, in receivers with internal and external coherence, the separated Doppler frequency

$$f_D = \frac{2W}{\lambda} \cos\theta \cos(\beta - \alpha),$$

where α is the drift angle of the aircraft.

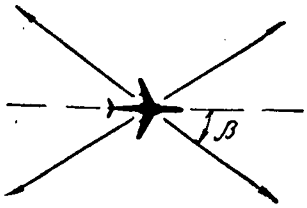


Fig. 3.55. Diagram of the Positions of the Beams from an Antenna on a Doppler Meter.

In the special case where the drift angle of the aircraft is absent, the Doppler frequency for each antenna beam will be the same:

$$f_D = \frac{2W}{\lambda} \cos\theta \cos\beta.$$

Three- and four-beam antennas are desirable because they make it possible to compensate automatically for errors in measurements which arise with longitudinal and transverse rolling of the aircraft.

At the same time, in cases when single beam or two-beam antennas are used, they must be placed on gyrostabilizing devices. In the opposite case, longitudinal or transverse rolling of the aircraft will change the slope angle of the antenna θ, thus leading to a change in the Doppler frequency.

In the case of a four-beam antenna, the longitudinal or transverse rolling of the aircraft produces a change in the slope angle of one pair of beams in a positive direction and changes the opposite pair in the negative direction by the same magnitude. If angles θ are then located on an approximately linear section of the cosine /328 curve, the frequency shift of the opposite antenna beams will be opposite in sign but approximately the same in magnitude, which can also be used for compensating roll errors in the system (Fig. 3.56, a).

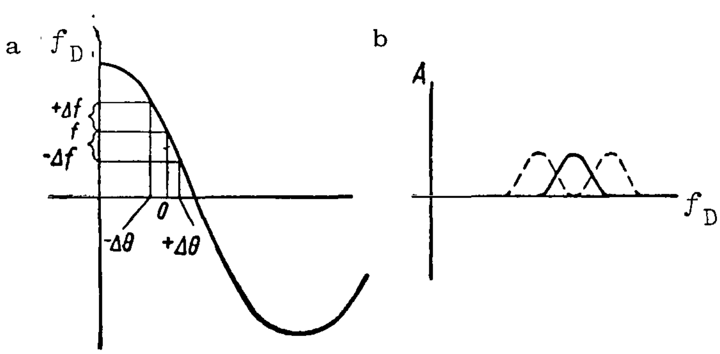


Fig. 3.56. Shifts in the Doppler Frequency with Tilting of the System: (a) Change in the Cosines of the Angles; (b) Frequency Shift.

For example, in the case of receivers with autocoherence, when the Doppler frequency increases in the front right-hand beam and decreases by the same magnitude in the rear left-hand beam, the beat frequency of one pair will simply be retained.

In systems with internal and external coherence, turning of the antenna leads to doubling of the frequency spectrum of the opposite beams (Fig. 3.56, b).

With a horizontal position of the longitudinal and transverse axes of the aircraft, the Doppler frequency in the forward and opposite rear beams will be the same. Tilting the system shifts the frequency spectrum of one of the beams forward, and that of the opposite beam backward to the same extent. However, if we add up these frequencies with time and divide them by the measurement time, the average frequency will turn out to be equal to the frequency of the horizontal position of the axis of the aircraft.

Doppler meters which are presently in use can be divided into four types, depending on the regime of emission and reception of signals:

1. **Pulse meters.** In transmitters, these meters produce high-frequency pulses in the same manner as is done in aircraft radars. Reception of reflected signals takes place in the intervals between pulse emission. In order to separate the Doppler frequency, autocoherence by beam pairs is employed.

A shortcoming of this method is the presence of "dead" altitudes, i.e., when the reflected signals arrive at the moment coin- /329
ciding with the emission of pulses, and not in the intervals between them. In addition, when flying over mountainous terrain, the distance to the Earth's surface according to opposite beams of the antenna may not be the same, thus leading to a failure of the arrival of reflected signals to coincide for these beams and producing a disturbance of their coherence.

Another shortcoming of pulse meters is the need for high voltages to drive the magnetron in analyzing the high-frequency pulses, thus necessitating a hermetic sealing of the transmitter units and subjecting them to a certain pressure.

2. **Meters using continuous radiation of high frequency.** In this case, the high frequency is radiated continuously by the transmitter. Reception of signals is accomplished with a separate antenna, having a certain by-pass coefficient with the transmitting antenna.

The reflected signals are combined in the receiving antenna with the frequency produced by the transmitting antenna, so that the beat frequency is separated out in an external coherence system.

The advantages of this method are the independence of the reception conditions for the signals of flight altitude and the local relief. In addition, devices of this type worked at relatively low powers in the receiver mechanism.

A shortcoming of this method is the need to have separate antennas for transmission and reception of the signals.

3. **Meters with continuously pulsed radiation.** These meters employ a constant regime of generation and transmission with pulsed emission of a portion of the high frequency into the antennas by means of commutating devices. The reflected signals are combined with the frequency developed by the transmitter in the intervals between the moments when the high-frequency segments are emitted (internal coherence). This means that it becomes possible to use part of the advantages of continuous emission (operation at relatively low voltages in the transmitter circuits) and the pulse systems (reception and transmission with a single antenna).

However, the shortcomings still remain which afflict pulsed meters, i.e., the presence of "dead" altitudes and the effect of the relief on reception conditions. In addition, there are also difficulties in using these meters at low flight altitudes, since at a very short signal path, the moments of transmission and reception are practically impossible to separate.

4. **Meters with frequency modulation of signal transmission.** Frequency modulation of signal transmission can be used either in a pulsed or continuous-pulsed regime of operation for the meter.

If the transmission of high-frequency pulses at a constant frequency at low altitudes makes it possible to superpose the moments of reception of signals on the moments of emission, while in moun- /330 tainous regions there may be disruptions in the coherence of the signals, when there is a change in the frequency of the transmission of the signals, and only a portion of them will contribute to the combination with the radiation moments. The majority of signals will be received in the intervals between the moments of emission, since the duration of the intervals is made sufficiently greater than the duration of the pulses. To a certain degree, in this case, the effect of disruption of coherence is reduced in mountainous regions.

The best properties are exhibited by continuous-pulsed meters with frequency modulation of signal transmission, since in this case all positive qualities of the continuous and pulsed systems are employed. However, the shortcomings of the pulsed systems remain, including difficulty in making measurements at very low flight altitudes.

Of the types of Doppler meters which we have discussed, the ones which are currently used most widely are the meters with continuous radiation and continuous-pulsed meters with frequency modulation of signal transmission.

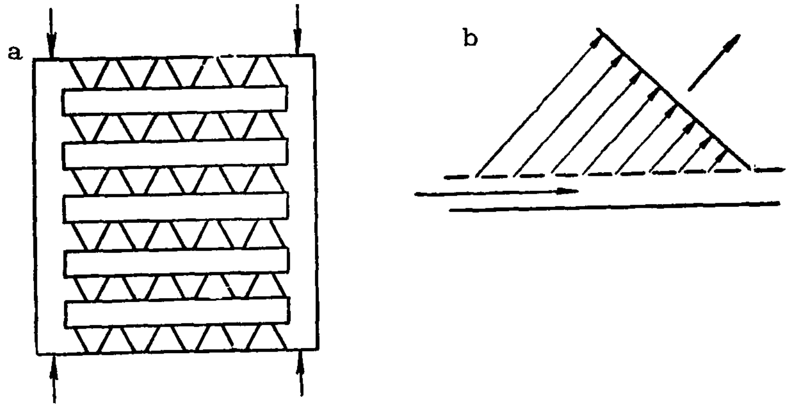


Fig. 3.57. Doppler-Meter Antenna: (a) Waveguide Lattice; (b) Diagram of Beam Formation.

In the first types of Doppler meters, beginning with the single beam versions, reflector-type antennas were used with an isosceles directional characteristic. Recently, antennas of the "waveguide network" type have come into use.

The principle of operation of these antennas is the following (Fig. 3.57, a).

Imagine a rectangular lattice, made up of waveguides, to one corner of which an electromagnetic wave of high frequency is applied.

On the upper walls of the transverse waveguide in this lattice, there are slots for emission of wave energy into space.

The wave energy, propagated along a transverse waveguide, emerges through the slits with a certain shift in time from one slit to /331
the next, so that there is an interfference of the waves emerging from the slits, as is the case in spaced antennas (Fig. 3.57, b).

The direction of the radiation maximum and the isophasal lines are located at an angle to the surface of the waveguide.

Since the electromagnetic energy propagated along the longitudinal wave guide reaches the next transverse waveguide also with a shift in time, a similar picture of interference with a tilting of the isophasal line also takes place along the waveguide lattice. As a result, an isophasal surface is formed above the waveguide lattice, having a slope in the direction of its diagonal toward the corner opposite the corner at which the electromagnetic waves enter the lattice. Consequently, one of the beams will be formed along the diagonal of the antenna.

If wave energy is also transmitted from the diagonally opposite corner of the lattice, two oppositely directed antenna beams can be formed simultaneously.

Flat, multi-beam antennas, especially when fixed in position, are very useful, since they can be placed below the radio-transparent housing flush with the skin of the aircraft and do not produce any additional aerodynamic resistance during flight.

Schematic Diagram of the Operation of a Meter with Continuous Radiation Regime

The high frequency processed by the transmitter passes through a commutation device to the transmitting antenna, where the beams for propagation of electromagnetic waves are formed in pairs. The commutating device is connected to the counter of the meter, to separate the frequencies of the first and second pairs of beams (Fig. 3.58).

A portion of the wave energy radiated by the transmitter reaches the receiving antenna, where it is combined with the received signals reflected from the Earth's surface, so that the Doppler frequency of the given pair of beams can be separated.

The separated Doppler frequency, after amplification, passes to the calculating device, at whose output is an indicator for the ground speed and drift angle of the aircraft.

As we already know, for the four-beam antenna of a Doppler meter with internal coherence, the separated frequency by beam pairs will be equal to:

(a) For the first pair,

$$f_{D_1} = \frac{2W}{\lambda} \cos\theta \cos(\beta + a);$$

(b) For the second pair, /332

$$f_{D_2} = \frac{2W}{\lambda} \cos\theta \cos(\beta - a).$$

For a Doppler meter, the sign of the angle is not important, but its absolute value is. Therefore, we can simply assume that with a positive drift angle, the drift angle in the right-hand pair of beams will be calculated from the angle β, while in the left-hand pair these angles will be added. With a negative drift angle, the calculation of the angles will be performed in the left-hand pair of beams, and combined in the right. Therefore, the last two formulas given above can be written in the form

$$\left.\begin{aligned} f_{D_1} &= \frac{2W}{\lambda} \cos\theta \cos(\beta + a); \\ f_{D_2} &= \frac{2W}{\lambda} \cos\theta \cos(\beta - a), \end{aligned}\right|$$

where the sign shows that the formulas change places for the left and right-hand pairs of beams when the sign of the drift angle changes.

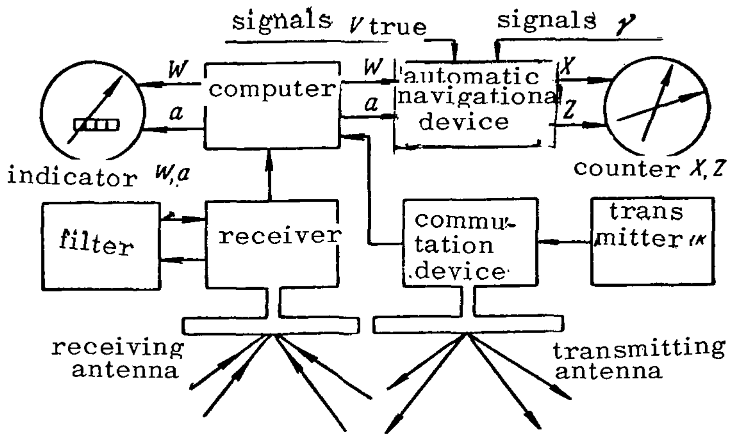


Fig. 3.58. Functional Diagram of A Doppler Meter.

Note. Since the pairs of beams are diagonal to the wave guide lattice, and each of them contains a left and right-hand beam relative to the longitudinal axis of the aircraft, the left or right-hand pair of beams here is referred to as a pair whose leading beam is directed to the left or right of the longitudinal axis of the aircraft.

Obviously, in the case of a fixed antenna on the aircraft (Fig. 3.59), the first problem for the calculating device of the Doppler meter is the determination of an angle at which /333

$$\frac{f_{D_1}}{f_{D_2}} = \frac{\cos(\beta + a)}{\cos(\beta - a)}.$$

Since the angle β is a constant value, and the frequencies f_{D_1}, f_{D2} are variable, the solution of a problem of this kind does not present any significant difficulties. The desired angle *a* is the drift angle of the aircraft.

The second problem for the computing device is the determination of the ground speed (*W*) with a previously known drift angle and Doppler frequency for a pair of beams:

$$W = \frac{f_{D_1}\lambda}{\cos\theta\cos(\beta+a)} = \frac{f_{D_2}\lambda}{\cos\theta\cos(\beta-a)}.$$

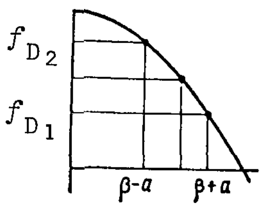


Fig. 3.59. Differece in Doppler Frequencies of the Right- and Left-Hand Beams of an Antenna.

The calculated drift angle of the aircraft and the ground speed are transmitted to the visual indicator of these parameters and also to the automatic navigational device for integration of the aircraft path in time.

The problem of the calculating device of the Doppler meter is significantly simplified by mounting a movable antenna on the aircraft. In this case, the direction of the antenna is set so that the Doppler frequencies of both antennas f_{D_1} and f_{D_2} will be the same, i.e., the bisectrix of the beams will coincide with the direction of the aircraft motion:

$$f_D = f_{D_1} = f_{D_2} = \frac{2W}{\lambda}\cos\theta\cos\beta.$$

Then the drift angle of the aircraft is determined by the course angle of the antenna setting, and the ground speed is found by the formula:

(a) With internal and external coherence

$$W = \frac{f_D\lambda}{2\cos\theta\cos\beta};$$

(b) With autocoherence

$$W = \frac{f_D\lambda}{4\cos\theta\cos\beta'}$$

This means that all coefficients entered into the formulas (with the exception of f_D) are constants while f_D is a variable quantity.

We should mention that during flight above the ocean, Doppler frequencies from pairs of beams in a Doppler meter are somewhat lower than above dry land at the same airspeeds. This is caused /334 by peculiar features of the reflection of electromagnetic waves from the surface of the water.

In flight above dry land, if the conditions for diffuse reflection of the waves are approximately the same over all areas in contact with the Earth's surface, and the maximum amplitude coincides with the center of the beam at the maximum of the radiation characteristic, then the reflection conditions above a watery surface will depend to a considerable extent upon the angle of incidence of the beam. Therefore, the leading edge of the beam will have a sharper angle of incidence (and therefore a lower signal amplitude), while the trailing edge of the beam will strike more obliquely and have somewhat greater amplitude. Consequently, the maximum amplitude of the signals shifts from the center to a region of lower Doppler frequency (see Fig. 3.54).

To compensate for errors in the operation of the meter above water, the circuit is designed to include a calibration element which is switched on from the control panel by turning a switch from the "land" position to the "sea" position.

Over a smooth watery surface (with a swell less than a scale value of one), the potential of the reflected signals becomes inadequate to ensure operation of the meter, and the latter then is turned off by switching the automatic navigational device to memory operation.

The channel of the Doppler frequency receiver is fitted with a filter intended to damp out all parasitic frequencies produced by other electronic devices mounted aboard the aircraft which could disturb reception of reflected signals from the Earth's surface. The filter must have a narrow passband within the region of Doppler frequencies of the received signals.

If the frequency of a carefully adjusted filter differs considerably from the midpoint of the range of Doppler frequencies being employed, it begins to introduce errors in the measurement of the Doppler frequency, shifting it toward the point of fine tuning of the filter. Therefore, filters are used with automatic tuning for the frequency of the signals employed.

Use of Doppler Meters for Purposes of Aircraft Navigation

Doppler meters for ground speed and drift angle are very effective in aircraft navigation. The following problems can be solved directly by using a Doppler meter:

(a) Maintainance of a given direction of flight along an orthodrome or loxodrome, automatically if desired. To do this, it is only necessary that the sum of the course (γ) and drift (a) angles /335
of the aircraft be constantly equal to a given flight path angle (ψ):

$$\psi = \gamma + a;$$

(b) The calculation of the path of the aircraft in terms of distance can be solved on the basis of the ground speed and time:

$$S = Wt.$$

The results of solving these problems by using Doppler meters are much more accurate than those obtained when other types of electronic devices are used, and there is less work involved.

However, accurate aircraft navigation requires constant monitoring of all variable factors in the ground speed, drift angle, and aircraft course, which are tedious for the crew. However, if we use individual, discrete values for calculating the drift angle, course, and ground speed, (e.g., every 15 min of flight time), as is done with aircraft radar, one of the basic advantages of Doppler meters will be lost: the constant supply of information regarding the motion parameters of the aircraft.

In view of the above, as well as the relative simplicity of automating aircraft navigation on the basis of Doppler measurements, the latter are practically impossible to use without combining them with automatic navigational instruments.

Automatic navigational instruments connected to Doppler meters calculate the aircraft path with time in an orthodromic or geographic system of coordinates.

To calculate the path of the aircraft in an orthodromic system of coordinates, the navigational devices are connected to transmitters of the orthodromic course (a gyro assembly for the course system, operating in the GSC regime). The automatic system includes a transmitter of the flight angle or (as it is usually called) the given chart angle (GCA).

The signals for the drift angle of the aircraft, obtained from the meter, and the course signals of the aircraft, obtained from the course system, are combined and their sum compared with a given path angle fed into the transmitter.

If the sum of the course and the drift angle of the aircraft is equal to the given path angle of the flight $\psi = \gamma + \alpha$, the ground speed is directed along the X-axis: $W = W_x$; $W_z = 0$.

If this equation is not satisfied, the vector of the ground speed is divided into two components:

$$W_x' = W\cos(\gamma + \alpha - \psi);$$

$$W_z = W\sin(\gamma + \alpha - \psi).$$

The vector components obtained for the ground speed along the axes of the coordinates are integrated over time and calculators are used to find the running values of the aircraft coordinates X and Z. /336

Calculation of the aircraft path and geographic coordinates can also be done directly on the basis of the signals from the Doppler meter and the course calculator. However, to do this it is necessary to have an exact knowledge of the true course of the aircraft and to express the division of the ground-speed vector of the aircraft according to the formulas:

$$\frac{d\varphi}{dt} = W\cos(\gamma + a);$$

$$\frac{d\lambda}{dt} = W\frac{\sin(\gamma + a)}{\cos\gamma}.$$

To ensure operation of the gyroscopic transmitter in a regime of true course, in addition to the moment which compensates for the diurnal rotation of the Earth

$$\omega_{\Phi} = \Omega \sin\varphi,$$

it is necessary to add the moment which compensates for the change in the true course with time due to the eastern or western component of the ground-speed vector of the aircraft:

$$\omega_{W} = \frac{W\sin(\gamma + a)\sin\varphi}{\cos\varphi} = W\sin(\gamma + a)\,\mathrm{tg}\,\varphi.$$

However, calculation of the aircraft course by this system cannot be considered adequate for three reasons:

(1) The path during flight with a constant true course is loxodromic, but this complicates the preparations and makes it more difficult to use the radio-engineering and astronomical methods during flight for correcting the coordinates of the aircraft. Therefore, in addition to the magnetic (true) compass on the aircraft, there must also be an orthodromic couse device.

(2) The constant dependence of the operation of the course system on the operation of the Doppler meter and a calculating device introduces inaccuracies into the aircraft navigational elements. For example, when the Earth is not visible, a flight can be made over dry land; however, if the aircraft then begins to travel over a smooth watery surface, the reflected Doppler signals will not only introduce errors into the accuracy with which the path is calculated with time, but will also incorporate errors in the operation of the course system.

(3) The errors which appear in the calculation of the aircraft course at the points of correction of its coordinates cannot be used directly for correction of the aircraft course, as can easily be done in an orthodromic system of coordinates.

A more logical calculation of the geographic coordinates of the aircraft would involve the orthodromic system of aircraft navigation, based on a constant conversion of the orthodromic course of the aircraft to the true course on the basis of the running coordinates of the aircraft:

$$\gamma_{true} = \mathrm{arctg}\,\frac{\mathrm{tg}\,\lambda_0 l}{\sin\varphi_l} + \Delta\phi_{ort}$$

where

$$\Delta\gamma_{ort} = \gamma_{ort} - \psi_{ort}.$$

The true course for the aircraft obtained in this manner can be used to calculate the geographic coordinates of an aircraft as was shown earlier; it can also be used for correcting the orthodromic course by astronomical means.

The advantages of a method of this kind are the independence of the true course from the ground speed and its automatic correction along with the correction of the aircraft coordinates. /337

However, we should mention that the calculation of the aircraft course and geographic coordinates should really be replaced by a constant conversion of the running orthodromic coordinates into geographic ones, e.g., by Formulas (1.64) and (1.65):

$$\sin\phi_{geog} = \sin\phi_{ort}\cos\theta - \cos\lambda_{ort}\sin\theta;$$

$$\sin\lambda_{geog} = \sin\lambda_{ort}\cos\phi_{ort}\sec\phi_{geog}.$$

In this case, the geographic coordinates will always agree strictly with the orthodromic ones, so that there will be output parameters from only one integrating device and automatic correction in the second system with correction of coordinates in one of them.

In general, the geographic coordinates are not of much interest as far as aircraft navigation is concerned. However, they are important for ensuring accurate operation of navigational transmitters (latitudinal correction of course systems, analysis of gyroverticals, reliable operation of inertial navigational systems, etc.). In addition, the geographic coordinates can be conveniently used in the presence of astronomical methods of aircraft navigation and for introducing the coordinates of reference points into the calculating device, used for correction of aircraft coordinates.

For purposes of aircraft navigation, automatic navigational devices are much more dependable for calculating the path of the aircraft in orthodromic coordinates.

In addition to the basic regime of operation by signals from a Doppler meter, automatic navigational devices as a rule have an operating regime with "memorized" navigational parameters.

The regime for operating by "memory" can be incorporated in one of the following two versions.

1. By "memorizing" the last values of the ground speed and drift angle of the aircraft. In this version, in the case when

there is an interruption in the arrival of Doppler signals for some reason, (e.g., when there are no waves in a flight over water), the path can be calculated by "memory" for a period of 15-20 min, only under the condition that the flight direction and airspeed have been recorded. With a changing flight regime for the aircraft, calculation by "memory" leads to large errors, since the ground speed and drift angle change on a new course or with a change in other parameters.

2. By "memorizing" wind parameters at flight altitude. In this variety, the calculating device is provided with special "memory" potentiometers, which constantly set the value of the wind parameters:

$$u_x = W\cos(\gamma + a - \psi) - V\cos(\gamma - \psi);$$

$$u_z = W\sin(\gamma + a - \psi) - V\sin(\gamma - \psi).$$

Then, if the signals should not be received from the Doppler meter, the path of the aircraft can be calculated by comparing the vector of the wind speed along the axis of the system of coordinates with the wind vector components added to it. If the given path angle of the flight then changes, the components of the wind vector are redistributed among the coordinate axes and the calculation regime is not disturbed. /338

However, in both the first and second methods of "memorizing" navigational parameters, no provision is made for an exact calculation of the aircraft path during a long period of time, since the wind parameters change with distance. In these cases, the navigational mechanism is used for calculating the path of the aircraft on the basis of discrete data obtained by measuring the ground speed and drift angle, e.g., by means of a aircraft radar or some other device, as is done (e.g.) when using the navigational indicator NI-50B.

In some types of navigational instruments, inertial or astro-inertial instruments are used as memory devices.

The problem does not involve a detailed study of inertial navigational devices, because the latter have not yet found wide application in civil aviation. In addition, the problem of the advisability of installing them is still not sufficiently clear, since the considerable complexity and stability of these instruments means that the range of problems which they can solve is still extremely narrow. Therefore, we will content ourselves with a brief mention of the operating principle of these instruments.

Inertial navigational devices are gyrostabilized platforms on which accelerometers and special gyroscopes are mounted which integrate the accelerations of the aircraft with time along the axes of the reference system.

In the case when the motion of the aircraft along one or two axes takes place with acceleration, a moment is applied to the axes of the gyroscope which is proportional to these accelerations, so that precession of the gyroscope axes takes place, i.e., there is integration of accelerations with time.

Since

$$W_x = \int_0^t a_x(t)\,dt$$

and

$$W_z = \int_0^t a_z(t)\,dt,$$

where a_x and a_z are the accelerations along the corresponding axes, we can use the position of the gyroscope axes to get an idea of the components of the aircraft speed along the axes of the coordinates.

The components of the ground speed along the axes of the reference system can be integrated in turn with time by means of a navigational instrument.

In an operating Doppler meter, the position of the axes of the integrating gyroscopes can be corrected by signals from this meter. In the case when the Doppler information does not arrive, the inertial device can be used for a long period of time to retain "remembered" values of the components of the speed along the axes of the coordinates, correcting them for any accelerations that arise in the way of wind changes, as well as in changes in the flight regime.

Aircraft navigation using Doppler meters and automatic navigational instruments becomes extremely simple and practical, but very careful preparations for flight and exact measurements of the coordinates of the aircraft at the correction points are required. An exact measurement of the aircraft course is extremely important /339
in this regard.

On the other hand, the fact that the crew is constantly aware of the ground speed, the drift angle of the aircraft, and its coordinates makes it possible to maintain a given flight trajectory for long periods of time according to the indications of the instruments. To do this, it is sufficient that the sum of the aircraft course and the drift angle be constantly equal to the given path angle, and that the Z-coordinate of the aircraft be equal to zero.

It is particularly easy to solve problems in aircraft navigation if the readings of the aircraft course and the drift angle are obtained from the indicator in the form of a sum, i.e., as the actual path angle of the aircraft flight. It is then sufficient to pilot the aircraft so that with Z equal to zero, the flight angle will actually be equal to the given one.

In a case when the path angle of the flight is not maintained precisely and the Z-coordinate of the aircraft is not equal to zero, or, if the improper operation of a system has caused the aircraft to deviate from the given flight path as revealed by correction of its coordinates, the path angle of the flight is set so that the aircraft approaches the given line of flight at an angle of 3-5°. When the Z-coordinate decreases to zero, the path angle of the flight becomes equal to the given value.

The aircraft can be placed on the given line of flight by using the autopilot. For this purpose there must be a calculating unit aboard the aircraft for relating the Doppler meter with the automatic navigational device and an autopilot which solves the simple problem:

$$\Delta Z + k\Delta\psi = 0,$$

where ΔZ is the lateral deviation from the line of flight, $\Delta\psi$ is the angle of approach to the line of flight, and k is the selected coupling factor.

The aircraft is then steered so that a lead in the path angle of the flight is taken when the aircraft deviates to a certain degree from the given line of flight with a certain coefficient. Then, in the presence of lateral deviation, the aircraft will automatically move into the line of flight, decreasing its lead as it approaches the latter.

Certain difficulties in aircraft navigation when using Doppler meters with automatic navigational devices are encountered in converting the computer to calculate the path in orthodromic coordinates of the previous stage, at the turning points along the route. The methods of conversion to the new system of calculation of coordinates is shown in Chapter II, Section 9. However, when using Doppler meters, it is better to set the aircraft coordinates to the reference system of the previous stage before beginning the /340
turn of the aircraft. For example, with $Z_1 = 0$, $X_1 = -\text{LLT}$.

$$X_2 = -\text{LLT} \cos \text{TA};$$

$$Z_2 = \text{LLT} \sin \text{TA}.$$

For purposes of simplifying the conversion of the path calculation into the system of coordinates of the next stage, double coordinate calculators are used, linked mutually with one another. In this case, the calculation of the aircraft path is performed by one of the calculators in the system of coordinates of the flight segment in which the flight is being made. The second calculator adjusts itself according to the path angle of the next path segment, and it can be used to calculate the aircraft coordinates in the reference system of this segment.

The transition of the aircraft to the next orthodromic segment of the path is accomplished by the indications of the second calculator, after which the first calculator is cleared and set for the next path segment.

As we have already pointed out, in the case of double calculators, their readings are mutually related, i.e., they are converted according to the formulas:

$$X_2 = X_1 \cos TA - Z_1 \sin TA;$$

$$Z_2 = X_1 \sin TA + Z_1 \cos TA.$$

Therefore, in correcting the coordinates of the aircraft on one of these computers, a correction is automatically made in the aircraft coordinate in the reference system of the next stage.

Thus, at each turning point along the route, the aircraft makes a turn in a previously prepared and corrected system of coordinates for the next stage of flight, thus completely getting rid of any undesirable features of the transition which might occur if only one calculator were used.

Preparation for Flight and Correction of Errors in Aircraft Navigation by Using Doppler Meters

Aircraft navigation using Doppler meters for measuring the ground speed and drift angle of an aircraft can be done very simply and rapidly. However, the required accuracy for aircraft navigation when using these devices can only be achieved with very careful preparation for flight, as well as careful correction for errors in aircraft navigation which arise during flight.

When using Doppler meters, there may be errors in measuring the following elements in aircraft navigation due to errors in the transmitters:

(a) Measurement of the aircraft course;

(b) Measurement of the drift angle and ground speed;

(c) In the programming of the given path angle and the distance of the flight stages; /341

(d) In the integration of the aircraft flight along the axes of the coordinates by the automatic navigational device.

The accuracy with which the aircraft course is measured is of extreme importance for aircraft navigation when using Doppler meters and is closely related to the proper programming of path angles for each flight stage. This is explained by the very high requirements for accuracy in determining path angles in preparing for flight.

Preparation for flight using Doppler meters must be carried out properly according to the third group of conditions in Chapter Two, Section 2.

For each flight segment, all parameters of the orthodrome must be determined, beginning with λ_{dis}:

$$\lambda_{dis} = \lambda_{01} - \lambda_1,$$

$$\mathrm{tg}\,\lambda_{01} = \mathrm{tg}\,\varphi_2\,\mathrm{ctg}\,\varphi_1\,\mathrm{cosec}\,\Delta\lambda - \mathrm{ctg}\,\Delta\lambda.$$

It is then necessary to determine the original azimuth of the orthodrome α_0 by the formula

$$\mathrm{tg}\,\alpha_0 \frac{\sin\lambda_{01}}{\mathrm{tg}\,\varphi_1},$$

and then the coordinates of the intermediate points on the orthodrome for plotting them on the chart:

$$\mathrm{tg}\,\varphi_i = \frac{\sin\lambda_{0_i}}{\mathrm{tg}\,\alpha_0}.$$

The distance between the turning points along the route along the orthodrome can be determined by the formula

$$\cos S_i = \cos\lambda_{0_i}\cos\varphi_i.$$

If we introduce into this formula the coordinates of the initial and final points of the flight segment, and (when necessary) any intermediate points, we can find the distance to those points from the starting point of the orthodrome. The distances between the points are determined by calculating the distances from the starting point of the orthodrome to them.

For programming the flight path angle we determine the azimuths of the orthodrome at the beginning and end of each segment according to the formula

$$\mathrm{tg}\,\alpha_i = \frac{\mathrm{tg}\,\lambda_{0_i}}{\sin\varphi_i}.$$

If it is proposed that we use astronomical methods for correcting the aircraft course in flight (e.g., in flight over water or terrain which has no identifying landmarks), the course correction points are marked and the azimuths of the orthodromes at the /342
correction points are determined by this formula.

Reference points are selected for correcting the coordinates of the aircraft during flight. Usually these are landmarks which show up clearly on radar or places where goniometric-rangefinding installations are located. Then the orthodromic coordinates of these points are determined, and the ground goniometric-rangefinding instruments are used to determine the azimuths of the orthodromic segments of the path on which these devices will be used, relative to the meridians on which the ground beacons are located.

The given path angle for the first flight segment is considered equal to the azimuth of the orthodrome at the starting point of this segment. The path angles of all subsequent path segments are considered to be equal to the sum of the path angle of the previous segment plus the angle of turn in the path at the turning point on the route.

Doppler meters have relatively low errors in measuring the drift angle of an aircraft, so that they can be compensated for in the total by the errors in aircraft course.

In general, besides the errors in measuring the drift angle, depending on the operating regime of the meter, the height and speed of flight, which have a more or less constant character, there are errors which have a fluctuating nature (oscillations in the meter readings from the average value). The principal reason for fluctuations is the varying conditions of reflection of electromagnetic waves from the Earth's surface.

When some point is encountered which reflects electromagnetic waves well, in the ellipse of reflection from the Earth's surface, the maximum of the amplitude of Doppler frequency is first displaced forward (for a rear beam, backward); then, as the point passes through the ellipse of reflection, the maximum of the amplitude shifts toward the average Doppler frequency and then backward, into a region of lower frequencies.

Thus, there is first a positive "firing" of the Doppler frequency, then a leveling off, and finally a negative "firing". For the rear beam, the "firings" of frequencies take place in reverse order.

The periods of fluctuating oscillations are short and depend on the time required for the reflecting points to pass through the ellipse of reflection. Practically speaking, they are located within the limits of 3-6 sec, so that they can be smoothed out to a considerable degree by selecting the proper rate of analysis for the readings of the drift angle and ground speed.

As far as the calculations of the aircraft path for distance and direction are concerned, the fluctuating oscillations do not have any noticeable effect on it, since after 3-5 min of flight the integral value of the positive fluctuations becomes equal to the integral value of the negative fluctuations.

The process of navigational exploitation of autonomous Doppler systems for aircraft navigation can be employed for adjusting the system itself, i.e., in correcting the aircraft coordinates /343 manually or automatically it is possible to determine and compensate simultaneously the systematic errors in the operation of the system as a whole.

In fact, if the aircraft (at the starting point of a flight segment) is located precisely on the desired flight line, but its *Z*-coordinate is equal to zero, and we keep the coordinate *Z* equal to zero during all subsequent stages of the flight, the aircraft will have to remain on this line constantly. If this is not the case, an error will crop up in the calculation of the aircraft path with respect to direction, i.e., a certain angle will develop between the given and actual flight path angles of the aircraft.

It is most likely that under the conditions of precise determination and setting of a given path angle on the transmitter, an error in calculation will arise as a result of improper measurement of the aircraft course, since gyroscopic devices can show drift in their readings with time. Therefore, the total correction which is required for proper calculation of the path should most logically be made in the readings of the course instrument.

If a portion of the errors in calculating is not related to the operation of the course instrument, then their contribution to the course errors will not make the accuracy of aircraft navigation any worse.

The latter statement is valid for a complex of instruments which permit calculation of the path in terms of direction, but it is theoretically not completely valid for instruments intended for distance finding of landmarks for the purpose of making corrections in the aircraft coordinates.

Nevertheless, if we consider that the total error in measuring the drift angle and calculating the paths in terms of direction with an automatic apparatus is no more than 0.2 to 0.3° as a rule, we must recognize that correction of the aircraft course by the results of calculating the path is much more accurate than correcting it by any other methods, including astronomical ones.

During flight, the actual aircraft coordinates are determined by the distances *R* and the path bearings of the landmarks (PBL), selected for this purpose by the formulas:

(a) In the measurement of aircraft radars

$$X = X_l - R \cos \text{PBL};$$
$$Z = Z_l - R \sin \text{PBL}.$$

(b) In the measurement of goniometric-rangefinding systems:

$$X = X_м + R \cos (A - \psi_м);$$
$$Z = Z_м + R \sin (A - \psi_м),$$

where X_M and Z_M are the orthodromic coordinates of a ground beacon.

Obviously, the formulas for the aircraft radar and the goniometric-rangefinding systems are invariable. The difference in the signs of the second terms on the right-hand sides is explained by the fact that the bearing of a landmark is obtained with the aid of an aircraft radar but the bearing of an aircraft relative to a ground beacon is obtained with the aid of a goniometric-rangefinding system. /344

At the moment when the distance and path bearing of a landmark or aircraft are determined from a ground beacon, the indicator readings for the aircraft coordinates are recorded. After determining the actual coordinates of the aircraft by means of a navigational slide rule, they are compared with the coordinates on the indicator recorded at the moment of distance finding, and the errors in calculating the coordinates are found:

$$\Delta X = X_{act} - X_{calc};$$

$$\Delta Z = Z_{act} - Z_{calc},$$

where X_{act} and Z_{act} are the coordinates of the aircraft on the basis of the measurement results and X_{calc} and Z_{calc} are the coordinates of the aircraft according to the readings on the calculator.

The corresponding corrections are then entered in the readings of the running orthodromic coordinates of the aircraft on the calculator.

The characteristic feature of the solution of these problems is the lack of a need to fix the time of measurement of the aircraft coordinates and the introduction of corrections in the readings of the calculators when they change, as is necessary when using all other radionavigational instruments.

This feature is completely characteristic for Doppler systems. The relationship to time here is maintained only with selection of regimes of speed for reaching checkpoints at a given time. In measuring the aircraft coordinates and all other elements of aircraft navigation, the time need not be taken into account.

Let us examine further the methods of getting rid of systematic errors in calculating the aircraft path and primarily the measurements of the aircraft course with the use of Doppler meters.

For a precise determination of the errors in measuring the aircraft course, we need to determine the actual coordinates of the aircraft at at least two successive points, with the measurement base on the order of 200-300 km.

At the first point, the actual coordinates of the aircraft are determined and the readings of the calculator are corrected. At the second point, the actual coordinates of the aircraft are determined once again and the error in calculation is found, which has been accumulated during the flight time along the base from the first to the second measurement point.

If we consider the error in the readings of the calculator at the first point to be equal to zero (since they have been corrected), the error in measuring the course is determined by the formula

$$\Delta\gamma = \operatorname{arctg}\frac{\Delta Z_2}{X_{1,2}},$$

where ΔZ_2 is the error in calculating the aircraft coordinates at the second point, and $X_{1,2}$ is the length of the measurement base between points 1 and 2. /345

This problem is easily solved on a navigational slide rule (Fig. 3.60).

By using a Doppler meter, it is possible to find not only the errors in measuring the aircraft course, but also the nature of their accumulation with time.

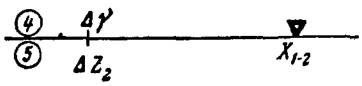


Fig. 3.60.

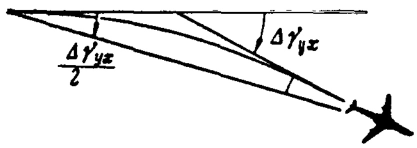


Fig. 3.61.

Fig. 3.60. Determination of the Error in Measuring the Course on the NL-10M.

Fig. 3.61. Determination of Gyroscope Deviation on the Second Measurement Base.

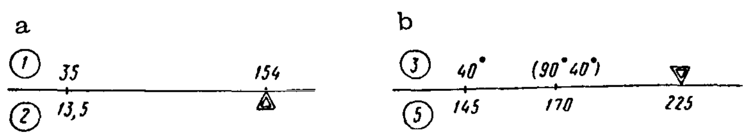


Fig. 3.62. Use of the NL-10M to Determine (a) Degree of Deviation of the Gyroscope and (b) Orthodromic Coordinates of the Aircraft from Ground Radio Beacons.

As we already know, the deviation of a gyroscope with time can be compensated by a suitable shift of the latitude on the compensator for the diurnal rotation of the Earth. If the deviation of the gyroscope is significant (2-3 deg/hr), it can be determined by changes in the errors in calculating the path on two adjacent bases, preferably of the same length (Fig. 3.61).

With considerable deviations of the gyroscope axis, the path of the aircraft turns out to be curvilinear if the Z-coordinate recorded on the calculator is equal to zero, and the final error in measuring the course will be greater than this average error which appears on the first base at the initial value ($\Delta\gamma_d$) divided in half.

Therefore, after introducing the corrections in the readings of the course instrument, an error remains in the measurement of the course which is equal to this value.

If the measurement is repeated on the adjacent base, of approximately the same length, the error found in the measurement of the course will consist of two values:

(a) The error in the initial setting, equal to $\Delta\gamma_d/2$;

(b) The average error due to the deviation of the gyroscope on the second base, also equal to $\Delta\gamma_d/2$.

Thus, the error which is found will constitute the magnitude of the gyroscope deviation during the flight time along the second base. /346

In order to determine the magnitude of gyroscope deviation per hour of flight, it is sufficient to divide the error which has been found into the flight time on the second base.

Example. The flight time of an aircraft on the first and second bases is 20 min each. On the first base, an error in measuring the aircraft course was found and compensated for. However, on the second base the error in measuring the course turned out to be equal to 1°. Find the magnitude of gyroscope drift per hour of flight.

Solution:

$$\Delta\gamma_d = \frac{1^\circ}{0.33\ \text{hr}} = 3\ \text{degrees/hr}.$$

If the deviations of the gyroscope are small (0.5-1°), they cannot be found by measurement from a second base. However, in this case, it is not necessary to shift the latitudinal potentiometer in making compensation. It is sufficient to correct the readings of the course periodically (along with the correction of the aircraft coordinates) by the results of the measurements on one base.

When necessary, a Doppler meter can be used to determine the small-scale variations in the gyroscope (0.5-2 deg/hr). To do this, both the aircraft coordinates and error in measuring the course are determined on the first base.

On subsequent bases, only the aircraft coordinates are determined and corrected. On the last base, the error in the aircraft course is again determined. The error which is found will constitute the deviation of the gyroscope from the moment of the end of the first to the end of the last base.

At the same time, the remaining error at the end of the first base is equal to $\Delta\gamma_d/2$, while the error found at the end of the last base,(e.g.) the fourth, is equal to:

$$\Delta\gamma = \frac{\Delta\gamma_{d_1}}{2} + \Delta\gamma_{d_2} + \Delta\gamma_{d_3} + \frac{\Delta\gamma_{d_4}}{2},$$

i.e., if the last base is equal to the first, the error which will be found by measuring the course will be equal to the deviation of the gyroscope in the second, third, and fourth bases. Thus, the flight time will be sufficient for showing up even small-scale deviations of the gyroscope per hour of flight.

Since the navigational use of Doppler meters does not pose any difficulties, but the detection of errors in calculating the path and measuring the aircraft course are much more difficult, we would like to conclude by providing several examples of how to determine these errors.

1. The last correction of aircraft coordinates was made at the point X = 156 km, where the error in the Z-coordinate was found to be zero.

The flight then continued with maintainance of the Z- coordinate on the computer equal to zero. At the point X = 330 km, according to the readings of the computer, the actual coordinates of the aircraft were determined on the basis of a radar landmark, having the coordinates X_l = 375 km, Z_l = 61 km. The polar coordinates of the landmark were as follows: PBL = 54°, R = 72 km. Find the errors in calculating the coordinates in measuring the aircraft course. /347

Solution: by using a navigational slide rule, we find the following:

$$R \cos \text{PBL} = 42.5 \text{ km};$$

$$R \sin \text{PBL} = 58 \text{ km}.$$

Consequently, the actual coordinates of the aircraft are as follows: X = 375-42.5 = 332.5 km; Z = 61-58 = 3 km, while the errors in calculating the coordinates are: ΔX = +2.5° km, ΔZ = + 3 km.

The error in measuring the aircraft course is

$$\Delta\gamma = \text{arctg} \cdot \frac{3}{332{,}5-156} = 0°58'.$$

In this case, the aircraft deviated to the right from the given path, so that the readings of the course instrument were reduced an/ it was necessary to make a correction equal to +0°58' or approximately +1°.

2. After correcting the coordinates in the aircraft course, considered in the first example, the aircraft traveled along a base equal to 180 km with a ground speed of 850 km/hr.

A second check of the aircraft coordinates revealed that the error in measuring the course was +0°35'. The flight was made within latitudinal limits of 50-60°. Find the degree of deviation of the gyroscope per hour of flight and the required shift in the latitudinal compensator to get rid of it.

Solution: The flight time of the aircraft along the base is equal to 13.5 min. The length of the second base is approximately equal to the first base, so that the deviation of the gyroscope along the second base is equal to the error found by measuring the course.

The deviation of the gyroscope was found by means of a navigational slide rule (Fig. 3.62, a).

Answer: the deviation of the gyroscope per hour of flight amounts to 154' or 2°34'.

At latitudes of 50-60°, for each degree per hour of deviation in the gyroscope, it is necessary to shift the latitudinal compensator by 6°. In our example, the gyroscope deviated in the direction of a reduction of the course indication so that the latitude on the compensator had to be set to the value: 6° x 2.6 = 15.6°. After the desired change in the setting of the latitudinal potentiometer is made, the deviation of the gyroscope should cease completely.

3. For correcting the aircraft coordinates, a goniometric-rangefinding system is employed. The orthodromic coordinates of a ground radio beacon are:

$$X_м = 187\ км; \quad Z_м = 142\ км.$$

The flight angle of an orthodrome segment, measured relative to the meridian of the point where the beacon is established, is equal to 64°. Find the orthodromic coordinates of the aircraft if its azimuth (A) is equal to 24° and R = 225 km.

Solution:

$$A - \psi_м = 320°;$$
$$\cos 320° = \cos 40° = \sin 50°;$$
$$\sin 320° = -\sin 40°.$$

By using a navigational slide rule, we find (Fig. 3.62, b), i.e.,

$$R \cos 320° = 170 \text{ км};$$
$$R \sin 320° = -145 \text{ км}.$$

Consequently, the orthodromic coordinates of the aircraft are

$$X = 187 + 170 = 357 \text{ км};$$
$$Z = 142 - 145 = -3 \text{ км}.$$

5. PRINCIPLES OF COMBINING NAVIGATIONAL INSTRUMENTS /348

In Chapters Two and Three of the present work, we discussed the complexes of navigational instruments, which make it possible in one way or another to automate the processes of aircraft navigation or measurement of individual navigational parameters.

The first navigational complex is the course system.

The basic principles of combining individual transmitters into a course system is the combination of the readings for purposes of automatic mutual correction (the MC, AC, GSC regimes), and also to combine the readings of individual instruments to improve the navigational values, constituting the sum of individual elements. For example: OBR = OC + CAR.

The second complex is the navigational indicator NI-50B, in which there is a course transmitter, a transmitter of the airspeed, and a manually-set wind transmitter.

The most complete of these complexes is the autonomous Doppler system of aircraft navigation, which works in conjunction with course transmitters and an automatic navigational device.

Thus, the basic reasons for combining navigational instruments are the following:

(a) Comparison of readings for purposes of mutual correction,

(b) Combination of readings for purposes of automatic summation.

Combination of individual transmitters into navigational systems not only makes it possible to solve navigational problems automatically or semi-automatically, but also makes it possible to realize their solution for automatic pilotage of an aircraft along the given trajectory. An example of such a realization is the automatic pilotage of an aircraft on the basis of signals from Doppler meters with automatic navigational instruments.

These complexes generally involve autonomous navigational instruments: transmitters for the course, airspeed, and drift angle. The only exception is the aircraft radiocompass, whose readings are combined with the readings of course instruments to obtain bearings. However, this is a result of a peculiar feature on the use of radiocompasses (for obtaining the bearing it is necessary to add the course angle of the radio station to the aircraft course). The use of simple combinations of navigational systems such as ground radars, radio distance-finders, externally directed goniometric and goniometric-rangefinding systems, fan-type beacons, and hyperbolic systems cannot be combined satisfactorily.

Nevertheless, it is desirable that these related navigational /349 systems, as well as the aircraft radar and astronomical navigational instruments, should also be combined into navigational complexes for the purposes of automatic correction of aircraft coordinates. However, the principle of combining these instruments into general navigational systems must be of a quite different nature than is the case for the complexes which we have discussed.

The first characteristic of combined navigational systems and aviational sextants is that they are intended only for determining discrete values of aircraft coordinates. Therefore, they can be used in navigational complexes as sources of information which duplicate the results of automatic calculators of the aircraft path, i.e., only for purposes of correcting previously obtained navigational parameters.

The second feature of these devices is that with a relatively high accuracy of coordinate measurement for the aircraft, they cannot be used to determine the first derivatives of these coordinates with time. Let us illustrate this with a concrete example.

Let us say that some navigational instrument, taking its instrumental errors into account (for electronic devices, considering the conditions for propagation of electromagnetic waves, and for astronomical ones, the accelerations of the level of the aircraft) make it possible to determine the successive coordinates of an aircraft with an error which does not exceed 1 km, so that the error in measurement can change in value and sign.

In this case, the error in determining the direction of the aircraft motion on the basis of two successive measurements may have a maximum value of

$$\frac{\Delta Z}{S}=\frac{2}{S}.$$

With a measurement base of 30 km (approximately 2 min of flight in a jet aircraft), the angular error in the measurements can reach

$$\frac{2}{30} = \frac{1}{15} \approx 4^\circ.$$

If the measurements are made more frequently, (e.g.) oftener than each minute of flight, the error in measuring the direction can reach 8°.

With continuous measurement of the aircraft coordinates, the numerator in our example can retain its value, but the denominator will tend toward zero, i.e., the error in determining the direction of flight or (what amounts to the same thing) the first derivative of the Z-coordinate with time, will be equal to infinity.

We can reach an analogous conclusion for the case of determining the ground speed of an aircraft (the first derivative of the X-coordinate with time) by continuous measurement of it at a succession of points where the LA is measured. /350

This example shows that communication and astronomical navigational systems can only provide a rough pilotage of the aircraft along a given trajectory. With a very precise measurement of the aircraft coordinates along the route (with errors no greater than 200-300 m) and a very careful damping of the readings (averaging for time), automatic pilotage will take place with variations of the course within limits of 5-6°, i.e., 5-10 times greater than would be obtained by the results of measuring the drift angle by a Doppler meter.

The only exception to this is the pilotage of an aircraft using strictly stabilized zones of landing beacons, where the errors in determining the deviations from a given trajectory are measured in several meters. Under these conditions, the pilotage of an aircraft can take place with variation of the course within limits of 1-2° with a very precise maintainance of the general direction of flight.

The third feature of these methods is the considerable diversity of special coordinate systems used (Chapter I, Section 7). To introduce these methods into navigational complexes, it is neccessary to have an automatic conversion of special coordinates into orthodromic coordinates, calling for the availability aboard the aircraft of very complex and precise mathematical instruments, to make the spherical conversions.

It is somewhat simpler in this regard to use goniometric-range-finding methods for short-range navigation and aircraft radars. Due to the limited radius of their operation, the polar coordinates of these devices can be converted into orthodromic ones by solving simple equations for plane representations, using simple calculating devices with low accuracy.

Combined navigational systems and astronomical methods can be combined into navigational complexes only for purposes of correcting the aircraft coordinates at discrete points, maintaining the regime of correction by the aircraft crew. The advisability of including each of these devices in the total complex or its independent use for correction of the coordinates of the aircraft by the crew is determined by the tactical characteristics of the system in the conditions for flight of the particular type of aircraft involved.

CHAPTER FOUR

DEVICES AND METHODS FOR MAKING AN INSTRUMENT LANDING

SYSTEMS FOR MAKING AN INSTRUMENT LANDING

Landing an aircraft under conditions of limited ceiling and /351 meteorological visibility in the layer of the atmosphere near the ground is the most complicated and difficult stage of the flight. Even under favorable meteorological conditions, a proper landing of the aircraft requires considerable attention and experience on the part of the crew.

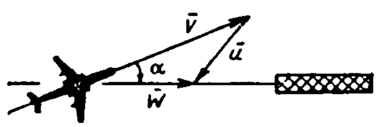


Fig. 4.1. Setting the Aircraft Course for Lining Up with the Runway.

Experience has shown that in order to land any kind of aircraft, it is necessary that it be located exactly on the landing path at a certain distance from the touchdown point (Fig. 4.1). It is also necessary that the course followed by the aircraft be selected so that the vector of the ground speed is directed along the axis of the landing and take-off strip (LTS).

However, it is not desirable to land an aircraft with a lead in the course being followed in order to compensate for the drift angle, since this causes considerable lateral stresses on the aircraft undercarriage when it begins to taxi along the runway. Therefore, the longitudinal axis must be lined up with the LTS immediately before landing, by making a flat turn without banking. Then (since the turn was flat) the aircraft will keep the desired direction of motion relative to the Earth's surface for a short period of time, showing lateral deviation relative to the air mass flowing over it.

This shift gradually dies out, eventually turning into a drift angle relative to the new course of the aircraft. Therefore, the selection of the approach angle must be made several seconds (no more than 5 or 7) before landing the aircraft.

It should be mentioned that the correct selection of an aircraft course while keeping it simultaneously on the given trajectory for landing poses considerable difficulties for the crew in /352 preparing to land.

In cases when the aircraft is not lined up with the runway, it is necessary to carry out a maneuver which will bring it on to the axis of the LTS, and which involves a considerable loss of time and also a loss of distance along axis LTS (Fig. 4.2).

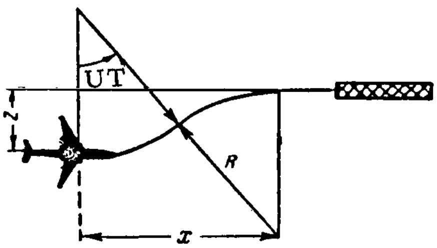


Fig. 4.2. S-Shaped Maneuver for Lining Up the Aircraft with the Runway.

Let us say that in a flight along the LTS axis, the crew has reached a point at which their lateral deviation is equal to Z.

Obviously, in order to line up the aircraft with the LTS axis in the most economical fashion and without any remaining deviation relative to the LTS axis, it is necessary to turn the aircraft toward the runway through a turn angle (TA) of a magnitude such that the lateral deviation of the aircraft from the LTS axis is reduced by a factor of two. Then the aircraft must be turned by the same amount in the opposite direction but through an angle such that the trajectory along which the aircraft is traveling when it emerges from the turn coincides with the LTS axis.

In order to avoid loss of the selected direction of the ground speed vector while making the turns, i.e., shifting the aircraft after placing it on the landing course, the turns made by the aircraft must be coordinated as much as possible.

It is obvious from Figure 4.2 that the magnitude of each of the two coordinated turns for bringing the aircraft on to the runway can be determined by the formula

$$Z = 2(R - \cos TA) = 2R(1 - \cos TA),$$

whence

$$\cos TA = 1 - \frac{Z}{2R},$$

where R is the turning radius of the aircraft with a given banking and airspeed during the turn.

Obviously, while the aircraft is making the maneuver to land, it must travel through a path along axis LTS

$$X = 2R \sin TA.$$

Example. When an aircraft is descending and is lined up with the runway on the desired course and with a horizontal ground speed of 280 km/hr, there is a lateral deviation from the LTS axis equal to 60 m.

Find the angles of the combined turns of the aircraft with a given banking of 8° and the path of the aircraft along the descent path during the completion of the maneuver. /353

Solution. The radius of the turns made by the aircraft are found by using a navigational slide rule (Fig. 4.3), which gives the answer 4500 m.

$$\cos TA = 1 - \frac{60}{4500} = 0.9867;$$

$$TA = 9°20';$$

$$\sin TA = 0.1625; \quad X = 2\ 4500\ 0.1625 = 1463 \text{ m}.$$

ANSWER: $R = 4500$ m; TA = 9°20'; $X = 1463$ m.

However, we must take into account the fact that the desired aircraft path in lining up with the runway must be chosen on the basis of the assumption that the turns are made with a constant banking angle, i.e., with a stable turn regime. At the same time, there is a delay in the maneuver produced by the reaction of the crew and mainly due to the inertia of the aircraft when entering and emerging from the turns.

Fig. 4.3.

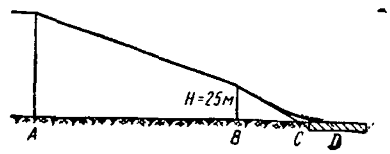


Fig. 4.4.

Fig. 4.3. Using the NL-10M to Determine the Turning Radius of an Aircraft.

Fig. 4.4. Landing Profile for a Jet Aircraft.

As special tests have shown, the delay in the maneuver occurs primarily along the descent path and has practically no influence on the desired magnitude of the angles of the combined turns. This is explained by the fact that when the aircraft is entering and emerging from a bank at the beginning and end of the maneuver, the axis of the aircraft practically coincides with the axis of the LTS and the aircraft has practically no lateral velocity at these points.

As far as the movement of the control surfaces when making the turns is concerned, the time required to move them is approximately two times less than the time required for the aircraft to enter and leave the turn, so that the lateral component of the aircraft speed at turn angles up to 12° has a magnitude less than one-fifth of the longitudinal velocity.

The delay time in the maneuver depends on the square of the horizontal velocity of the aircraft. At glide speeds of 280 km/hr, the delay time is equal to 4.5 sec of flight time on the average, or 350 m of the aircraft's flight along the LTS axis. This means that in our example, the required travel of the aircraft in lining up with the runway is equal to approximately 1800 m.

At the same time that the course is being selected which must be followed in order to make the landing, the crew must begin some distance away from the landing point to set up the desired descent /354
trajectory in the vertical plane (Fig. 4.4).

In Figure 4.4, Point A is the point of transition from horizontal flight along the landing path to the descent regime of the aircraft.

Point B is the point where the landing distance begins, which is also called the critical point for safe transition to making another pass. After this point has been passed, a second attempt at landing cannot be made, so that the aircraft must make a final selection of the aircraft course before this point is reached and the deviation of the aircraft from the given trajectory (upward and downward) must not exceed certain limits. Before this point is reached, a decision must be made either to make the landing or circle around the airport once again.

After the starting point for the landing distance has been passed, the crew carefully observes the altitude. To do this, a leveling point C is selected along the approach to the airport (this is a conditional designation for the point where the descent trajectory of the aircraft crosses the Earth's surface), toward which the further descent of the aircraft is aimed.

With proper descent and a constant pitch angle of the aircraft, this point is projected at a constant level on the cockpit window. If the approach is being made too rapidly, this point shifts upward on the glass, and if the aircraft is coming in too slowly it moves downward.

Before reaching Point C (at an altitude of 8-15 m, depending on the type of aircraft) the aircraft levels off and then lands at Point D.

The descent trajectory of the aircraft in the vertical plane is called the *glide path*. The aircraft is kept on a fixed glide path by selecting the proper angle of pitch for the aircraft and the correct amount of power to the engines. This process is much simpler in principle than the selection of the course to be followed by the aircraft, since it does not require maneuvering but only the proper setting of the pitch angle and the levers which control the motors. However, it complicates landing as a whole because both processes must be carried out simultaneously while a given horizontal airspeed is being maintained.

Unlike all other navigational devices, the systems used in making an instrument landing are intended specially for keeping the aircraft on a given descent trajectory before landing in the horizontal and vertical planes.

The proper operation of these devices and the maneuverability of the aircraft determine the minimum permissible distance from the LTS at which the aircraft can be piloted by instruments or by instructions from the ground, with correction of any errors that may occur after changeover to visual flight. The more precisely the desired trajectory is maintained by instruments, the closer the transition to visual flight will lie to the landing point and the lower the altitude at that point.

The limits within which an aircraft can be piloted by instruments without the airport being visible and with no terrestrial landmarks in sight which could show approaches to the airport is called the *weather minimum* for landing the aircraft. /355

At the present time, there are three principal types of systems for making instrument landings:

(a) A simplified landing system which involves lining up the aircraft with radio stations.

(b) A course-glide landing system.

(c) A radar landing sytem.

A necessary complement to each of these systems is the system of landing lights at the airport.

Simplified System for Making an Instrument Landing

The complex of devices in the simplified system for making an instrument landing on the basis of information from two master radio stations includes the following:

(1) Two master radio beacons, located on the LTS axis, whose standard designation is the short-range master station (SRMS), located 1000 m from the end of the LTS, and the long-range master station (LRMS), located 4000 m from the end of the LTS.

(2) Two USW marker beacons with a narrow vertical propagation characteristic for electromagnetic waves, located on the same sites as the LRMS and SRMS.

(3) The lighting of the approaches to the LTS and its outline.

(4) The complex of aircraft radio navigational and pilotage-navigational equipment as a whole. This includes:

(a) One or two radio compasses,
(b) A marker receiver,
(c) Course control of the aircraft,
(d) A barometric altimeter,
(e) A radio altimeter for low altitudes,
(f) An airspeed indicator,
(g) A gyrohorizon,
(h) A vertical speed indicator (variometer).

We have already discussed in great detail (in Chapters Two and Three) most of the ground and aircraft equipment which is included in the simplified system for making instrument landings. In this chapter, we will provide only a brief description of the operating principles of the pilotage and special landing equipment, which were not discussed in the other chapters, since this equipment has a very limited application for purposes of aircraft navigation and its use is very simple from the standpoint of the methodological /356
errors which must be taken into account.

In particular, we shall acquaint ourselves with the operating principles of the following pieces of equipment: marker devices, radio altimeters for low altitudes, the gyrohorizon and variometer.

Marker Devices

In order to make a landing with the simplified system, it is very important to know (admittedly, at separate points) the distance remaining until the end of the runway.

As we know, aircraft radio compasses do not permit a precise determination of the moment when an aircraft flies over the control radio station; this is due to the special characteristics of the operation of the open antenna aboard the aircraft. To solve this problem, marker beacons and aircraft marker receivers have been devised.

Marker radio beacons are transmitters with a directional transmission characteristic vertically upward, sometimes with a slight deviation toward the LTS so that the limit of the directional characteristic of the radiation is located to one side of the LTS and as close as possible to the vertical. In this case, an aircraft which is flying over the beacon towards the LTS will receive the signals from the marker transmitter at the moment when it is exactly above the beacon.

For purposes of recognition, the transmission from the marker beacon is not continuous but in the form of frequent short pulses (SRMS) or longer, less frequent signals (LRMS). These signals are heard aboard the aircraft for a period of 3-6 sec after it has flown over the vertical limit of the radiation characteristic and before it crosses the second, deflected limit of the characteristic.

A still simpler device is the aircraft marker receiver. It is set to one frequency which is the same for all beacons. Therefore, it is very simple in design, has small dimensions, and requires no attention for use except to be switched on and off.

When used in a complex together with course-glide devices, the marker receiver is turned on by a switch which is combined with the course-glide equipment, so that the crew does not have to interfere in its operation at all. In many cases, the marker receiver is combined with the switch for the radio compasses, the purpose being to ensure a low consumption of electrical energy, and allow stability and high reliability in the operation of this receiver.

The marker receiver is connected to a light signal (a red light on the instrument panel in the cockpit marked "marker") and to a device which gives a simultaneous sound signal by means of a bell. Thus, when the aircraft flies over the marker, the lamp flashes and a series of short rings is heard.

Low-Altitude Radio Altimeters /357

At the present time, low-altitude radio altimeters based on the principle of frequency modulation are the ones most widely employed.

A schematic diagram of such a radio altimeter is shown in Figure 4.5.

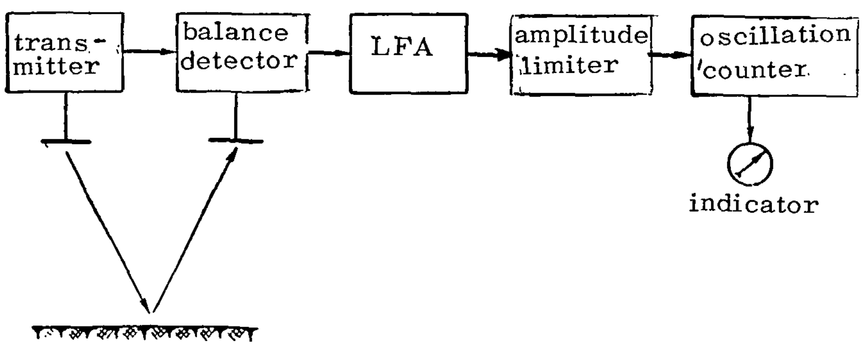


Fig. 4.5. Diagram of Low-Altitude Radioaltimeter.

The radio altimeter transmitter has a modulating device which produces a saw-tooth wave. For this purpose, we can use (e.g.) a variable membrane capacitor with mechanical oscillation of the membrane.

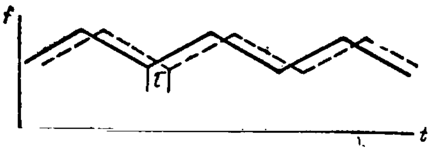


Fig. 4.6. Frequency Characteristic of Radioaltimeter.

The frequency of the signals reflected from the ground and picked up by the receiving antenna has the same saw-tooth characteristic, but is shifted in time by a value τ, required for the electromagnetic waves to travel from the transmitting antenna to the ground and back again to the receiving antenna (Fig. 4.6).

It is clear from the figure that the frequency difference between the emitted and received waves at any moment in time (with the exception of the segments between the extreme values of the frequency characteristic) will be strictly linear with respect to the flight altitude. For a complete retention of the linearity, these segments can be cut out by cutting off the receiving section with a Π-shaped voltage at the end points of the emitted frequency.

The emitted and received frequencies are combined in the balancing detector, where a low frequency is formed which is proportional to the flight altitude.

Following amplification, the low frequency is converted to /358 rectangular oscillations which are calibrated both in terms of amplitude and duration. Thus, the counting circuit will receive pulses which are of uniform magnitude, and whose number per unit time will depend on the flight altitude.

The number of calibrated pulses is summed and fed in the form of a direct current to the indicator, whose pointer shows the altitude in meters.

In the simplified landing system, the radio altimeter plays only an auxiliary role as an indicator of a dangerous approach to the ground, since its readings depend upon the nature of the relief and cannot be used for checking the rate of descent. To set up the descent trajectory of the aircraft, barometric altimeters are used.

In more complete landing systems, the radio altimeter can be used to give a trajectory value as well, but only in the last stage of descent before landing above a given final area of safety adjoining the LTS.

Since those landing systems which ensure descent of the aircraft by instruments until the point where the landing distance begins use the radio altimeter only to signal a dangerous approach to the ground, we can exclude them for convenience from the group of basic pilotage instruments located in the center of the field of vision of the pilot, and use audible signals. If an aircraft is making a descent and reaches the limit of permissible altitude above the ground, the audible signal warns the crew of the necessity to terminate descent.

Gyrohorizon

The artificial indicator of the position of the horizon relative to the axis of the aircraft (gyrohorizon) is a common pilotage instrument, intended for piloting the aircraft when the true horizon is not visible. However, it is very important in guiding the aircraft along a landing trajectory, where it is used for maintaining a desired landing trajectory.

In principle, the design of the gyrohorizon is simpler than that of the gyrosemicompass, e.g., unlike the latter, the gyrohorizon has a vertical axis of rotation for the gyroscope, and a gravitational correction device suspended from the bottom of the gyro assembly. This serves to keep the gyroscope axis constantly vertical in the aircraft.

The external frame of the gyrohorizon is located horizontally, while its axis of rotation coincides with the longitudinal axis of the aircraft. Therefore, we can immediately determine the existence and magnitude of a lateral rolling of the aircraft by the position of the external frame relative to the axis of the aircraft.

For this purpose, a silhouette of the aircraft has been pasted on the glass which covers the dial, and a horizontal strip which moves up and down imitates the position of the visible horizon.

Figure 4.7 shows the schematic diagram of the gyrohorizon.

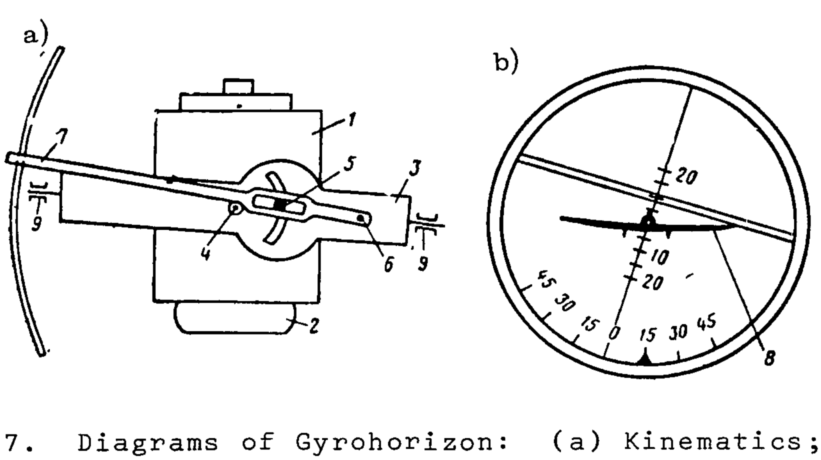


Fig. 4.7. Diagrams of Gyrohorizon: (a) Kinematics; (b) Indicator.

Gyro assembly 1, with a vertical axis of the gyroscope rotor and the gravitational correction device 2 mounted at the bottom, are suspended in the horizontal external frame 3 on the gyro assembly bearings 4. The carrier for the horizon line 5 is fastened to the casing of the gyro assembly and displaced somewhat forward relative to the horizontal axis of the gyro assembly (along the direction of the aircraft's flight from the forward part of the instrument). The axis of the line is fastened at the front to the external frame, also along the flight direction of the aircraft. Therefore, when reducing the angle of pitch of the aircraft, the strip of the gyrohorizon 7 moves upward, remaining parallel to the horizontal axis of the gyro assembly. When the pitch angle increases, the horizon line moves downward as the true horizon does.

During lateral rolling of the aircraft, the casing of the gyrohorizon (along with the silhouette of the aircraft) rotates relative to the bearings of the external frame 9 in the direction in which the aircraft is rolling, which provides an indication of the rolling of the aircraft relative to the horizontal strip. For estimating and maintaining given longitudinal and lateral rolling of the aircraft, a scale is located between the outer frame and the horizon line and shows scale divisions for estimating the magnitude of the rolling in degrees.

The gyrohorizon, fitted with the kinematic system described above, can be used within limited degrees of longitudinal and transverse rolling of the aircraft. Obviously, the rear bearing of the outer frame, i.e., the one located between the outer frame and the scale, must be mounted on a support in the unit. This support acts /360 as a pivot for the lever supporting the horizon line, e.g., when the aircraft rolls over on one wing.

In the case of considerable changes in the pitching angle of the aircraft (e.g., in a Nestrov loop), a support will hold the lever for the strip in a notch on the outer frame.

The projection of one of these supports limits the degree of freedom of the gyroscope, thus leading to a "dis-location" of its indications, and a very long period of time is required to readjust them by gravitational correction.

To ensure "nondislocation" of the operation of the gyrohorizon, the gyroscopic section protrudes outside the housing of the instrument, i.e., constitutes a separate gyroscopic instrument, a gyrocompass without a limited degree of freedom. The readings of the gyrovertical are transmitted to the horizon indicator by means of master and slave selsyns.

We should also note that gyrohorizons or gyroverticals are transmitters which indicate longitudinal and lateral rolling for the operation of autopilots, acting as transmitters of turn angles of the aircraft in the horizontal plane, in which gyroscopic semicompasses are used.

Variometer

A *variometer* is a device which measures the rate of vertical descent or climb of an aircraft.

The operating principle of a variometer is based on the deceleration of a current of air which equalizes the pressure inside the body of the unit with the external static pressure. This means that when vertical movement occurs, a pressure drop develops within the body of the unit and in the static tube (Fig. 4.8).

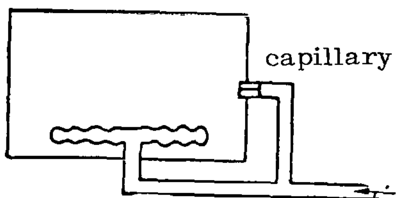


Fig. 4.8. Diagram of Variometer.

The pressure from the static pressure intake passes directly into the manometric chamber of the instrument. Within the body of the instrument, this pressure passes through a capillary opening, i.e., with retardation. Therefore, when the aircraft gains altitude, the pressure in the unit will be somewhat higher (when the aircraft descends, somewhat lower) than inside the manometric chamber. This pressure drop is proportional to the vertical speed of the aircraft.

To measure this drop, the variometer is fitted with a transmitter mechanism, similar in principle to the mechanism of the altimeter or speed indicator. The indicator scale is graduated directly in terms of vertical speed, as expressed meters/sec.

Angle of Slope for Aircraft Glide /361

The proper selection of an angle of slope for gliding is very important for all instrument landing systems, and especially for the simplified systems guided by master radio stations, both from the standpoint of making a safe landing and the meteorological minimum at which a landing can be made.

When making an approach to land, it is very important that the flight altitude (H) correspond to the remaining distance (S) to the point where the aircraft touches down:

$$H = S_{\text{rem}} \operatorname{tg}\theta$$

where θ is the glide angle.

A simplified system of instrument landing makes it possible to determine the remaining distance to the landing point only when passing over the LRMS and SRMS.

The point at which the aircraft begins to descend from the altitude established for circling above the field is determined by calculating the time, and is therefore insufficiently exact. A descent between the LRMS and SRMS is also made by calculating the path of the aircraft with time, but this calculation takes only a short period of time and is performed after a certain point has been passed; it is therefore more accurate.

According to the standards adopted in the USSR, the flight altitude for circling over an airport (for aircraft with gas turbine engines) has been set at 400 m; for piston-engine aircraft, it is 300 m. In both cases, however, the true flight altitude above the local terrain surrounding the airport must be no less than 200 m. This altitude reserve is retained even when coming straight in for a landing, until the beginning of descent in the designated glide pattern.

From the moment when descent begins in a glide, and until the aircraft passes over a certain marker (LRMS), the altitude reserve above the relief is kept at a minumum of 150 m. After flying over the LRMS, and before reaching the SRMS, the height of the aircraft above the terrain is reduced from 150 to 50 m. During this maneuver, however, it is necessary to keep in mind the fact that there may be a possible premature loss of altitude, in case of an unexpected strong head wind. For this reason, it is considered that the minimum flight altitude between the LRMS and the SRMS (flight altitude above the SRMS) must be at least 50 meters above the heighest point in the vicinity, beginning at half the distance between the LRMS and SRMS and extending to the point where the SRMS is located.

These same altitude reserves are maintained even when using more complete landing systems, although in this case the given glide path for the aircraft is defined in space and the probability of a premature descent is sharply reduced. In this case, however, the basic method for checking the proper descent is the measurement of /362 the barometric altitude when flying over the marker points, thus guaranteeing safety of flight in case the landing instruments aboard the aircraft or on the ground should malfunction.

In cases when the approaches to an airport are free of obstructions, the angle of slope in the glide path is set equal to 2°40'. The flight altitude relative to the level of the airport in this case is set at 200 m above the LRMS and 60 m above the SRMS.

Typical Maneuvers in Landing an Aircraft

Simplified systems for bringing an aircraft in for a landing are used at airports with a low traffic density, where the installation of complex landing systems would not be justified. Consequently, it is difficult to know in advance whether these airports will have provision for radar control, to set up the approach and landing pattern on command from the ground. Hence, the approach for landing is made with the same devices which are used in landing the aircraft along a straight line. For this reason, a successful accomplishment of the maneuver under these conditions will be assured if the starting point for the maneuver is one of the marker points of the system.

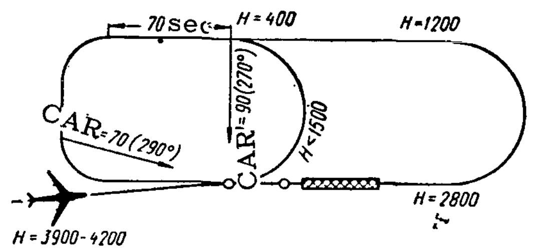


Fig. 4.9. Large and Small Rectangular Landing Patterns.

Usually a LRMS is used for this purpose, since at the majority of airports, it is the main control facility at the airport. There are then three possible ways to bring the aircraft to the starting point for the maneuver:

(1) An approach of the aircraft to the LRMS, with a path angle close to the landing course.

(2) An approach to the LRMS, with a path angle nearly perpendicular to the landing course.

(3) An approach of the aircraft to the LRMS, with a path angle nearly the reverse of the landing course.

In directing the aircraft toward the LRMS at path angle close to the landing course, the approach for landing can be made along a more or less straight-line course (Fig. 4.9).

A large rectangular route is covered in this case, if the /363
aircraft approaches the airport at a great altitude (for aircraft with gas turbine engines, this is 3900 to 4200 m), and an additional length of time is required for the aircraft to descend before landing.

In this case, in making the approach to the LRMS, the aircraft makes a turn to the path angle for landing (in the following, the path angles will be referred to as magnetic), at which the aircraft descends to 2800 m (relative to the pressure at the level of the airport where it is landing).

At an altitude of 2800 m, the double turn begins (first and second turns without a straight line between them) at 180° with a descent to 1200 m. Flight then continues with a magnetic path angle (MPA) opposite to the landing angle, with descent to the altitude set for circling over the airport.

In aircraft with gas turbine engines, limits have been set for the horizontal airspeed with the undercarriage lowered. Therefore, in a flight with a MPA opposite to the landing angle, the flight altitude for circling the field is maintained for 5 to 6 km until the LRMS is passed, so that at the moment when it actually is passed, the speed of the aircraft in horizontal flight can be cut to the speed established for lowering the undercarriage.

After passing over the traverse of the LRMS, the flight continues opposite to the landing direction for 70 sec, prior to starting the third turn (usually at a flight altitude of 400 m, up to CAR = 120° to the right and up to CAR = 240° on the left straight-line paths). The undercarriage is lowered in this path segment.

After a period of 70 sec flying time from the moment when the traverse of the LRMS is passed or until CAR-120 (240°) is reached, the third turn is made. Since the horizontal airspeed in the vicinity of the third turn is much less than in the vicinity of the doubling of the first and second turns, the radii of the third and fourth turns (with a banking angle of 15 to 17°) are then much less than the radius of the double turn. Therefore, between the third and fourth turns there is a period of straight-line flight which lasts 50 to 55 sec. This straight-line segment is used for preliminary lowering of the wing flaps before landing, and also acts as a "buffer", which compensates for errors in aircraft navigation in cases when the effect of a side wind in making the maneuver from the starting point until the end of the third turn has not been estimated sufficiently precisely.

In these cases, the "buffer" line can be extended or shortened somewhat, but the last (fourth) turn must be always made on time.

At airports where the nature of the local terrain or complex wind conditions render flight along a straight line at 400 m impossible (for aircraft with gas turbine engines), but the established flight altitude is 600 or 900 m, the duration of the flight from the traverse of the LRMS to the beginning of the third turm is increased, so that after the aircraft emerges from the fourth /364 turn it is located below the glide path established for a given approach direction and has a segment of horizontal flight to the end of the glide path which is only 20 to 30 sec long. This time is needed to prepare the crew for landing and for extending the flaps fully.

For example, if the flight altitude along a straight-line course is set at 600 m, and the slope angle of the glide path is 2°4', the fourth turn must be executed no closer than 15 km from the end of the LTS, since the aircraft (at an altitude of 600 m) enters the glide path at a distance of 13 km from the end of the LTS, and 2 km are required for the horizontal flight segment before entering the glide path.

Consequently, the start of the third turn under calm conditions, after passing the traverse of the LRMS, lasts 2 minutes and 30 seconds of flying time (at V = 350 km/hr), with CAR approximately equal to 135° (225°). If the flight altitude along the straight-line path is set at 900 m, the flying time from the traverse of the LRMS to the beginning of the fourth turn is increased to 3 minutes and 30 seconds, so that it is advisable to increase the glide path up to 4° for the purpose of shortening the time involved in making the descent.

In cases when an aircraft is approaching an airport with a path angle close to the landing angle, at an altitude of 1500 m or less, the double first and second turns are made immediately after passing the LRMS. The descent to circling altitude and reduction of speed to lower the undercarriage in this case are performed in the designated turn. Hence, the large rectangular flight pattern in converted to a small one, and the maneuvering time is shortened to about 4.5 min.

If the aircraft approaches the airport at the altitude established for circling the field, the radii of all four turns are made approximately the same, so that in order to create the "buffer" line between the third and fourth turns, the first and second turns of the aircraft are executed in succession with a time interval between the end of the first and the beginning of the second turn which equals 40 sec.

The fourth turn on the large and small rectangular patterns is made along the course angle. In aircraft with gas turbine engines, the CAR at the beginning of the fourth turn (when turning to the right) must be equal to 70° (and -290° when turning to the left).

The landing approach for aircraft with piston engines is made according to the small rectangular pattern, with different first and second turns, and the same time parameters between the first and second turns (40 sec), from the traverse of the LRMS to the /365
beginning of the third turn (70 sec) (at a flight altitude of 300 m). The fourth turn for these aircraft begins at CAR = 75 or 285°.

However, due to the lower airspeed along the straight-line segments and the smaller turning radii, the linear dimensions of the maneuver for aircraft with piston engines are much less than for aircraft with gas turbine engines. In addition, due to the shorter time for each turn, the total time for executing the maneuver for aircraft with piston engines is shorter (for example) by 1 minute.

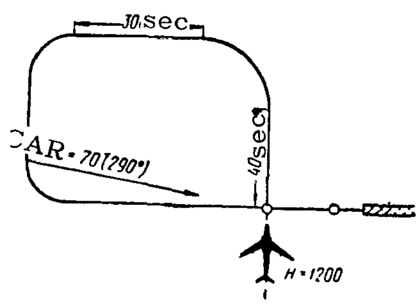


Fig. 4.10. Landing Maneuver When Approaching the LTS Axis at a 90° Angle.

When an aircraft is approaching an airport at an MPA which is perpendicular to the landing angle, the landing altitude for aircraft with gas turbine engines is usually set at 1200 m above the level of the airport (Fig. 4.10). After passing the LRMS, the aircraft continues on a course which lasts for 40 sec until descent, and the second turn is also executed with loss of altitude.

After completing the second turn, the flight lasts 30 sec until the beginning of the third turn, when the undercarriage is lowered.

A similar maneuver is executed by piston-engine aircraft, with the sole difference that the flying time from the LRMS to the beginning of the second turn is set at no less than 1 min, since the airspeed of these aircraft in all stages of the landing approach until emergence from the fourth turn is roughly the same.

If an aircraft approaches an airport with an MPA which is close to the reverse of the landing angle, the crew of a gas turbine aircraft travels along a small rectangular pattern with different sides for the first and second turns (Fig. 4.11,a). In the case of aircraft with piston engines, the so-called standard turn is executed in this instance (Fig. 4.11,b) on the landing course.

These maneuvers agree in terms of the magnitude and direction of the turns; in the former, however, there is a straight-line segment between the second and third turns for lowering the undercarriage, while there is a "buffer" line between the third and fourth turns. In addition, if the aircraft approaches the airport at the flight altitude for circling the field, and all the turns of the aircraft are made without loss of altitude (the radii of all the turns being the same), then between the first and second turns there will also be a period of flight along a straight line for a period of 40 sec.

In the case of a standard turn, all four turns will be made in /366 succession without there being any straight-line segments between turns.

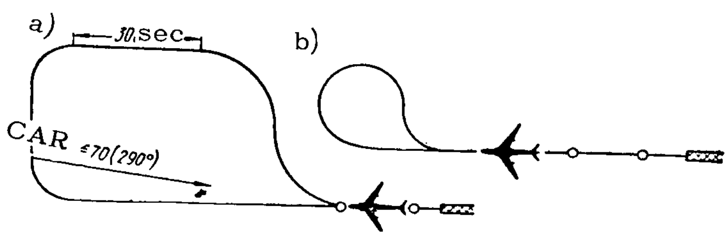


Fig. 4.11. Landing Maneuver with a Course Opposite to the Landing Course. (a) Along a Straight-Line Path; (b) Standard Turn.

Analogous maneuvers for approaching to make a landing can also be made by using more complete landing systems. However, airports that have such systems, as a rule, are also equipped with radar devices to monitor the aircraft maneuvering in the vicinity of the airport. Therefore, the beginning of the landing maneuver need not necessarily be made at the marker point on the LTS axis, thus making it possible to come in for a landing along the shortest path from any direction.

A small or large rectangular pattern is usually used as the basis for setting up a landing approach along the shortest path. However, it is not generally completed, usually beginning at the point of tangency of the entrance into the maneuver to one of its turns.

Calculation of Landing Approach Parameters for a Simplified System

In the preceding section, we discussed the typical maneuvers for landing an aircraft when approaching the airport from any direction. The execution of these maneuvers does not pose great difficulty for the crew of the aircraft, since the flight is made with a sufficient altitude reserve and sufficient speed, while the demands on the accuracy of making the maneuver are not very high.

The main difficulty lies in flying along a given descent trajectory in the glide path, due to the very high demands on the maintenance of flight direction, altitude, and horizontal glide speed, depending on the remaining distance to the touchdown point. In the case of aircraft with gas turbine engines, there is the additional need to reduce the airspeed gradually as the airport is approached.

In order to facilitate the task of descending along a given trajectory to a certain degree, as well as to avoid serious errors in flight along the landing path, some preliminary calculations are made, of which the following is the most important. /367

If the landing approach is made in a dead calm, the geometric dimensions of the maneuver (and consequently, the point where the descent begins along the landing path) are determined by simple relationships between the airspeed, time, turn radii, flight altitude in circling the field, and established steepness of the glide path.

The calculated data for making a landing in a calm are usually plotted on special landing patterns, devised for each airport. Under actual conditions, however, it is necessary to take into account the head wind and side wind components (for the landing course), which can have a very great effect on the making of a landing.

Calculation of Corrections for the Time for Beginning the Third Turn

In preparing to land, especially with the aid of a simplified system, it is necessary to ensure that the aircraft emerges from the fourth turn onto the landing approach always at the same distance from the LTS. Obviously, in order to solve this problem, it is necessary to consider only the head-wind component for the landing course.

In making an approach to land along a rectangular pattern, the last reliable point for determining the *X*-coordinate of the aircraft (the distance along the axis of the direction of the airport) is the traverse of the LRMS, while in a standard turn it is the passage over the LRMS with an MPA opposite to the landing angle.

If we do not take the wind into account when coming in for a landing, the aircraft will enter the landing path at a distance from the LTS which exceeds the distance for calm conditions by the value

$$\Delta X = u_x t,$$

where t is the flying time from the traverse of the LRMS to the emergence from the fourth turn, or from the moment when the aircraft passes over the LRMS until it emerges from the standard turn.

Example: The flying time from the traverse of the LRMS to the emergence from the fourth turn in a calm is 4 min, divided into these stages:

Traverse of the LRMS to beginning of third turn...70 sec
Third turn...60 sec
Buffer line...50 sec
Fourth turn...60 sec

The speed of the head-wind component on the landing course is u_x = 15 m/sec. /368

Find the value of ΔX for emergence from the fourth turn.

Solution:

$$\Delta X = 240 \times 15 = 3600 \text{ m}.$$

In order for the distance for emergence from the fourth turn to remain the same as in a calm, it is necessary to shorten the flying time from the traverse of the LRMS to the beginning of the third turn by a value

$$\Delta t = \frac{3600}{V+u_x} .$$

In our example, for an airspeed of 400 km/hr (110 m/sec) and a course opposite to the landing course,

$$\Delta t = \frac{3600}{110+15} \approx \frac{3600}{125} = 29\,\text{sec}$$

Thus, a flight from the traverse of the LRMS to the start of the third turn would last 41 sec instead of 70 sec.

By combining the formulas for obtaining the values for ΔX and Δt, we finally obtain the formula for determing the value Δt:

$$\Delta t = \frac{tu_x}{V+u_x}.$$

For our example,

$$\Delta t = \frac{240 \cdot 15}{110+15} = 29\,\text{sec}$$

The problem for a standard turn is solved in the same way. In this case, the time t is the time from the passage over the LRMS in a course opposite to the landing course, to the end of the standard turn; the value of Δt is calculated from that for a calm in flying from the LRMS to the start of the standard turn.

Calculation of the Correction for the Time of Starting the Fourth Turn

The beginning of the fourth turn in coming in for a landing is usually determined from the course angle of the LRMS. For example, when executing a maneuver to the right:

$$\text{ctg}\,\text{CAR} = \frac{R}{X},$$

where R is the radius of the turn made by the aircraft, and X is the distance along the LTS axis from the LRMS to the starting point of the fourth turn.

Under the influence of a side wind, the fourth turn is begun earlier if the lateral component of the wind on the landing course is favorable between the third and fourth turns, later if this component is unfavorable.

Obviously, if we take the wind into account: /369

$$\text{ctg}\,\text{CAR} = \frac{R + t \cdot u_z}{X}$$

where t is the time of the fourth turn and u_z is the lateral component of the wind speed.

For example, with a turning radius of 4500 m and X = 12.5 km,

$$\text{ctg}\,\text{CAR} = -\frac{4500}{12\,500};$$
$$\text{CAR} = 70°.$$

If the lateral component of the wind appears on the "buffer" line and is favorable, with a speed of 10 m/sec, then for a turning time of 60 sec we will have:

$$\text{ctg}\,\text{CAR} = \frac{4500 + 60 \cdot 10}{12\,500};$$
$$\text{CAR} = 68°,$$

i.e., the turn must begin 2° earlier than under calm conditions.

Calculation of the Moment for Beginning Descent Along the Landing Course

Under calm conditions, the distance at which the aircraft emerges from the fourth turn is determined by the flying time from the LRMS to the start of the third turn.

For example, with V = 111 m/sec (400 km/hr), this distance will be:

$$X = 70 \cdot 111 \approx 8\ \text{км to LRMS}$$

The fourth turn will be completed at approximately this distance if a correction for the effect of the wind is made in the time for starting the third turn. Consequently, with a standard location of the LRMS, the distance from the point where the aircraft comes out of the fourth turn to the touchdown point is 12 km.

The distance for beginning the descent along the glide path is determined by the formula

$$X_d = H_c\,\text{ctg}\,\theta,$$

where X_d is the distance for beginning the descent and H_c is the flight altitude for circling the field.

For example, at a circling altitude of H_c = 400 m and a slope angle for the glide path θ = 2°40':

$$X_d = 400 \cdot \text{ctg}\,2°40' = 8500\ \text{м}.$$

Thus, after coming out of the fourth turn at a distance of 12 km from the LTS, the aircraft must follow the landing path without losing altitude for a period of time /370

$$t_h = \frac{X_e - X_d}{V - u_x},$$

where t_h is the time of horizontal flight along the landing path.

For example, if the horizontal airspeed after coming out of a turn is 360 km/hr (100 m/sec), and the head wind is moving at 15 m/sec, the time for horizontal flight in our case will be

$$t_h = \frac{12\,000 - 8500}{100 - 15} = \frac{3500}{85} = 41 \text{ sec}$$

Under calm conditions, the time for horizontal flight in this case will be:

$$t_h = \frac{3500}{100} = 35 \text{ sec}$$

Practically speaking, the descent of the aircraft must begin 5 to 6 seconds before this time has actually elapsed, since a certain period of time is required to guide the aircraft into its landing regime.

Calculation of the Vertical Rate of Descent Along the Glide Path

The vertical rate of descent of an aircraft along the glide path is determined by the simple formula

$$V_y = W_x \operatorname{tg} \theta = (V_x - u_x) \operatorname{tg} \theta.$$

For example, with a mean horizontal rate of descent of 290 km/hr (80 m/sec), a head wind of 15 m/sec, and a slope angle in the glide path of 2°40':

$$V_y = 65 \cdot \operatorname{tg} 2°40' = 3 \text{ м/sec}$$

The calculation of the vertical rate of descent is of particular interest for piston-engine aircraft, whose horizontal glide is about 50 m/sec.

Since the head wind can be as high as 25 m/sec on landing, the vertical glide speed for these aircraft can change by a factor of 2, i.e., from 2.3 to 1.15 m/sec.

In the case of aircraft with gas turbine engines, the ratio of the maximum rate of descent to the minimum rate, with the same steepness of glide, is 1.5.

Determination of the Lead Angle for the Landing Path /371

A knowledge of the approximate value of the drift angle, and consequently the necessary lead angle for the landing path of an aircraft, considerably facilitates the choice of the course to be followed along a given descent trajectory.

The value of the drift angle along the landing path can be determined by the approximate formula

$$\operatorname{tg} US = \frac{u_z}{V_x - u_x}.$$

In flight along a given descent trajectory, however, the horizontal airspeed, altitude, and wind are variables, so that it is sufficient to use the following rule in finding the drift angle:

(a) For aircraft with gas turbine engines, at glide speeds of 270-290 km/hr, the lead angle is considered to be equal to 0.7° for each 1 m/sec of side wind.

(b) For aircraft with piston engines, (glide speeds of 180-200 km/hr), the lead angle is considered to be 1° for each 1 m/sec of side wind.

For example, with a side wind along the landing path of 8 m/sec, coming from the right, the lead angle will be:

- for aircraft with gas turbine engines, 5.5° to the left;
- for aircraft with piston engines, 8° to the left.

The calculations given above for the time of starting the third turn, the course angle for beginning the fourth turn, the time for beginning the descent, the vertical rate of descent, and the lead angle for the landing path, must all be made by the crew of the aircraft before approaching the airport on the basis of landing-condition information. All calculations must be complete before the landing maneuver begins.

Landing the Aircraft on the Runway and Flight along a Given Trajectory with a Simplified Landing System

While making preparations for landing, the crew must prepare the course to be followed by the aircraft along all the straight-line segments of the approach pattern, with the exception of the line between the third and fourth turns, beginning with a calculation of the drift angle.

The radiocompass must be set by the LRMS; if there are two sets of radiocompasses, the second must be set by the SRMS.

Along the line between the third and fourth turns, the course to be followed is always equal to the MPA of the "buffer" segment, so that the start of the fourth turn will be determined by the CAR. The slight drift of the aircraft which occurs at this time, as we have seen, is compensated by redefining the time for starting the third turn. /372

When the course angle of the LRMS becomes equal to the calculated value, the fourth turn is executed with a banking angle of 15° before acquiring the calculated landing path.

If all the calculated data are correct, the aircraft will come out of the turn precisely on the landing path with the desired course. At the moment when the aircraft emerges from the fourth turn, the timer is switched on to determine the time for beginning descent in the glide path.

In the majority of cases, however, due to errors in the operation of the radiocompass, improper maintenance of the course and air speed of the aircraft, errors in determining the side-wind component, and failure to bank at the proper angle when turning, the acquisition of the glide path by the aircraft is not accurate.

The accuracy with which the aircraft acquires the landing path is determined by a comparison of the magnetic bearing of the LRMS with the MPA for landing. IF MC + CAR = MPA_1, but the combined reading of the radiocompass is MBR = MPA_1, the aircraft will be exactly on the axis of the LTS.

If MBR is greater than MPA_1, the aircraft will be to the left of the given landing path. With MBR smaller than MPA_1, the aircraft will be to the right of the given landing path.

The difference between MPA_1 and MBR is called the acquisition error α.

Example: MPA_1 = 68°, with a calculated drift angle of +3°; the aircraft emerged from the fourth turn with MC = 65°, the course angle for the turn over the LRMS was 358°; find the acquisition error.

Solution:

$$\alpha = 68 - (65 + 358) = 5°,$$

i.e.,the acquisition error is 5° to the right.

For lining up the aircraft with the landing path, the course followed by the aircraft is usually changed by doubling the acquisition error. In our example, the course to be followed must be reduced 10°, so that the CAR of the LRMS becomes 8°; the flight is continued at this course until the value of the course angle increases to the magnitude of the acquisition error, i.e., becomes 13°.

When the pointer of the radiocompass is on the 13° mark (on a combined indicator, a bearing of 68°), with a slight lead (no more than 1 to 2°), the aircraft makes another turn to the calculated landing path, and the CAR of the LRMS becomes equal to the calculated drift angle of the aircraft (3° in the example).

As the aircraft continues to follow the landing path on the calculated course, the CAR will remain equal to the calculated drift angle if the course of the aircraft has been properly selected.

If the CAR is increased, the aircraft will drift to the left of the LTS axis, and the path being followed will have to be increased for acquisition of the desired line of flight, and decreased later on, although it will remain somewhat greater than the calculated value (the CAR is then less than the calculated drift angle). If /373
the CAR is then to remain constant, the course to be followed must be selected properly.

Similar operations in selecting a course are carried out when the aircraft deviates to the right of the desired line of flight. These operations will have the form of a mirror image of the operations described above, i.e., when the CAR is reduced, it is also necessary to reduce the course to be followed in acquiring the desired line of flight, then increase it somewhat, but still keep it below the calculated value.

In the case when the course angle of the LRMS continues to change, after the first operation to correct the course by acquiring the line of the given course, the operations are repeated using the familiar method of half corrections.

Thus, the readings of the radiocompasses, beginning with the LRMS and then the SRMS, are used to maintain the given direction of the descent trajectory.

When the aircraft is calculated to have reached the point for beginning its descent, it is shifted to a descent regime with a calculated rate of descent. The vertical rate of descent is maintained by observing the variometer readings and those of the gyrohorizon, while maintaining the established regime of horizontal airspeed on the basis of the instrument-speed indicator.

The gyrohorizon must be used to maintain the vertical rate of descent, because the readings of the variometer are less stable than those of the angle of pitch of the aircraft obtained with the aid of the gyrohorizon indicator. The readings of the variometer must be averaged over the time.

In addition, the variometer has slight delays in the readings with a change in the angle of pitch of the aircraft. Therefore, the gyrohorizon is employed to select the angle of pitch for the aircraft at which the average readings of the variometer are equal to the calculated values, and this angle is maintained by the readings on the gyrohorizon.

If the horizontal airspeed is then increased or decreased relative to the given value, it is regulated by changing the thrust of the engines and simultaneously changing the angle of pitch slightly to maintain the calculated rate of descent.

A failure to maintain the calculated settings for the glide path, or errors in calculations, may cause the aircraft to pass over the LRMS earlier at the required altitude, so that the descent of the aircraft is terminated and the aircraft is once again placed in the regime of descent at the moment it passes over the LRMS. However, if the given altitude has not been attained when passing over the LRMS, the vertical rate of descent is increased at the stage of the flight between the LRMS and the SRMS.

Similarly, the descent of the aircraft is terminated if it reaches the altitude set for passing over the SRMS before the sound of the SRMS is heard, marking the location of the latter. /374

The minimum weather for the ceiling when landing with a simplified system, in the case of aircraft with piston engines, is not set any lower than the altitude for passing over the SRMS; in the case of aircraft with gas turbine engines, it is significantly higher. Therefore, the aircraft can be allowed to descend only in the case when the crew of the aircraft can see the lights of the approaches to the LTS and the end of the runway.

Course-Glide Landing Systems

The simplified system for landing an aircraft as described in the preceding section, using the master radio stations, has a number of important deficiencies:

(a) The measurement accuracy of the aircraft bearing, using an aircraft radiocompass and course meter, is very low, so that it does not make it possible to land the aircraft (especially those with gas-turbine engines) with low weather minima.

(b) The operation of radiocompasses during flight in clouds and precipitation is highly subject to atmospheric disturbances, thus complicating a landing with these devices as guides.

(c) The simplified system requires constant checking of the position of the aircraft along a given descent trajectory in terms of direction only; the descent of the aircraft in a given glide path is accomplished by maintaining the vertical rate of descent of the aircraft and calculating the time, thus complicating the landing procedure and not ensuring safe descent under especially difficult conditions.

If we consider that the period of landing the aircraft with low ceiling and low meteorological visibility is the most difficult and dangerous stage of the flight, it is necessary to devise more complete systems of instrument landing. One such system is the course-glide landing system.

The geometric essence of course-glide systems is the use of radio-engineering methods to define two mutually perpendicular planes in space (Fig. 4.12):

(a) A vertical plane which intersects the Earth's surface along the LTS axis.

(b) An inclined plane which represents the glide path of the aircraft.

If the aircraft is in one of these two planes, the readings of the corresponding pointer on the indicator (direction or glide) must be equal to zero.

When the aircraft moves out of one of these planes, the corresponding pointer shifts from zero. The shift of the pointer must be linear within certain limits (i.e., proportional to the deviation of the aircraft from the given plane).

Obviously, the given trajectory for the descent of the aircraft is the line of intersection of these two planes. When the aircraft is on the given trajectory, both indicator pointers must point to zero on the indicator. /375

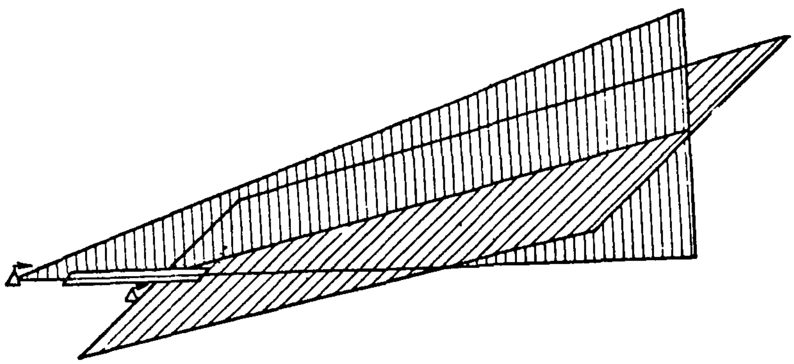


Fig. 4.12. Radio-Signal Planes of a Course-Glide Landing System.

For the best visual determination of the position of the aircraft relative to a given descent trajectory, the pointers on the indicator are made in the form of strips, one horizontal for glide and one vertical for direction. The movement of the strips then occurs in a direction which is opposite to the deviation of the aircraft from a given trajectory (Fig. 4.13).

The center of the instrument, with a silhouette of an aircraft shown on the scale, shows the position of the aircraft relative to the course plane and the glide plane. Thus, for example, in Fig. 4.13 the aircraft is located below the given glide path and to the left of the LTS axis. To set the aircraft on the desired trajectory, it must be turned in the direction of the planes, i.e., upward (to increase the angle of pitch) and to the right.

The indicator for the direction and glide has the traditional name of "Landing System Apparatus", or LAS for short.

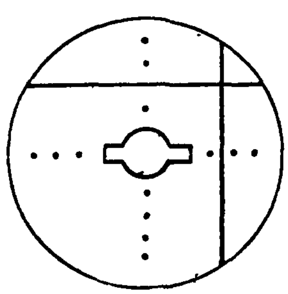
Ground Control of Course-Glide Systems

The principal pieces of equipment in a course-glide landing system are two ground beacons which form the course zone and the glide zone marking the given trajectory for the descent of the aircraft.

Both beacons operate on meter or centimeter wavelengths.

The antennas of the beacons that use meter waves are crossed horizontal dipoles (horizontal frames) in course beacons and horizontal dipoles in glide beacons. /376

Thus, the electromagnetic waves from the beacons are horizontally polarized, which to a certain degree reduces their effect on the directional characteristics of the antennas on the ground control facilities at the airport.



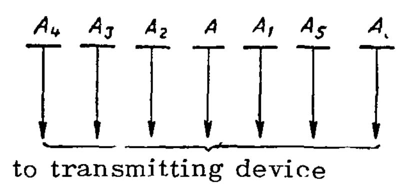


Fig. 4.13. Fig. 4.14.

Fig. 4.13. Indicator of Course-Glide Landing System.

Fig. 4.14. Diagram of Location of Antennas of Course Radio Beacon.

However, the Earth's surface plays a role in the formation of the course zone and the glide zone by these beacons. The course zone then becomes multilobed in the vertical plane, with the major lobe being the working lobe, which has a glide angle of the bisectrix which corresponds roughly to the slope angle of the glide plane of the aircraft. The Earth's surface is of still greater importance for the formation of the glide zone, whose slope angle depends on the height of the antenna above the ground.

The involvement of the Earth's surface in the formation of the beacon zones imposes limitations on the possibilities of the beacons in terms of ensuring the accuracy with which the aircraft can be landed. This is especially true for the glide zone, whose location can change with the state of the Earth's surface (wet or dry ground, grass cover, snow). The accuracy of the location of the course zone is subject to the influence of the local relief and equipment located within the limits of the directional characteristic of the antennas.

The most important of these shortcomings can be overcome to a great extent by employing beacons which operate on the centimeter wavelength, using reflecting antennas to form very narrow directional characteristics.

At the present time, however, these beacons have not been adopted sufficiently widely and are not used in enough locations. Therefore, we shall give a brief description of the course-glide systems only for the meter wavelengths.

In addition to the beacons, which form the course and glide zones, the course-glide system for landing also includes marker devices, whose locations can coincide with those for the markers in a simplified landing system.

In foreign practice, the first (long-range) marker is located /377
7 km from the end of the LTS; at a slope angle for the glide path of 2°30' and a circling altitude of 300 m, this marks the point at which the aircraft begins to descent in a glide. However, no significant advantages are gained by placing the marker at this spot, since the flight altitude of the aircraft when circling the field depends on the type of aircraft, while the slope angle for the glide path depends on the nature of the surrounding terrain. This means that the point for beginning the glide does not always coincide with the standard location of the aircraft (7 km).

For purposes of checking for the correctness of the location of the glide zone, it is better to choose a marker located 4 km from the end of the LTS, since at this point the aircraft will already have the selected rate of descent for following the glide path, and the altitude of its location will be determined more precisely.

An inherent part of the course-glide landing system is also the lighting system for the approaches to the runways and along the edges of the runway itself.

The *course beacon* is a transmitting device with an antenna system which consists (as a rule) of five or seven horizontal antennas (Fig. 4.14).

Antenna A has a radiation characteristic which is directed externally in the horizontal plane, and is powered by a transmitter operating without modulation on the meter wavelength.

Antennas A_1 and A_2 receive amplitude-modulated frequencies from the transmitter, one at 90 Hz and the other at 150 Hz.

Antennas A_3 and A_4 (as well as A_5 and A_6, in some types of beacons) serve to regulate the directionality of the radiation characteristic, as well as the direction of the radio-signal zone of the entire system.

The combined result of the electromagnetic oscillations of the entire antenna system forms the directional radiation characteristic of the electromagnetic waves in the horizontal plane; an example of this is shown in Fig. 4.15. Figure 4.15.a, shows the shape of the radiation characteristic in the horizontal plane; the left side is the one modulated by the 150 Hz frequency, while the right side is modulated by the 90 Hz frequency.

Along axis AB, where the radiation characteristics intersect the modulation frequencies of 150 and 90 Hz, the modulation depth of the carrier frequency by both low frequencies is the same (i.e., the difference in modulation depth is zero).

When the aircraft moves to the left of axis AB, the depth of the modulation with the 90 Hz frequency increases and that with the 150 Hz frequency decreases. The picture is reversed when the aircraft moves to the right of the axis.

The dotted lines in Fig. 4.15 show the projections of the radiation lobes of the electromagnetic waves in the vertical plane (as shown in Fig. 4.15,b) on the horizontal plane.

Line AB is the common axis with a difference in modulation /378
depths which is equal to zero for all lobes. However, the principal operating lobes are the first ones, located nearest to the ground.

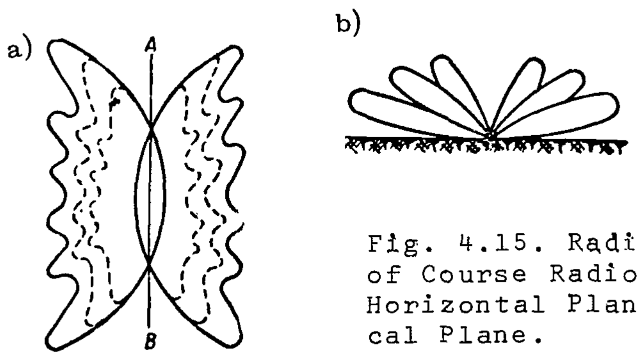


Fig. 4.15. Radiation Characteristics of Course Radio Beacon: (a) in the Horizontal Plane; (b) in the Vertical Plane.

The radiation characteristic of the course beacon is regulated in such a way that the axis of the zero difference in modulation depth coincides exactly with the axis of the LTS. Hence, it is necessary that the difference in modulation depths within limits of 3-4° of the equal-signal axis increase linearly with the lateral deviation. With further deviation from the LTS axis (within limits up to 10°), the difference in modulation depths must also increase, but not in proportion to the lateral deviation; it can maintain its value or decrease, but without changing sign in the entire hemisphere to the left or right of the radio-signal axis.

The distance for possible reception of the beacon signals in the sector 10° from the equal-signal axis in the working lobe of the zone must be within the limits of 45 to 70 km.

The *glide beacon* is also a transmitting device, operating in the meter wavelength, but at a frequency different from that of the course beacon.

The antenna system of the glide beacon consists of only two antennas (an upper and a lower), mounted on a common mast. The upper antenna is double, as shown in Fig. 4.16,a.

Both the upper and lower antennas receive an amplitude-modulated frequency, but with different modulation frequencies (for example, 90 and 150 Hz).

Each of the antennas, together with the ground forms an independent working lobe with its own modulation frequency (Fig. 4.16,b). The points of intersection of the working lobes in the vertical plane also form a radio-signal axis *AB* with a zero difference in the modulation depth.

Since the characteristic of the antenna directionality in the /379
horizontal plane is rather broad, the surface with a zero difference in modulation depth is conical, with *AB* as the generatrix. Therefore, the glide path can be an ideal straight line only in the case when the antenna system of the beacon is located at the point where the aircraft touches down on the runway.

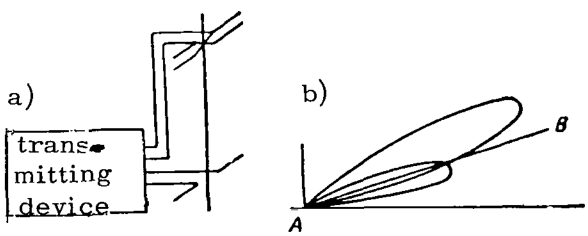


Fig. 4.16.

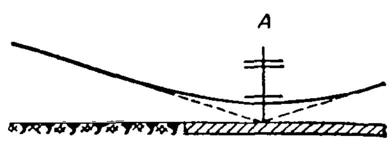


Fig.4.17

Fig. 4.16. Glide Radio Beacon: (a) Diagram of Antenna Location; (b) Radiation Characteristic.

Fig. 4.17. Hyperbolic Trajectory for Glide Plane.

However, the glide beacon cannot be located on the LTS axis or even in the immediate vicinity of the LTS, since it would constitute a flight hazard. Therefore, the intersection of the cone with the zero difference in the modulation depth for the glide beacon of the equal-signal plane of a course beacon gives a hyperbolic trajectory which does not touch the ground (Fig. 4.17).

As the aircraft approaches along the landing path toward the traverse of the glide beacon, the glide path begins to "float" above the ground, moving upward after passing over the beacon.

Since the location and shape of the directional characteristic of the two antennas of the glide beacon depends on the height of the antennas above the ground, the characteristic and the position of the line of their intersection in the vertical plane is regulated by the change in the height of the upper and lower antennas above the ground.

As in the case of the course beacon, the increase in the difference of the modulation depth with deviation from the glide surface upward or downward must be linear with this deviation. However, the curvature of the curve of the change in the difference in modulation depth will not be symmetric in this case, as it is for the course beacon. A steeper curve for the change in the difference of modulation depth is found above the glide surface, and a less steep curve is found below the surface.

The operating range for a glide beacon in a sector of ±8° from the LTS axis must be at least 18 to 25 km.

Aircraft-Mounted Equipment for the Course-Glide Landing System /380

The following units make up the aircraft-mounted equipment for the course-glide landing system:

(a) Antenna and receiver for course-beacon signals.
(b) Antenna and receiver for glide-beacon signals.
(c) Control panel.
(d) Landing-system apparatus (LSA)

The receivers of signals from the course and glide beacons contain essentially the same elements, with the exception of the AAC (automatic amplification control), which is not shown in the figure.

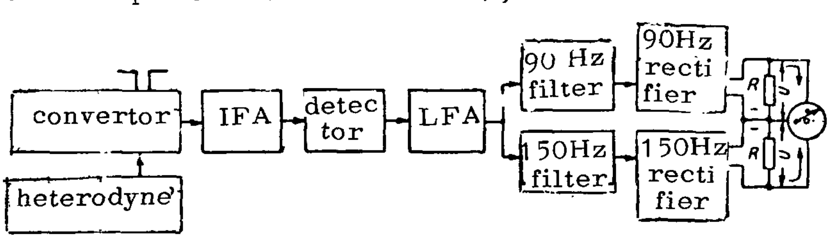


Fig. 4.18. Diagram of Aircraft-Mounted Glide Radio Beacon.

The glide-beacon receiver uses the circuit for the reinforced AAC. The latter is not employed in course-beacon receivers, since it would add the microphone commands relayed via this beacon to the aircraft when the communications receivers are out of order.

The signals from the course and glide beacons are picked up by the antennas and amplified by the HFA. The selection of the frequency channel is made on the basis of the first intermediate frequency by quartz returning of the heterodyne from the control panel. The signals are then amplified by the IFA and LFA channels, so that the signals pass through 90 Hz and 150 Hz filters to the rectifiers, then to the emergency blinker, and finally to the receiver ground. The indicator for the course or glide zone is connected in a bridge circuit between the rectifiers for the 90 and 150 Hz signals.

If the signals do not reach the receiver or there is some malfunction in the receiver blocks somewhere ahead of the 90 and 150 Hz filters, the readings of the LSA indicator on that channel will be zero; if the equipment is operating properly, it means that the aircraft is located precisely in the corresponding zone. Therefore, the LSA system includes the emergency blinkers. When no current is flowing in the 90 and 150 Hz rectifiers, the current through the emergency blinker windings will not flow, and a signal indicating that the apparatus is malfunctioning will be displayed on the indicator.

The design circuit for the receivers of the signals from the glide and course beacons includes potentiometers for electrical balance of the LSA indicators. Each of the rectifiers receives signals which do not pass through the 90 and 150 Hz filters. The indicator pointer should then point to zero. If the balance of the currents in the rectifier is upset, it causes the regulating potentiometer to rotate. /381

The balancing potentiometer of the receiver for the signals from the glide beacon is usually mounted on the receiver housing, while the receiver for signals from the course beacon is mounted on the control panel.

For smoothing the short-period oscillations of the course and glide indicator of the LAS, due to local disturbances in the radio-signal zone, the indicator circuit contains a special sealed unit damping capacitors in the circuit for turning on the apparatus.

Location and Parameters for Regulating the Equipment for the Course-Glide Landing System

The radio beacon for the course zone of the course-glide system for landing an aircraft is mounted at a distance of 600 to 1000 m from the end of the runway, along an extension of the axis of the LTS.

The beacon for the glide zone is mounted to the side of the LTS (as a rule, to the left of the landing path), at a distance of

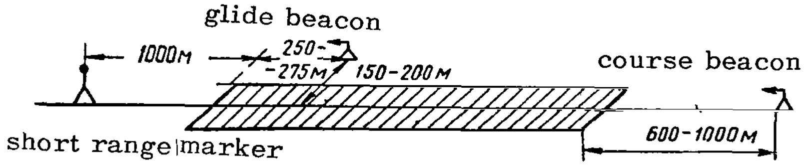


Fig. 4.19. Diagram of Location of Ground-Based Equipment for Course-Glide System.

150 to 200 m from its axis and 250-275 m from the end of the runway.

The axis of the zone of the course beacon coincides with the LTS axis. A control point is chosen for measuring the parameters for regulating the system on the LTS axis.

The control point is selected as a point where the antenna receiving signals from the glide beacon aboard the aircraft will be located at the moment when the aircraft touches down on the runway. It is considered that this point is located at an altitude of 6 m above the surface of the LTS, and is plotted from the location of the glide beacon, 75 m toward the end of the LTS (i.e., the distance from the end of the runway to the control point is 180 to 200 m).

The slope angle for the glide path is calculated from a theoretical plane located 6 m above the surface of the LTS. The vertex of the slope angle of the glide path is the control point (CP). /382

The width of the zone of the course and glide beacons is reckoned from the angles of deviation from the given descent trajectory, calculated respectively from the point where the course beacon is located and from the control point, within the limits of which the strips of the landing-system apparatus deviate from the zero position to the limits of the scale.

Obviously, the angle of deviation of the LSA strip depends on the difference in the modulation depths in the beacon zones, as well as on the sensitivity of the receiver aboard the aircraft. Therefore, the angular width of the zones of the course and glide beacons is regulated by the sensitivity of the receivers mounted aboard the aircraft, which are used as standards.

The standards for the width of the course-beacon zone are set as follows:

(a) The angular width of half the zone must be located within 2 to 3° of the LTS axis.

(b) The linear width of half the zone at a distance of 1350 m from the control point (1150 m to the end of the runway) must be equal to 150 m. An expansion of the zone from the nominal value to 45 m and a narrowing to 30 m is considered permissible.

The horizontal scale of the LSA (see Fig. 4.13) from the center to the scale stop has 6 divisions. The first division is the white circle on the silhouette of the aircraft, the second is the end of the vane, the third, fourth, and fifth are points on the horizontal axis of the scale, while the sixth is the scale stop.

The vertical scale also has six divisions, of which the second division here is the first point on the vertical axis of the apparatus.

Each division of the horizontal scale of the LSA corresponds to a deviation of the aircraft from the LSA axis (relative to the point where the course beacon is located) within limits of 20 to 30' or 25 m (+7, -5 m) from the LTS axis at a distance of 1350 m from the control point.

The angle width of the zone of the glide beacon is linked to the slope angle of the glide path, which is determined by the conditions of the formation of the zone. The width of the zone beneath the glide path is then somewhat greater than above the glide path.

The standards for regulating the glide zone are the following:

(a) The position of the upper limit at an angle to the axis of the zone within the limits from 0.19 to 0.21 θ, i.e., approximately 1/5 of the slope angle for the glide path.

(b) The location of the lower limit at an angle to the axis of the zone within limits from 0.29 to 0.31 θ (somewhat less than 1/3 of the slope angle for the glide path).

Accordingly, one division of the vertical scale of the LSA in the upper part is equal to about 0.030 θ, while in the lower part it is about 0.05 θ, where θ is the slope angle of the glide path.

Landing an Aircraft with the Course-Glide System /383

Setting up the maneuver for an aircraft approaching an airport to descend with the use of the course-glide system is performed according to the same rules as in the simplified system for landing an aircraft.

The complement of equipment for the course-glide system for landing an aircraft is usually supplemented by one or two master radio stations with marker beacons, located in the system for simplified landing, which is used for setting up the maneuver for bringing the aircraft in for a landing and to a certain degree reserves the course-glide system for cases of malfunction of the ground or airborne equipment, as well as during times when equipment is being repaired or adjusted.

If in addition to the course-glide and master beacons, the airport is equipped with radar for observing the aircraft, the maneuver for landing in minimum weather can be made along the shortest path for each landing direction and takeoff direction.

By the same rules which govern the simplified landing system, preliminary calculations are carried out which ensure a simpler and more exact action of the crew in flight along a given descent trajectory.

A portion of the preliminary calculations, such as (for example) the determination of the moment for starting the descent in a glide, cannot be done in this case if we keep in mind the fact that the given glide path is defined in space. The calculations of the drift angle of the aircraft and the vertical rate of descent along the landing path are of somewhat less importance in this case.

When the maneuver for making a landing is made on command from the ground, the need for such calculations as the determination of the moment for making the third turn no longer exists. However, the moment for beginning the fourth turn must in all cases be determined by the crew of the aircraft, with the maximum accuracy possible.

In setting up the maneuver for landing, the strips of the LSA can be located on any divisions of the scale and no attention need be paid to their readings; however, when approaching the fourth turn, both strips must be located on the scale stops. The strip for the course zone rests on the stop on the side opposite the direction of the maneuver, the strip for the glide zone rests on the stop at the top. The emergency blinkers must then be off.

The strip for the course zone must move away from the scale stop during the fourth turn. The movement of the strip away from the stop is called *deflection*.

When the fourth turn is made correctly, deflection of the strip for the course zone occurs at the moment when the turn angle is held until the aircraft acquires the calculated landing course (Fig. 4.20,a). For aircraft with piston engines, this turn aisle is about 45°; for aircraft with turbojet or turboprop engines, it is about /384
30°.

With a residual turn angle of 45° for aircraft with piston engines (30° for aircraft with gas turbine engines), if deflection of the course-zone strip does not occur, it means that the fourth turn is being made with a lead.

In this case, it is desirable to significantly reduce the banking angle during the turn or even to stop turning and follow the LTS axis at the residual turn angle until the LSA strip deflects.

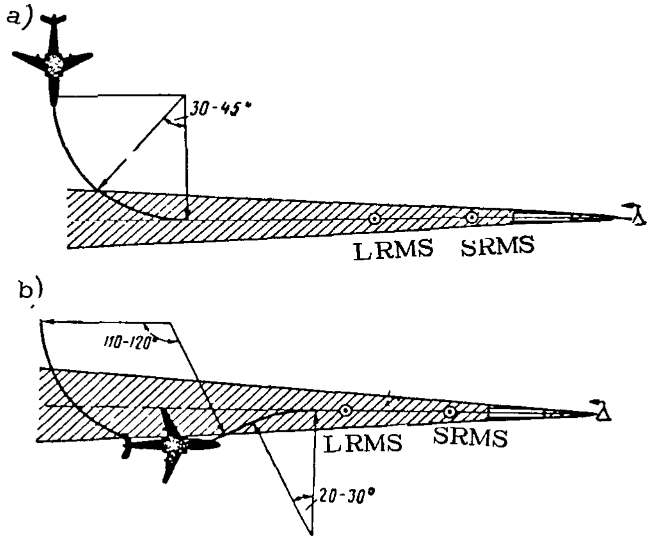


Fig. 4.20. Acquisition of the Landing Path by an Aircraft: (a) With Proper Turn; (b) With Turn Begun Late.

When the course-zone strip deflects, the turn must be continued until the landing course is acquired. When the landing course is acquired, the course-zone strip must be located near the zero marking (center of the scale).

In cases when the fourth turn is made with a delay (Fig. 4.20,b), the deflection of the LSA strip takes place earlier than 45 or 30° before acquisition of the landing course. In this case, the turn must last until the landing course and beyond, at a landing angle opposite to the LTS axis, depending on the magnitude of the transition of the course strip through the center of the scale.

For example, if the descaling occurs at the very beginning of the fourth turn, it is necessary to increase the banking angle in the turn up to 20°, and the aircraft will continue to turn to the opposite angle for landing (20° in aircraft with piston engines and 30° for aircraft with gas turbine engines). /385

With less delay in turning, the opposite angle for approach can be within the limits of 5 to 20°.

With reverse deflection of the course-zone strip, the aircraft makes a reverse turn onto the landing course, with a simultaneous flat turn onto the LTS axis. After the aircraft has acquired the

LTS axis, the flight continues for a time until deflection of the glide-zone strip takes place at a constant altitude.

At the moment when the glide-zone strip moves away from the upper stop, the aircraft shifts to a descent regime with a smooth acquisition of the desired glide path downward.

Directional Properties of the Landing System Apparatus

The selection of the desired course and the vertical rate of descent are sources of considerable difficulty for the crew and require a certain degree of training. However, these difficulties do not arise from principles of piloting the aircraft along the LSA, but rather from the necessity of simultaneously observing several devices and instruments and selecting a flight regime in the vertical and horizontal planes simultaneously.

Nevertheless, with a proper reaction of the crew to a change in the positions of the strips on the LAS, the landing maneuver should be successful in all cases and not very difficult.

In piloting the aircraft by the LSA, two of its principal characteristics must be employed:

(1) The indicating characteristic, i.e., the indication of the position of the aircraft relative to a given descent trajectory.

(2) The command characteristic, i.e., the ability to predetermine the actions of the crew in selecting the flight regime.

Inasmuch as the first property of the LSA is obvious, let us examine the second.

The course and glide zones are rather narrow in space, sufficiently so that the limits of these zones can be considered parallel over short segments of the trajectory.

Let us say that an aircraft at a given moment is located to the side of the LTS axis, and the ground speed vector of the aircraft does not coincide with the direction of this axis (Fig. 4.21). Obviously, the ground speed vector of the aircraft can be divided into two components: a longitudinal one W_x and a lateral one W_z.

The longitudinal component W_x is not involved in the selection of the course to be followed. The principal role is played by the lateral or transverse component, W_z.

The component W_z determines the rate of motion of an LSA strip along the horizontal scale of the apparatus. With the strip fixed at any scale division, the component W_z is equal to zero, which agrees precisely with the selected aircraft course, i.e., its path is practically parallel to the axis LTS. /386

The regulation of the LSA is set so that the change in the course of the aircraft (1.5 to 2°) makes the motion of the vertical

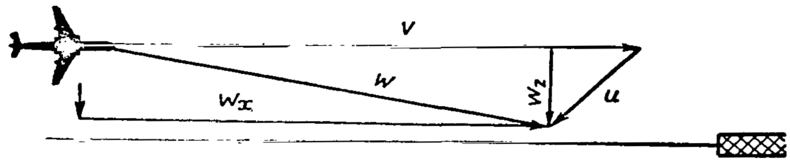


Fig. 4.21. Division of Ground Speed Vector into Longitudinal and Lateral Components along the Landing Path.

strip LSA visible to the eye. For example, in aircraft with piston engines, a change in the course by 2° produces a lateral shift of the aircraft of 2 m/sec. This means that the most dangerous region of flight (1200 to 1500 m to the end of the runway), the LSA strip crosses each scale division in 11 to 12 sec, i.e., a sufficiently noticeable value equal to half a scale division after each 5 to 6 sec of flight. If a turn is made in the direction of the motion of the LSA strip by 2°, its motion can be halted at any division on the scale.

On this basis, the principle of selecting the course for the aircraft by the LSA must be the following:

If the vertical strip is located at a significant distance from the center of the instrument (on the third or fourth division), it is then necessary that the rate of its shift to the center of the instrument be significant. To do this, it is sufficient to turn the aircraft in the direction in which the strip is moving, by 4 to 6°.

As the strip approaches the center of the apparatus, its rate of motion must be arrested by turning the aircraft 1 to 2° in the direction shown by the arrow. At the moment when the strip reaches the center of the instrument, its motion is arrested by a final turn of the aircraft by 1 to 2°, and the aircraft will be set on the LTS axis, with the course already selected.

This method requires a very precise flight of the aircraft along the axis of the course zone, with periodic changes in the course within the limits of 1 to 2°.

An analogous method is employed to set the vertical rate of descent of the aircraft, with simultaneous acquisition of the desired glide plane and subsequent flight along it.

Maintenance of the descent regime of the aircraft along a given trajectory by the readings of the LSA continues up to the moment when the aircraft emerges from the clouds and makes a transition to visual flight, after which a visual estimate of altitude is made and the aircraft touches down on the runway. /387

Directional Devices for Landing Aircraft

Determination of the rate of shift of the strips calls for increased vigilance in observing each of them. In addition, local irregularities in the course zone and glide zone at individual points disturb the regularity of the process; this must be taken into consideration by the crew and carefully separated from the generally established tendency.

All of this requires considerable caution and training on the part of the crew for making a descent along a given trajectory.

Recently, special directional devices for piloting an aircraft in the course and glide zones have begun to be employed widely.

Unlike the LSA, the directional properties of these devices are *not* expressed by the derivatives of the positions of the strips on the instrument with time, but directly by the positions of these strips.

The most widely employed directional devices at the present time are those which are based on various laws of control, with an indication which is linked to the banking of the aircraft during a coordinated stable turn, or to the angle of pitch at a set rate of descent.

In pilotage of the aircraft in the horizontal plane, these laws represent a definite link between the course and the banking of the aircraft in a turn, with lateral deviation from the radio-signal plane of the course zone and the first derivative of this deviation with time.

$$K_\gamma \Delta\gamma + K_\beta \beta + K_z Z + K_{V_z} V_z = 0,$$

where $\Delta\gamma$ is the angle of approach to the landing path, β is the banking angle of the aircraft in the turn, Z is the lateral deviation of the aircraft from the zone axis, V_z is the rate of lateral shift of the aircraft, and K are the coefficients for the corresponding parameters.

A similar law is employed for piloting an aircraft in the vertical plane:

$$K_a \vartheta + K_y Y + K_{V_y} V_y = 0,$$

where ϑ is the angle of pitch of the aircraft, Y is the deviation of the aircraft from the glide path in the vertical plane, and V_y is the rate of vertical motion of the aircraft.

Since the linear values Z and Y and their first derivatives /388
cannot be measured directly in polar systems, their values are replaced by angle values (α and $\Delta\theta$) and their derivatives. The values α and $\Delta\theta$ and their derivatives α and $\Delta\theta$ are measured by the differences in modulation depths and their derivatives in the zones of the course and glide beacons.

Obviously, by selecting the proper banking angle and pitch angle for the aircraft, the aircraft can be positioned so that both strips on the directional indicator are located on zero.

The coefficients for the converted position parameters for the aircraft axes are selected so that whatever deviations the aircraft may make from the given trajectory (if the indicator strips remain on zero), the aircraft will still travel along the given landing and glide path with a predetermined trajectory (whose course depends upon the coefficients selected). This means that the landing course and vertical speed must be selected simultaneously, since they are required for flying the aircraft along a given trajectory.

Hence, instead of adjusting the rate of motion of the strips in accordance with their motion toward the center of the instrument as in a normal LSA, in directional instruments the crew need only bring the indicator strips to the center of the instrument by changing the banking angle of the aircraft as well as its angle of pitch; this significantly facilitates the task of piloting an aircraft.

To further reduce the work of the crew, directional instruments are usually combined with a gyrohorizon indicator. In this case, the entire attention of the pilot is concentrated practically on the readings of only one instrument. However, directional instruments based on the rules stated above have some important shortcomings, which to a certain degree reduce the accuracy of piloting an aircraft relative to piloting by the indications of an LSA.

The proper selection of coefficients for making a turn and the angle of pitch of the aircraft can be made only at a certain distance of the aircraft from the ground beacons. During measurement of the distance, the linear width of the course and glide zones changes, thus leading to a failure of the system regulation parameters to agree with the dynamic flight trajectory of the aircraft. This shortcoming can be completely overcome if the system is regulated not only by the angular deviation of the aircraft from the radio-signal axis, but by calculation of the distance remaining to the ground radio beacons:

$$Z = L_c \operatorname{tg} \alpha;$$
$$Y = L_g \operatorname{tg} \Delta\theta,$$

where L_c and L_g are the distances to the course and glide radio beacons.

Control can then be effected in a rectangular system of coordinates, and therefore with constant agreement of the regulation of the system with the dynamic trajectory of the aircraft's flight. /389

In polar coordinates, shortcomings in the operation of the directional system can be eliminated by a special selection of converted signal coefficients (not proportional to the values of the signals in various sections of the trajectory) in accordance with the tactical characteristics of aircraft of various types.

It should also be mentioned that in directional systems, the indication of the position of the aircraft relative to a given descent trajectory is lost. This means that on board the aircraft, in addition to the directional devices, there must still be a conventional LSA indicator, which is used as a standard to check the accuracy of pilotage according to the directional indicator.

So-called paravisual directional instruments are also beginning to be used nowadays; in principle, they represent a reinforcement of the directional properties of the LSA.

In this case, the usual LSA indicators are located in the center of the field of the pilot's vision, while at the periphery of his vision there are imitators of the motion of the strips according to the first derivatives α and $\Delta\theta$, which link the indication shown with the longitudinal and lateral rolling of the aircraft according to the laws of the design of directional instruments.

Radar Landing Systems

From the tactical standpoint, radar landing systems have no special advantages over course-glide systems; on the contrary, their use is less convenient, since there are no instruments aboard the aircraft for indicating the position of the aircraft and no commands for piloting it relative to a given descent trajectory.

The accuracy with which an aircraft can be landed by means of radar landing systems is roughly equal to that of landing it with course-glide systems. Nevertheless, radar landing systems are widely employed, along with course-glide systems.

The primary reason why radar landing systems have been employed so widely is the need for a constant check on aircraft making their landing approaches by course-glide systems, for the purposes of pointing out errors made by the crew and preventing the very dangerous consequences of error.

The second reason is the need to give the crew assistance in landing the aircraft if they should request it, if for some reason the course-glide system cannot be used. The same reasoning applies /390
in retaining the course-glide system in case the ground control is not functioning.

The radar landing system consists of a complex of devices for observing the flight of approaching aircraft (radar screen, USW radio distance-finder) and those actually making a landing (landing radar). In addition, the system includes communication apparatus for transmitting information and necessary commands to the aircraft.

The landing radar is the heart of the radar landing system, so we shall pause to examine the principles of its operation.

Unlike ground radar installations with circular screens, the landing radars have a sector screen, i.e., there is no rotating directional characteristic of the antenna, but one which scans (oscillates) in a certain sector. Accordingly, the scanning line on the radar screen also oscillates.

The landing radar has two antennas:

(a) The course-sector antenna, with a wide characteristic in the vertical plane and a narrow one in the horizontal.

(b) The glide-sector antenna, with a wide characteristic in the horizontal plane and a narrow one in the vertical; the scanning of the characteristic of this antenna takes place in the vertical plane.

The scanning of the directional characteristics of the landing-radar antenna can be achieved either by mechanical oscillation of the antenna reflector or by special devices which change the phase of the wave along the chord of the antenna reflector, thus causing the plane of the wave front to oscillate (so that all the wave-propagation characteristics also oscillate).

The scanning sectors of the directional characteristics of the antenna are made narrow;

(a) For a course sector of 15°: to either side of the LTS axis.

(b) For a glide sector, 9° wide: +8° upward and -1° downward from the plane of the horizon.

A peculiar feature of the landing radar is the special design of the scanning on the course and glide screens. Thus, instead of the circular distance marks on conventional circular radar screens, the distance marks on landing radars are straight lines, i.e., the delay in the distance marks is made proportional not to R, but to $R/\cos\alpha$, where α is the angle of deviation of the scanning line from the axis of the scanning sector. Hence, a rectangular system of coordinates is formed on the screen from the polar system of coordinates for the aircraft.

In addition, the radar screen has a transverse scale three times larger than the distance scale for the course sector and five times larger than that for the glide sector. This means that there is a corresponding relationship between the increase in the scale indicating the position of the aircraft relative to the given trajectory for the same screen radius.

A general view of the screens of the course and glide sectors /391
is shown in Fig. 4.22.

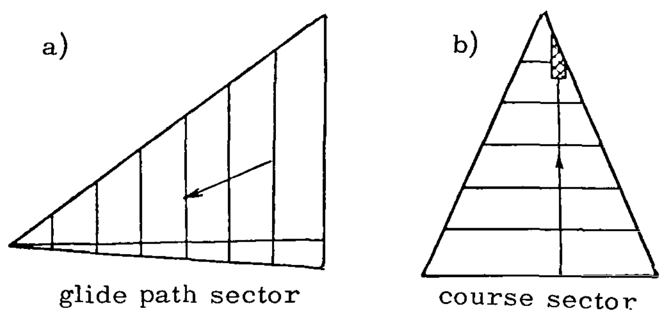


Fig. 4.22

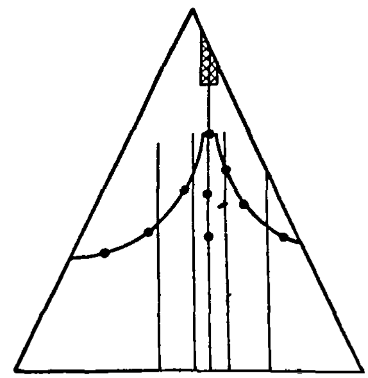


Fig. 4.23

Fig. 4.22. Landing-Radar Screen: (a) Glide Sector; (b) Course Sector.

Fig. 4.23. Pattern on Course Screen of Landing Radar.

The landing radar is mounted on the traverse of the center of the LTS, at a distance of 100 to 150 m to the side, so that the conditions for using it when landing at either end of the runway will be the same.

In the immediate vicinity of the landing radar, there is a circular-scan radar for observing aircraft near and far from the airport.

In setting up the landing maneuver, immediately before completing the fourth turn, the short-range radar approach system is used, also called control-tower radar (CTR). Its screen can be used to show landing maneuvers for aircraft approaching from all directions.

All turns of the aircraft are made on command from the flight supervisor, as are the course corrections on the straight-line segments between the turns, if the given flight directions are not maintained sufficiently accurately.

Observation of an aircraft with the landing radar begins while it is making the fourth turn, using only the course sector screen.

In order to ensure that the aircraft lands precisely on a given descent trajectory, the required pattern is superposed on the landing radar screen. This pattern on the screen serves three purposes:

(1) To show the given trajectory for the aircraft's descent.

(2) To provide auxiliary lines for giving commands to the crew of the aircraft.

(3) To show the boundary lines for safe flight altitude and the permissible zones for landing the aircraft. /392

Since the landing radar is usually used for two directions of landing and takeoff, and can even be used for three or four is if other runways intersect, the patterns for the screens are printed on removable celluloid sheets which can be changed when shifting the landing radar to a new landing direction.

The screen for the course sector of a landing radar (Fig. 4.23) usually shows the following:

1. The given landing path (axis of LTS), beginning at the end of the runway and extending to the limit of the screen. The following points are marked on this line: the beginning of descent along a set glide path and the locations of the LRMS and SRMS landing systems within the range of the master radio stations. The SRMS is usually fitted with a corner reflector, which produces a bright spot on the screen and is used in setting the radar for the given landing direction and as a control to check the accuracy of the setting of the radar after it is turned around.

2. The lines delimiting the zone of possible aircraft landings. These lines are defined on the basis of the assumption that the aircraft, being on a course close to that for landing, can be lined up with the LTS axis prior to the start of the landing distance only in the case when

$$X > 2R \sin UT,$$

where

$$yP = \arccos\left(1 - \frac{Z}{2R}\right),$$

where X is the remaining distance to the start of the landing distance, Z is the lateral deviation from the landing path, and R is the turning radius with a banking angle of 10°.

The order in which these lines are plotted is the following:

(a) Several points of deviation of the aircraft from the landing path are given (e.g., 30, 100, 200, 500, 1000, 2000, and 4000 m) and the required turn angles to correct these deviations are determined: /393

$$\cos UT = 1 - \frac{Z}{2R};$$

(b) The required course for lining up the aircraft with the LTS axis is determined:

$$X = 2R \sin UT.$$

To this path, we add the distance traveled by the aircraft (in 4 sec for piston-engine aircraft, 7 sec for gas turbine aircraft), required for receiving commands and carrying out the maneuver to line up the aircraft with the runway.

(c) The path obtained for the aircraft is measured from the starting point of the landing distance (as rule, from the SRMS), and we obtain the minimum attainable distances of the selected points for the lateral deviations of the aircraft.

By connecting the points by a smooth curve, we obtain the limit of the possible landing zone of an aircraft, with permissible lateral deviations.

In the course of landing an aircraft, if it shows up outside the indicated limits, the landing cannot be allowed and the command is given to make another pass at the field.

The boundary lines are usually plotted for two typical glide speeds of aircraft:

for those with piston-engines, 200 km/hr;

for those with gas turbine engines, 280 km/hr.

The turn radius is calculated for a coordinated turn with a banking angle of 10°, with the lines for starting the turn plotted for making a landing at approach angles of 10 and 30°.

If the aircraft has a significant deviation from the LTS axis after emerging from the fourth turn, we can in principle use any angle of approach to the LTS axis which makes it possible to line up the aircraft with the landing path before the landing distance is reached.

However, as experience has shown, it is simplest to line up the aircraft with the landing path by using only two values for the approach angles: 10° if the deviation of the aircraft from the given line path is less than 500 m, and 30 m for deviations exceeding 500 m. Then the landing-radar screen can be bounded by a total of two auxiliary lines for beginning the turn onto the landing path.

In this case, the distance from the LTS axis to the auxiliary line can be determined by the formula

$$Z = R(1 - \cos UT)$$

However, experimental data show that there is an appreciable delay in the aircraft's acquiring the landing path, due to the /394
time involved in transmitting commands and due to the reaction of the aircraft and crew in making the turn. Therefore, it is better to plot these lines on the basis of statistical data obtained from experience, as determined from a large number of aircraft landings.

According to these data, the turn to the landing course must begin:

(a) For an approach angle of 10°, in aircraft with piston engines, 150 m from the LTS axis (5 mm on the screen scale); for aircraft with gas turbine engines, it is 250 m from the LTS axis (8 mm on the screen scale).

(b) With an approach angle of 30°, these distances are 450 and 750 m, respectively (15 and 25 mm on the screen scale).

The markings on the glide screen of the landing radar are shown in Fig. 4.24.

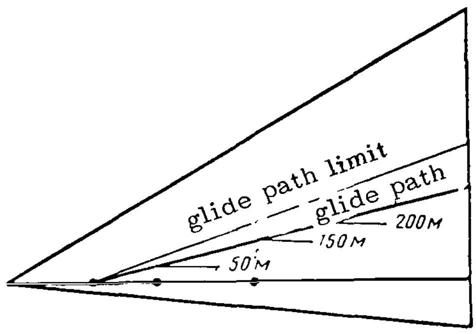


Fig. 4.24. Pattern on Glide Screen of Landing Radar.

In this case, the descent trajectory for the glide path is set at the airport. Above this glide path are two boundary lines for landing the aircraft; for aircraft with gas turbine engines it is 4°, and for aircraft with piston engines it is 5°.

If the blip representing an aircraft appears above the boundary line designated for a given type of aircraft, the landing of the aircraft will be complicated. Therefore, when controlling the landing of an aircraft, it should not be allowed to go beyond the limits of these boundary lines.

Below the established glide path, there are boundary lines for permissible descent of the aircraft below the glide path, i.e., the lines limiting the flight altitude above the local terrain: 200 m prior to beginning descent in a glide, 150 m before passing over the LRMS, and 50 m before passing over the SRMS. In addition, there may also be flight altitudes for circling the field, set at 300, 400 and 500m.

These lines are used for aircraft coming in for a landing according to the CGS (course-glide system).

In the case where the blip marking an aircraft intersects one of these lines, further descent of the aircraft is to be considered dangerous and the intervention of the flight supervisor operating the landing radar is required.

Bringing an Aircraft In for a Landing with Landing Radar /395

The method of bringing an aircraft in for a landing with a landing radar is very simple and quite effective at the present time.

The setting up of the landing maneuver and the calculations of the elements of the descent is made by the same rules as in using the simplified or course-glide landing systems.

The moment for starting the fourth turn is determined on the basis of the blip representing the aircraft on the flight supervisor's screen. No commands are given to the crew during the fourth turn.

After the aircraft emerges from the fourth turn, the calculated landing path must be followed for 10 to 15 sec. If the blip on the landing-radar screen is parallel to the LTS axis, the calculated course of the aircraft is equal to the landing course and the aircraft need merely be lined up with the landing path. If the blip is at an angle to the LTS axis, the calculated course of the aircraft is not equal to the landing course, but it is very easy to determine the desired course correction by visual inspection, since the angle of the blip is equal to three times the angle of the course error. For example, with a blip angle of 10°, the course correction must be 3°.

Having thus determined the required correction in the course to be followed, the supervisor gives a command to the crew, telling them to acquire the desired landing path at an angle of 10 or 30°, thus setting the course to be followed.

At the moment when the blip crosses the corresponding auxiliary line, a command is given to turn the aircraft onto the landing course, considering the correction given.

In the majority of cases, when these two commands are given, it is sufficient to line up the aircraft with the landing path on the desired course. If a tendency is observed during flight along the landing path for the aircraft to shift laterally, it can be corrected by commands for small changes in the aircraft course (by 2 or 3°), with indication each time of the course which must be followed.

When the blip approaches the point where the aircraft is to begin its descent in a glide, a command is given to descend at a calculated vertical speed. If it then develops that the aircraft is deviating from the given glide path (either upward or downward), the flight supervisor corrects the vertical speed, giving new values for it and ensuring that the aircraft travels exactly along the given path.

An advantage of the radar landing system is the relative simplicity of the supervisor's task in directing the aircraft to a landing and the uncomplicated actions of the crew in carrying out the supervisor's commands, with no previous training required. These advantages are also reinforced by the fact that the flight supervisor, /396 who constantly watches over several aircraft coming in for a landing and gives them instructions, acquires a very great amount of experience in the course of his work, a great deal more than that which the crew can acquire from the landings of their own aircraft alone. In addition, the supervisor, in the course of his work in guiding one aircraft after another to a safe landing, acquires a peculiar "feel" for estimating the navigational difficulties on a given day (selection of the required vertical speed and landing course on the basis of his experience with aircraft that have landed earlier).

Therefore, in practice, the accuracy of landing an aircraft with a radar system is no worse than with a course-glide system. Nevertheless, the main shortcoming of the system (a lack of indication for the crew as to the position of the aircraft on a given descent trajectory) creates a certain degree of inaccuracy in making the landing, and in this respect the radar landing system is inferior to the course-glide system.

CHAPTER FIVE

AVIATION ASTRONOMY[1]

1. The Celestial Sphere

The *sky* appears to the observer as an immense hemisphere.

The *celestial sphere* is an imaginary sphere of arbitrary radius, whose center is the eye of the observer (Fig. 5.1). /397

An observer on the Earth's surface can see only the half of the celestial sphere which is located above the horizon, since the other hemisphere is located below the horizon.

If the Earth were transparent, an observer located at any point on its surface would see not one but two domes which together form the celestial sphere.

Special Points, Planes, and Circles in the Celestial Sphere

Zenith and nadir. If a line is plotted perpendicular to the location of the observer (through the center of the celestial sphere), it will intersect the imaginary limits of the celestial sphere at two points (see Fig. 5.1). The point which is located above the observer is the *zenith* (Z). The opposite point is the *nadir* (Z').

True horizon. If a plane is defined through the center of the celestial sphere and is perpendicular to the vertical line ZZ', we /398 can call it the *plane of the horizon.*

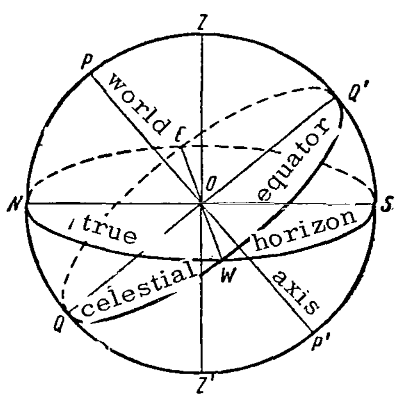


Fig. 5.1. Celestial Sphere

The plane of the horizon intersects the celestial sphere along the circumference of a great circle (the points NESW) which is called the *true horizon.*

World axis. The imaginary line PP', around which the apparent rotation of the celestial sphere takes place, is called the *world axis.* It passes through the point of the observer, located at the center of the celestial sphere, and intersects the arbitrary limits of the celestial sphere at two

[1]This chapter was written by M.I. Gurevich.

diametrically opposed points PP′. The world axis is inclined to the horizon at an angle which depends on the latitude of the observer.

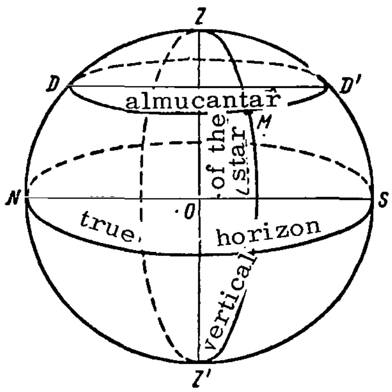


Fig. 5.2. Vertical and Almucantar.

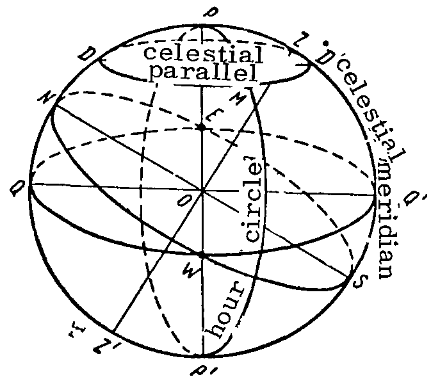


Fig. 5.3. Celestial Meridian, Hour Circle, and Celestial Parallel.

Celestial poles. The points where the imaginary world axis intersects the arbitrary limit of the celestial sphere are called the *celestial poles*. Point P is called the *superior (north) celestial pole*, and the opposite point P′ is called the *inferior (south) celestial pole*. Only the north celestial pole is visible in the Northern Hemisphere, and only the south celestial pole is visible in the Southern Hemisphere.

Celestial equator. The plane which passes through the center of the celestial sphere and is perpendicular to the world axis is called the *plane of the celestial equator*. The great circle QEQ′W, along which the plane of the celestial equator intersects the celestial sphere, is called the *celestial equator*.

The celestial equator divides the celestial sphere into northern (QPQ′) and southern (Q′P′Q) parts.

The plane of the celestial equator is inclined to the plane of the true horizon at an angle which also depends on the latitude of the observer. /399

Vertical. The great circle on the celestial sphere whose plane passes through the vertical line is called the vertical. Every vertical passes through the zenith Z and the nadir Z′. The plane of the vertical is perpendicular to the plane of the true horizon (Fig. 5.2).

The vertical which passes through the east and west points (E and W, respectively) is called the *primary vertical*.

The great circle ZMZ′ of the celestial sphere, which passes through the zenith of the observer and a certain star (Point M, Fig. 5.2), is called the vertical of that star.

Almucantar. The small circle DMD′ on the celestial sphere, whose plane is parallel to the plane of the true horizon, is called the *almucantar*.

The almucantar which passes through a given star is called the *almucantar of that star*.

Hour circle. The great circle PMP′ of the celestial sphere, whose plane passes through the world axis, is called the *circle of declination* (Fig. 5.3). Since the world axis is perpendicular to the celestial equator, the plane of the hour circle is also perpendicular to the equator.

The hour circle which passes through a given star is the *hour circle of that star*.

Celestial meridian. The vertical PZP′Z′, which passes through the celestial poles, is called the *celestial meridian* (since its plane coincides with the plane of the meridian of the observer). The celestial meridian divides the celestial sphere into the eastern and western hemispheres.

The north point N and south point S. The celestial meridian crosses the true horizon at two points, called the *north and south points*.

Meridian line. The plane of the celestial meridian crosses the plane of the true horizon to form the *meridian line*. Obviously, the ends of the meridian line coincide with the north and south points. (N and S, respectively). This line is called the "noon line" in Russian because the shadows of vertical objects fall along this line at noon.

The east point E and west point W. If we plot a straight line in the plane of the horizon perpendicular to the meridian line (see Fig. 5.3) and face north, the east point E will lie on the right at the point where the plane intersects the circumference of the true horizon, while the west point will be located on the left.

As the figure shows, the east and west points are 90° distant from the north and south points. The same figure also shows that the east and west points (E and W, respectively) mark the points of intersection of the celestial equator with the true horizon.

Celestial parallel. The small circle on the celestial sphere, /400
whose plane is parallel to the plane of the celestial equator, is called the *celestial parallel* (similar to the terrestrial parallels).

Diurnal circle of a star. The small circle on the celestial sphere, drawn through a star parallel to the celestial equator, is called the *diurnal circle of the star.*

Astronomical coordinates. As we know, in order to determine the location of any point on the Earth's surface, it is sufficient to know the two angular coordinates of this point, the latitude and longitude.

In astronomy, the location of stars on the sphere is accomplished by means of two angular systems of celestial coordinates: the apparent system of coordinates and the equatorial system of coordinates.

In each of these systems, the position of a point (star) on the celestial sphere is determined by two celestial coordinates. Let us examine the systems of celestial coordinates individually.

Systems of Coordinates

Apparent System of Coordinates

The main circles relative to which coordinates are determined in this system (Fig. 5.4) are the true horizon and the meridian of the observer. The coordinates themselves are called the altitude of the star (h) and the azimuth of the star (A).

Altitude of a star. The angle between the plane of the true horizon and a line from the center of the sphere to the star (angle M'OM, Fig. 5.4) is called the *altitude of the star.* The altitude of a star can also be measured by the arc of the vertical from the true horizon to the location of the given star (M'M).

The altitude of the star is measured from 0 to 90° (positive values toward the zenith from the horizon, negative values from the horizon toward the nadir).

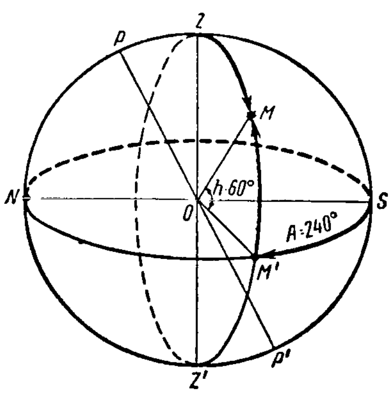


Fig. 5.4. Horizontal System of Coordinates.

Zenith distance. Instead of the star, we can also use the so-called zenith distance of the star as a coordinate, measured along the arc ZM.

As we can see from Figure 5.4, the zenith distance is the arc from the zenith to the location of the given star. It is easy to set up a formula to express the relationship between the altitude and the zenith distance of a star, since the two add up to 90°: $h + Z = 90°$, /401

$h = 90° - Z$, $Z = 90° - h$. Obviously, the value of the zenith distance will be somewhere between 0 and 180°.

Azimuth. The second coordinate in the apparent system of coordinates is the azimuth of the star. The *azimuth of a star* is the spherical angle between the plane of the meridian of the observer and the plane of the circle of the vertical of the given star.

The azimuth is calculated differently in different areas of astronomy: from the south point or from the north point toward the east and west. In aviation astronomy, the azimuth is always calculated from the north point along the horizon in an easterly direction (clockwise) from 0 to 360°. We can therefore define the *azimuth* in aviation astronomy as the angle measured along the arc NSM′ of the true horizon from the north point through the east (the east point) to the vertical of the star (see Fig. 5.4), from 0 to 360°.

Hence, the first system of coordinates for celestial luminaries is called the apparent system. The coordinates of this system are the altitude of the star (h) and the azimuth of the star (A).

The altitude and azimuth will suffice completely to determine the location of a star on the celestial sphere. For example, the star M, with $h = 60°$ and $A = 240°$, is indicated on the sphere (see Fig. 5.4).

Equatorial System of Coordinates

The equatorial system of coordinates is the second system of coordinates which is used to determine the location of a star on the celestial sphere. The main circles relative to which calculations are made in this system are the celestial meridian and the celestial equator.

The coordinates in this system are the declination of the star (δ) and the hour angle of the star (t); see Figure 5.5.

Declination of the star. The arc of the circle marking the distance from the equator to the location of the given star, or the angle between the plane of the equator and a line from the center of the sphere to the star, called the *declination of the star*.

Declination is measured by the arc of a circle which marks the distance from the equator to the location of the given star, from 0 to ± 90°. If the star is located in the Northern Hemisphere, its declination is considered positive, while if it is in the Southern Hemisphere, it is considered negative.

It is clear from Figure 5.5 that if the star is located on the equator, its declination will be equal to zero, while the declination of the north celestial pole is + 90° and that of the south celestial pole is -90°.

Polar distance. Occasionally, instead of the declination, the polar distance is used as a coordinate, measured along the arc PM. /402 The *polar distance* is the arc of the circle which marks the distance from the north celestial pole to the location of the star.

The relationship between the declination and the polar distance is expressed by the formula

$$\delta + PM = 90°$$

or

$$PM = 90° - \delta,$$

or

$$\delta = 90° - PM,$$

i.e., the declination and polar distance together add up to 90°. Therefore the point of the south pole has a polar distance equal to 180°.

Hour angle of a star. The arc of the celestial equator Q'M' (Fig. 5.5) between the south point of the equator and the hour circle of a given star is called the *hour angle of a star* (t).

In aviation astronomy, the hour angle is measured from the south point of the equator along the equator in the easterly and westerly directions from 0 to 180°.

The western hour angle is represented by the letter W, for example, t = 135° W; the eastern hour angle is represented by the letter E, for example, t = 60° E. In making calculations, the western hour angle must sometimes be calculated from 0 to 360°. If the western hour angle is found to be greater than 180°, it is related to 360°, but in this case the result is given as an eastern hour angle. For example, t = 265° W or t = 360° - 265° = 95°E.

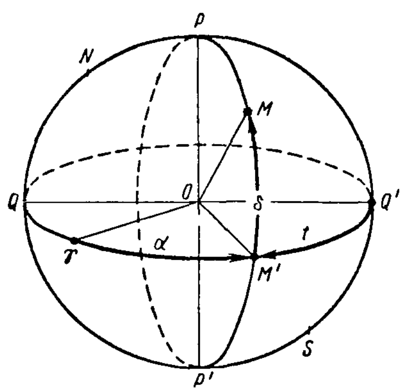


Fig. 5.5. Equatorial System of Coordinates.

Right ascension of a star. Instead of the hour angle, it is sometimes more convenient to use another coordinate, the right ascension of the star (α). The *right ascension of a star* is the angle as measured along the equator from the point of the vernal equinox (γ) to the hour circle of the given star (see Fig. 5.5).

The *point of the vernal equinox* is the imaginary point of the intersection of the ecliptic with the celestial equator, when the Sun passes from the Southern Hemisphere into the Northern Hemisphere. The opposite point on the ecliptic is called the *point of the autumnal equinox* (Ω).

In ancient Greece, the stars were used to reckon time. The constellation Aries was located at the point of the vernal equinox, and was represented by the symbol (γ). Due to the precession of the Earth, Aries has now moved away from the point of the vernal equinox. This point has remained unmarked, though its name has been retained, and its position in the sky is determined by using some other star which is a fixed distance from the point of the vernal equinox. /403

Right ascension is calculated from the point of the vernal equinox along the equator up to the hour circle of a given star in a clockwise direction (as seen from the north celestial pole), from 0 to 360°.

Like the hour angle of a star, the right ascension of a star can be reckoned in either degrees or hours, minutes, and seconds. This is because both of these coordinates (especially the hour angle) are closely related to the measurement of time.

Thus, the equatorial system of coordinates can be used to determine the location of a star on the celestial sphere.

If we know the declination and the hour angle or the right ascension, we can determine the location of a star on the sphere. For example, the star M, with $\delta = +50°$, $t = 45°$, is shown on the sphere (see Fig. 5.5).

Graphic Representation of the Celestial Sphere

In solving textbook problems in aviation astronomy, it is often necessary to sketch the celestial sphere and plot the stars on it according to their coordinates. Let us use a concrete example to study the order in which the celestial sphere is sketched.

Example. 1. The latitude of the observer is $\phi = 60°$N, the altitude of the star h = 70°, and its azimuth $A = 240°$.

Draw the celestial sphere and plot the position of the star on it. (Fig. 5.6,a).

Solution. (1) Use a compass to draw the celestial meridian in the form of a circle of arbitrary radius.

(2) Draw a vertical diameter (perpendicular line) and mark the zenith and nadir (Z and Z′, respectively) at the points where it crosses the circumference.

(3) Perpendicular to the vertical line, through the center of

the sphere, draw a large circle which will be the true horizon of the observer.

(4) Draw the world axis such that the angle it forms with the plane of the horizon will be equal to the latitude of the observer, i.e., ϕ = 60°N; mark the points where the world axis crosses the circumference (the north celestial pole P and the south celestial pole P′).

(5) At the points where the true horizon intersects the meridian of the observer, mark the north point N (close to the north celestial pole) and the south point S (close to the south celestial pole).

(6) Perpendicular to the point of intersection of the celestial equator with the true horizon, mark the east point E (on the right, as viewed by someone facing north) and the west point W (on the left).

This completes the sketching of the celestial sphere. We have yet to plot the position of the star on the sphere on the basis of its coordinate data, as follows:

(1) From the north point N, plot the azimuth of the star (equal to 240°) along the circumference of the horizon, judging the angle by eye. /404

(2) Through this point M′, draw the circle (semicircumference) of the vertical.

(3) Along the circle of the vertical, from the plane of the horizon, plot the altitude of the star, equal to 70°, judging the distance by eye.

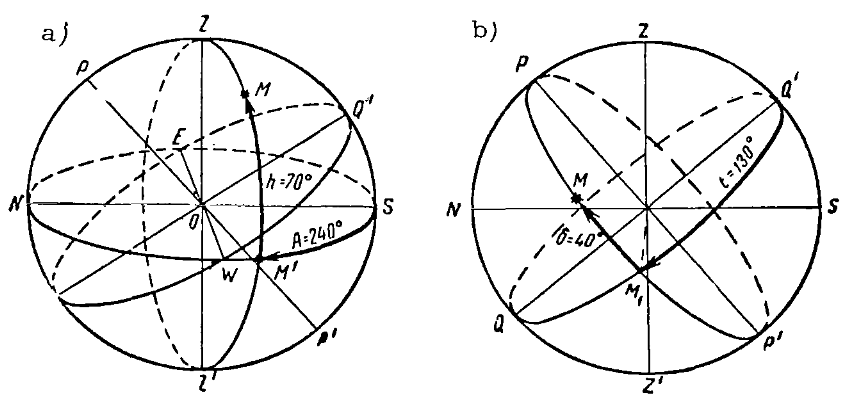


Fig. 5.6. Examples of Graphic Construction of the Celestial Sphere; (a) At a Latitude of 60°; (b) At a Latitude of 50°.

The result of this construction will be the celestial sphere as seen by an observer at 60°N and the position of a star on the sphere according to its apparent coordinates.

Example. 2. The observer is located at a latitude of 50°. Sketch the celestial sphere for this observer and plot on it the position of a star with the following equatorial coordinates: hour angle $t = 130°$, declination $\delta = +40°$.

Solution. (1) Sketch the celestial sphere in the same order outlined in Example 1.

(2) From the south point on the equator Q′, proceeding along the circumference of the equator in a westerly direction, plot an hour angle $t = 130°$ by eye (Fig. 5.6,b).

(3) Through this point (M′), draw the hour circle (PM′P′).

From the plane of the equator, along the hour circle, measure off the declination $\delta = +40°$ and mark the location of the star on the sphere (point M).

The result of this construction is the hour circle for an observer located at a latitude of $\phi = 50°$N; the star has been plotted on the sphere on the basis of its equatorial coordinates.

2. Diurnal Motion of the Stars

If one observes the heavenly bodies, even in the course of a single night, he will see that the appearance of the sky changes with the passage of time. Several constellations which were located near the eastern portion of the horizon gradually rise higher above the horizon, while others, which were located closer to the western horizon, gradually set. Some stars which were visible in the sky set in the west, while others, which were not visible before, rise in the east. The entire sky seems to move constantly /405 during the course of the night from east to west. At the same time, one can see that the mutual positions of the constellations and stars do not change in the course of the motion of the entire sky (see Supplement 3).

The reason for this apparent motion of the stars (or of the sky) is the diurnal rotation of the Earth on its axis *from west to east*.

In order to facilitate a study of the diurnal rotation of the stars, we will assume for the sake of discussion that the Earth is fixed and the celestial sphere rotates on the world axis at the same rate that the Earth actually rotates on its axis, but in the opposite direction, from east to west (in other words, the way it actually looks to us). Since the entire celestial sphere rotates on the world axis, all the points (stars) located on the sphere will turn along with it, i.e., it is clear that each star describes a sort of circle around the world axis.

Diurnal parallel of a star. All of the stars rotate together with the celestial sphere around the world axis. From this it is clear that every star, fixed permanently in the sky, describes a circle of some size in the course of 24 hours.

The circle described by a star in 24 hours in the course of its movement around the world axis is called the *diurnal circle of the star*. This circle is also called the *celestial parallel*.

Since the entire celestial sphere rotates around the world axis, it is easy to see (and important to remember) that the diurnal rotation of the heavenly bodies takes place parallel to the celestial equator, i.e., the diurnal parallel of the star (the path of the star around the world axis in 24 hours) is always located parallel to the celestial equator.

The magnitude of the diurnal parallel of the star depends on the location of the star in the sky. Obviously, stars which are located closer to the celestial poles (and have higher declination values) have a small diurnal circle. The closer a star is located relative to the celestial equator (the smaller its declination), the larger its diurnal circle will be. The largest diurnal circle belongs to those stars which are located on the celestial equator, and whose declination is zero.

Motion of the Stars at Different Latitudes

If we observe the diurnal motion of the stars at different latitudes, we will see that the sky and stars turn relative to the observer's horizon at different angles. This phenomenon becomes understandable if we recall the location of the world axis relative to the horizon at different latitudes.

The world axis is located relative to the horizon at an angle /406
which is equal to the latitude of the location. From this it follows that the higher the latitude of a location, the closer the celestial poles PP′ will be located to the zenith Z and the nadir Z′, and the smaller the angle will be between the true horizon and the celestial equator. Conversely, the lower the latitude of the location, the further the celestial poles will be from the zenith and nadir, and the angle between the true horizon and the celestial equator will be larger.

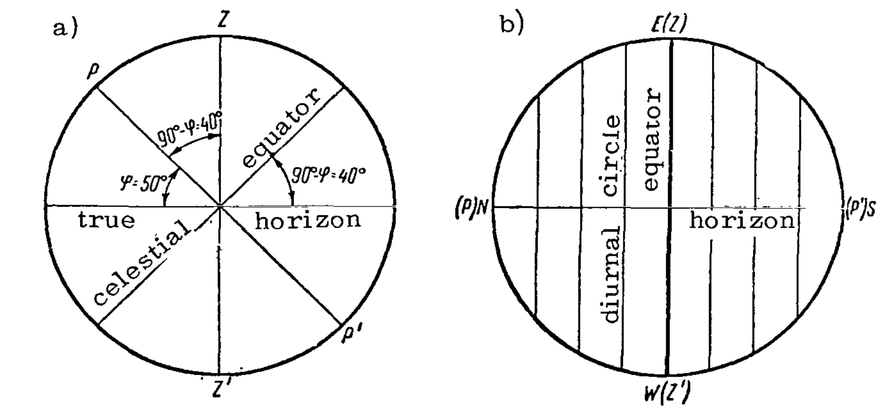
Figure 5.7,a shows the angle between the true horizon and the celestial equator for an observer located at a middle latitude, e.g. 50° (angle $90° - \phi = 40°$). Figure 5.7,b shows the angle between the true horizon and the celestial equator for an observer located on the Equator (angle $90° - \phi = 90°$), while Figure 5.7,c shows the angle between the true horizon and the celestial equator for an observer located at the North or South Pole. (The angle 90° - ϕ = 0, the true horizon is parallel to the celestial equator, the zenith point Z coincides with the north celestial pole P, and the nadir /407 Z' coincides with the south celestial pole P').

It is clear in all three figures that the angle between the true horizon of the observer and the celestial equator is always equal to 90° minus the local latitude (90 - ϕ).

We can draw the following conclusion from the above: the slope of the diurnal parallel of stars relative to the true horizon of the observer depends on the latitude of the observer. The higher the latitude of the observer, the smaller the slope of the diurnal parallels of the stars relative to the horizon; the lower the latitude, the greater the slope.

Rising and Setting, Never-Rising and Never-Setting Stars

If we know that the position of the celestial equator (and consequently the diurnal parallels of the stars) relative to the true horizon of the observer depends on the latitude of the observer, it will be clear why some stars at a certain latitude rise and set



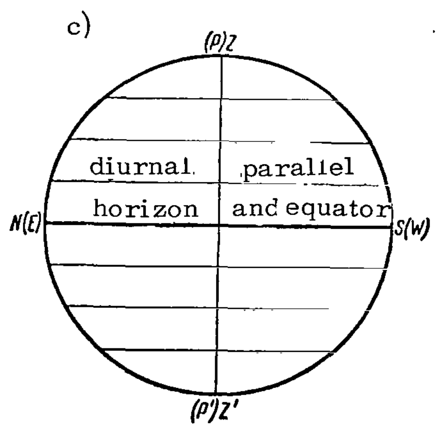


Fig. 5.7. Angles Between the True Horizon and the Celestial Equator; (a) At a Latitude of 50°; (b) On the Equator; (c) At the Poles.

at the horizon, others never set, and still others never rise.

A star never sets if its declination is greater than 90° minus the latitude of the location, i.e., if $\delta \geq 90° - \phi$.

For example, see Figure 5.8,a. Given the latitude of the observer $\phi = 60°$, the declination of the star $\delta = +45°$. From this it is clear that $90° - \phi = 90 - 60° = 30°$. Since the declination $\delta = +45°$, i.e., greater than $90° - \phi$, it is clear that the star cannot set below the horizon of the observer. In Figure 5.8,a we have sketched the celestial sphere for an observer located at a latitude of 60°. We mark off the declination of the star along the meridian of the observer (i.e., the hour circle) so that $\delta = +45°$, and then lay out the diurnal circle (diurnal parallel) of the star parallel /408

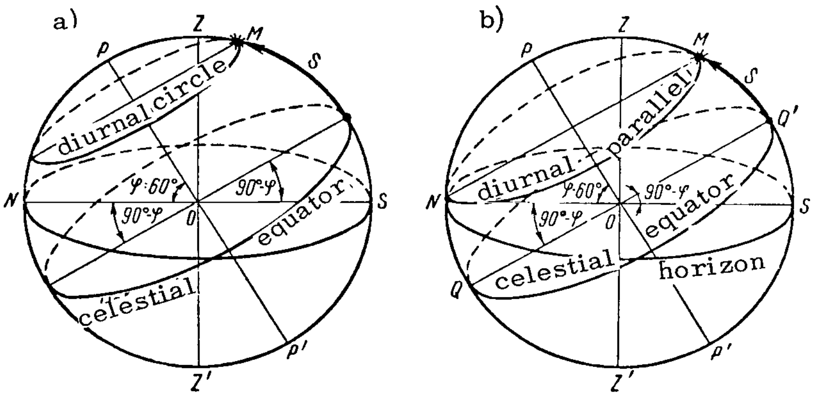


Fig. 5.8. Examples of Never-Setting Stars; (a) The Star Never Sets Below the Horizon; (b) The Star Touches the Horizon.

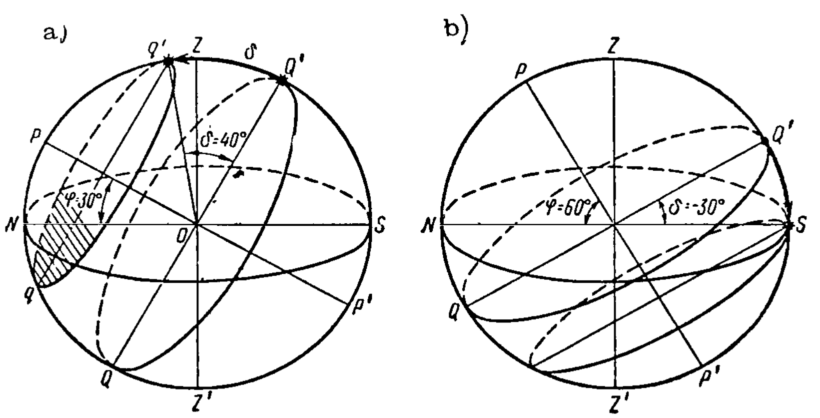


Fig. 5.9. Examples of Stars that Set; (a) The Star Rises and Sets; (b) The Star does not Rise.

to the celestial equator. As we can see from the figure, this circle is located above the horizon of the observer, and so a star which moves along this circle in the course of 24 hours will never set below the observer's horizon.

The star touches the horizon, but does not go below it, in the case when its declination is equal to 90° minus the latitude of the observer, i.e., if $\delta = 90 - \phi$.

Take Figure 5.8,b for example. The latitude of the observer $\phi = 60°$, the declination of the star $\delta = +30°$. From this it is clear that $90° - \phi = 90 - 60° = 30°$. In accordance with what we have said, if $\delta = 90° - \phi$, the star will touch the observer's horizon but will not set below it.

In Figure 5.8,b we have sketched the celestial sphere for an observer located at a latitude of 60°. Along the meridian of the observer (i.e., the hour circle), we have plotted the declination of a star $\delta = +30°$, and have then drawn the diurnal parallel of this star parallel to the celestial equator. As we can see, the diurnal parallel of the star touches the observer's horizon, but does not cross it, i.e., a star moving along its diurnal parallel in the course of 24 hours goes down to the horizon and then rises again in the course of its diurnal journey.

A star rises and sets when its declination (in terms of absolute value) is less than 90° minus the latitude of the location, i.e., if $\delta < 90° - \phi$. /409

Let us consider the following example: The latitude of the observer is $\phi = 30°$, the declination of the star is $\delta = +40°$. In Figure 5.9,a we have sketched the celestial sphere for an observer located at a latitude of 30°. Along the meridian (hour circle) we have marked off the declination of the star $\delta = +40°$ and have drawn the diurnal parallel of the star parallel to the equator (q - q'). As we can see from the diagram, a star which moves in the course of 24 hours along its diurnal parallel will be located below the horizon for a certain time (the shaded part of the diurnal parallel), and will be above the horizon the rest of the time.

A star never rises if its declination is equal to or greater than 90° minus the latitude of the observer and has a sign which differs with latitude (the latitude is positive and the

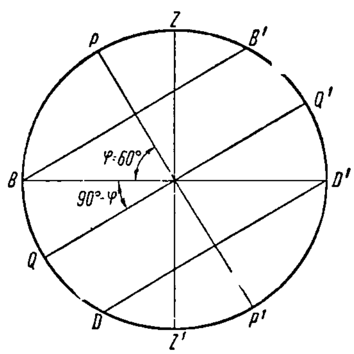


Fig. 5.10 Division of the Celestial Sphere into Regions with Rising and Setting, Never-Setting and Never-Rising Stars.

declination is negative, or vice versa), i.e., if $-\delta \geq 90° - \phi$, or $\delta = \phi - 90°$. For example, the latitude of the observer $\phi = 60°$N, the declination of the star $\delta = -30°$.

In Figure 5.9,b we have sketched the celestial sphere for an observer at a latitude of $\phi = 60°$N. Along the meridian of the observer (hour circle) we have marked the declination of the star $\delta = -30°$ (below the equator) and the diurnal parallel of the star parallel to the equator. As we can see from the diagram, a star which moves along its diurnal parallel will always be below the horizon and will never rise. This is completely understandable, since the declination of the star is negative. If the star had a negative declination still greater than $90 - \phi$, its diurnal circle would be located still further below the horizon.

Consequently, the entire celestial sphere of an observer located at a given latitude can be divided into three parts:

(1) The portion of the celestial sphere with stars that never set.

(2) The portion of the celestial sphere with setting and rising stars.

(3) The portion of the celestial sphere with stars that never rise.

All three portions of the celestial sphere are shown in Figure 5.10 for an observer at a latitude of 60°N. The circumference is the plane of the celestial meridian, ZZ′ is the vertical line of the observer, PP′ is the world axis. The straight line BB′ is the section of the plane of the celestial meridian as cut by the diurnal circle of the star, touching the horizon of the observer but not passing below it (a star whose $\delta = 90° - \phi$). This is the boundary of the region of stars that never set with that of the ones which rise and set for a given latitude of the observer. The straight line DD′ is the section of the plane of the celestial meridian as cut by the hour circle of the star in the Southern Hemisphere, touching the horizon but not going below it (a star whose declination is $-\delta = 90° - \phi$). This is the boundary of the region of stars that never rise with that of the ones which rise and set for a given latitude of the observer. /410

Motion of Stars at the Terrestrial Poles

In order to get a better idea of the nature of the diurnal motion of stars at the terrestrial poles, let us construct a form of celestial sphere for an observer located at the North Pole. In this case, the altitude of the Pole above the horizon will be equal to the latitude of the observer. Since the observer is located at the North Pole, $\phi = 90°$N and consequently the altitude of the Pole above the horizon will be 90° (Fig. 5.11).

The world axis coincides with the vertical line, i.e., the north celestial pole P coincides with the zenith Z and the south celestial pole P′ coincides with the nadir Z′, while the plane of the celestial equator will coincide with the plane of the horizon. This means that all stars, depending on their diurnal rotation, will move parallel to the horizon and their altitudes will not change as the celestial sphere rotates. All stars located in the Northern Hemisphere (having positive declinations) will move above the horizon, while the stars which are located in the Southern Hemisphere (and have negative declinations) will move below the horizon, i.e., all stars with $\delta > 0°$ will never set and all those with $\delta < 0°$ will never rise.

Motion of Stars at Middle Latitudes

Let us examine the nature of the diurnal motion of the stars at middle latitudes, when $0 < \phi < 90°$.

Figure 5.12 shows the appearance of the celestial sphere at a latitude close to 45°. Due to the inclination of the world axis, all stars move at an angle to the horizon (parallel to the celestial equator). At middle latitudes, a considerable number of stars rise and set (between /411
the parallels NK and DS). Stars which are located farther than these parallels from the celestial equator either never rise or never set.

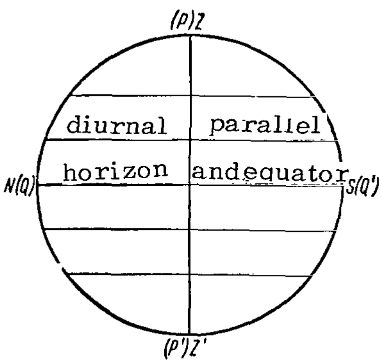


Fig. 5.11. Motion of the Stars at the Terrestial Poles.

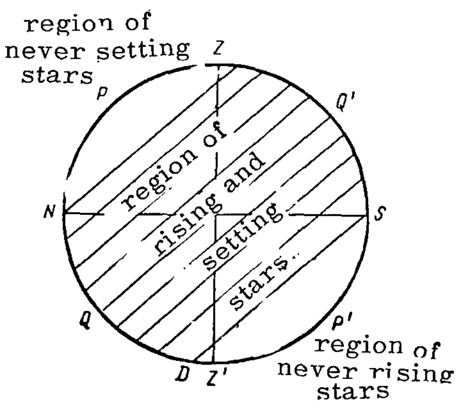


Fig. 5.12. Motion of the Stars at Middle Latitudes.

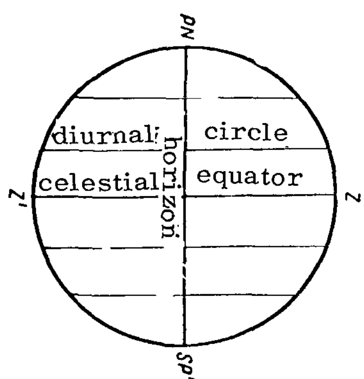


Fig. 5.13. Motion of the Stars at the Equator.

Motion of Stars at the Equator

Since the latitude of an observer located on the Equator is equal to zero, it is clear that the world axis lies in the plane of the horizon and coincides with the meridian line on the plane of the horizon, while the terrestrial poles PP′ coincide with the north and south points N and S, respectively.

Hence, the plane of the celestial equator, as well as the planes of the celestial parallels, are located perpendicular to the plane of the observer's horizon. However, since we know that the diurnal motion of the stars occurs parallel to the celestial equator, we can see from Figure 5.13 that there are no stars visible at the Equator which never rise or never set. All stars are located above the horizon for 12 hours and below the horizon for 12 hours, since all the parallels of the stars are cut in half by the horizon.

Culmination of Stars

The diurnal parallel of a star crosses the celestial meridian at two points (Fig. 5.14,a). These points are called the culmination points. The moment of passage of a given star through the celestial meridian is called the *moment of culmination*, or it is said that the star culminates.

The *upper culmination of a star* is the moment when the star is at its greatest altitude above the horizon. The *lower culmination of a star* is the moment when the star is at its lowest altitude above the horizon. In the case of stars that set, the lower cul- /412
mination takes place below the horizon.

Upper culmination of a star can take place on the southern portion of a meridian (between the scuth point and the zenith), and on the southern portion of the meridian (between the zenith and the north celestial pole), depending on the relationship between the

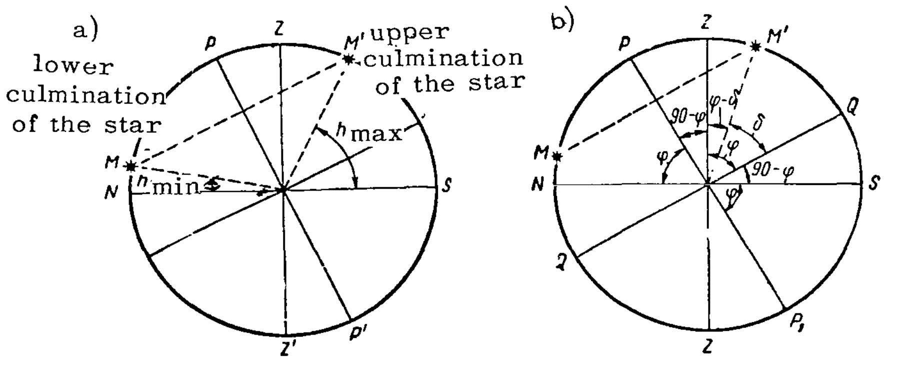


Fig. 5.14. Culmination of Stars on the Southern Section of the Meridian: (a) in the Apparent System of Coordinates; (b) in the Equatorial System.

latitude of the observer and the declination of the star.

A star culminates on the southern part of the meridian (between the south point and the zenith) when the latitude of the observer is greater than the declination of the star, i.e., if $\phi > \delta$.

A star culminates on the northern part of the meridian (between the zenith and the north celestial pole) when the latitude of the observer is less than the declination of the star, i.e., if $\phi < \delta$.

In Figure 5.14,b the celestial sphere has been sketched in simplified fashion, i.e., the circles of the horizon, equator, and parallels are not represented as circles but as diameters and chords. As we can see from the diagram, the latitude of the observer is greater than the declination of the star: $\overset{\smile}{NP} > \overset{\smile}{QM}$, i.e., $\phi > \delta$, so that the upper culmination of the star (point M′) lies on the southern part of the meridian (between the zenith Z and the south point S).

Let us determine the altitude of the star in this case.

The altitude of the star (h) is the arc SM′, but the arc SM′ $= \overset{\smile}{SQ}' + \overset{\smile}{Q}'Z - \overset{\smile}{M}'Z$, and $\overset{\smile}{SQ}' = 90° - \phi$; $\overset{\smile}{Q}'Z = \phi$ and $\overset{\smile}{M}'Z = \phi - \delta$.

By substituting these values, we will obtain $h = 90° - \phi + \phi - (\phi - \delta)$ or $h = 90° - \phi + \delta$.

In Figure 5.15,a the celestial sphere has also been sketched in a simplified form. Here the latitude of the observer (NP) is less than the declination of the star (Q′M′), i.e., $\phi < \delta$, so that the upper culmination of the star (point M′ occurs on the northern part of the meridian (between the zenith and the point of the north celestial pole). Let us determine the altitude of the star in this /413

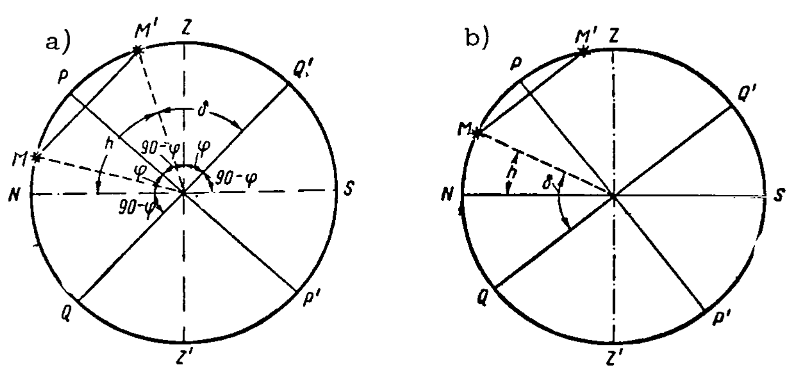


Fig. 5.15. Culmination of a Star on the Northern Section of the Meridian: (a) Coordinates of Upper Culmination; (b) Coordinates of Lower Culmination.

case. The altitude of the star (h) is NM′, but

$$\breve{NM'} = 180° - \breve{M'Q'} - \breve{Q'S}$$

or

$$h = 180° - \delta - (90 - \varphi),$$

i.e.,

$$h = 90° - \delta + \varphi.$$

If the star does not set, we will sometimes be interested in its altitude at the moment of lower culmination.

As we see from Figure 5.15,b, $\breve{MQ} = \breve{QN} + \breve{NM}$, but $\breve{MQ} = \delta$; $\breve{QN} = 90 - \phi$; $\breve{NM} = h$.

By substituting the values of these arcs, we will obtain $\delta = 90° - \phi + h$, so that $h = \phi + \delta - 90°$, i.e., the altitude of the star at the moment of lower culmination is equal to the latitude of the observer plus the declination of the star minus 90°.

Problems and Exercises

1. What must be the declination of a star if at the latitude of Moscow ($\phi = 55°48'$) (a) it never sets, (b) it rises and sets, or (c) it never rises?

Solution. (a) In order for a star never to set, we must have $\delta > 90 - \phi$. If we substitute the value of the latitude of Moscow (55°48′), we will obtain $\delta > 90 - 55°48'$, i.e., δ must be greater than +34°12′. Consequently, all stars which have a declination greater than +34°12′ will never set at the latitude of Moscow. Typical stars in this category are Capella, Alioth, Vega, Deneb, and Polaris. /414

(a) Stars rise and set, as we know, if the absolute value of their declination is less than 90° - ϕ, i.e., $\delta < 90° - \phi_1$. In our example, $\delta < 34°12'$. Stars in this category for the latitude of Moscow are Regulus, Arcturus, Altair, etc.

(c) In order for a star never to rise, its declination must be equal to or greater than 90° - ϕ and varies with the latitude of the observer, i.e., $-\delta \geq 90° - \phi$. In our example, δ must be equal to or greater than 34°12′. In addition, it must also be negative (inasmuch as we are talking about north latitude).

(2) Calculate which of the following stars: Aldebaran, Alpherants, Capella, Sirius, Procyon, Arcturus) will never rise, rise and set, and never set at the latitude of Leningrad ($\phi = 59°59'$N).

3. Calculate the altitude of the star Dubkhe at Moscow ($\phi = 55^\circ 48'$) at the moment of upper culmination.

4. At what altitude does the star Sirius culminate (upper culmination) at Leningrad?

5. Show mathematically that all stars whose $\delta > 0$ do not set at the Poles, while those which have $\delta < 0$ never rise.

3. The Motion of the Sun

The Annual Motion of the Sun

The Sun participates in the diurnal motion along with all the other stars. The apparent diurnal motion of the Sun is also the result of the diurnal motion of the Earth in rotating on its axis. However, the Sun also has its own so-called intrinsic motion in the course of a year, called the *annual motion of the Sun.*

The annual motion of the Sun is difficult to observe directly. However, if the stars were visible in the daytime, and we were to observe the mutual positions of the Sun and stars for a certain period of time, we would see that the mutual positions of these bodies would change in the course of time, while the mutual positions of the stars and constellations in the sky would not change.

The direction of this intrinsic annual motion of the Sun is opposite to the diurnal motion of the stars, i.e., from west to east.

The annual motion of the Sun is apparent (as is the diurnal motion), and occurs as the result of the annual rotation of the Earth around the Sun.

As we did in describing the diurnal motion of the sky and stars, we will consider that the Sun is moving and the Earth stands still.

Due to the existence of so-called annual motion of the Sun, the diurnal motion of the Sun has some unusual aspects, such as:

(a) The interval of time between the rising and setting of Sun varies continuously in the course of a year. /415

(b) The azimuth of the points of the rising and setting of the Sun changes constantly during the year, and the points where the Sun rises and sets move about, shifting from one quarter of the horizon to another. For example, in summer the Sun rises in the northeast, and sets in the northwest. In winter, it rises in the southeast and sets in the southwest.

(c) The meridional altitude of the Sun changes constantly in the course of a year.

Ecliptic. In the course of its intrinsic motion, the center of the Sun moves along a great circle of a sphere called the *ecliptic* (Fig. 5.16,a). The plane of the ecliptic intersects the plane of the celestial equator at an angle of 23°27′ at two points: at the point of the vernal equinox (γ) and the point of the autumnal equinox (♎).

Tropic year. The Sun completes a journey around the ecliptic (through 360°) in 365.2422 mean days.

The interval of time between successive passages of the center of the Sun through the point of the vernal equinox is called the *tropic year*.

Sidereal year. In the course of its annual motion, the Sun makes a full rotation relative to the stars in a period of time somewhat longer than the tropic year (i.e., in 365.25636 days). This time interval, equal to the period of time required for the Earth to rotate around the Sun, is called the *sidereal year*. After this interval, the Sun will have returned to its original position among the stars.

Motion of the Sun Along the Ecliptic

On March 21, in the course of its annual motion, the Sun crosses the celestial equator at the point of the vernal equinox. This date is called the *date of the vernal equinox*. When the Sun passes through the point γ, its declination and right ascension are equal to zero. After March 21, the Sun continues its motion and passes into the Northern Hemisphere, and its declination begins to increase (i.e., becomes positive). Thus, after three months (on June 22) the Sun is at the point K (see Fig. 5.16,a), which is called the *point of the summer solstice*. At this point, the Sun is at its highest position above the celestial equator. The

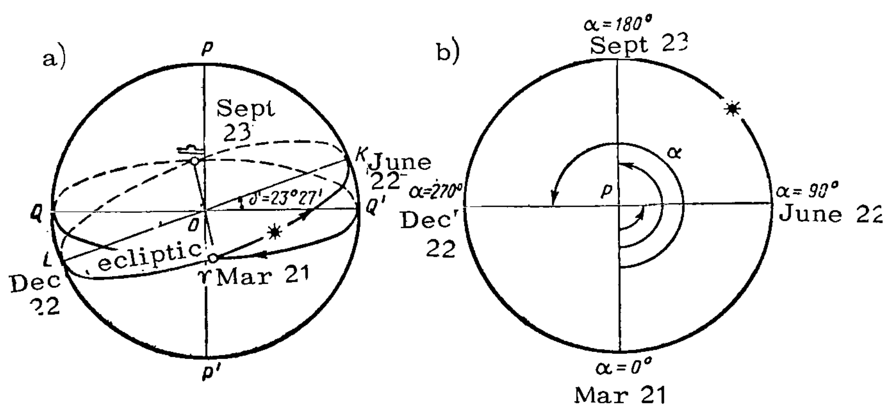


Fig. 5.16. Annual Motion of the Sun: (a) Motion along the Ecliptic; (b) Coordinates on the Dates of the Equinoxes and Solstices.

declination of the Sun at this point is +23°27′, and its right ascension is 90° or 6 hours. For several days, its altitude at noon remains nearly constant, i.e., +23°27′, so that the point K, which corresponds to the constellation Capricorn, has been named the *date of the summer solstice*. The points where the Sun rises and sets on this date will be at their maximum distances from the east and west points on the horizon. After the date of the summer solstice, the Sun begins to approach the celestial equator, its declination begins to decrease, and by September 23 it again intersects the celestial equator at the point of the vernal equinox ($\underline{\Omega}$, in the constellation Libra).

When the Sun passes through the point of the vernal equinox ($\underline{\Omega}$), its declination becomes equal to zero, while its right ascension becomes 180° or 12 hours.

TABLE 5.1

Date	Occurs on	Coordinates	
		Declination (δ)	Right Ascension (α)
Vernal equinox	March 21	0°	0°
Summer solstice	June 22	+23°27′	90° or 6 hours
Autumnal equinox	September 23	0°	180° or 12 hours
Winter solstice	December 22	-23°27′	270° or 18 hours

September 23 is called the *date of the autumnal equinox*. All of the events of the date of the vernal equinox are repeated on this date.

After September 23, the Sun passes into the Southern Hemisphere and its declination becomes negative. On December 22, the Sun is at its lowest position relative to the celestial equator and is at /417
the point of the winter solstice (the point L, in the constellation Leo). This date is called the *date of the winter solstice*. On the date of the winter solstice, the Sun has a declination of -23°27′, while its right ascension is 270° or 18 hours. The points where the Sun rises and sets on this date are farthest south from the east and west points on the horizon.

After December 22, the Sun begins its rise along the ecliptic, and on March 21 it has again risen to the point of the vernal equinox, where its declination and right ascension are once more equal to zero.

Thus, we can draw up a special table for the annual motion of the Sun along the ecliptic, showing its coordinates (Table 5.1; Fig. 5.16,b).

In the course of a year, as it moves through the sky (among the stars), the Sun passes through 12 constellations, called the signs of the zodiac. They have received this name because the majority of them bear the names of animals (Aries, the Ram; Taurus, the Bull, etc.), and the word *zoôn* in Greek means "animal". As the Sun moves among the stars in the course of a year, it is in the following positions: on the date of the vernal equinox (March 21), in the constellation Pisces (the Fishes); on the date of the summer solstice June 22, in the constellation Gemini (the Twins); on the date of the autumnal equinox (September 23), in the constellation Virgo (the Virgin), and on the date of the winter solstice (December 22), in the constellation Sagittarius (the Archer).

Diurnal Motion of the Sun

The Motion of the Sun at the North Pole

Due to the constant change in the declination of the Sun, its altitude at the terrestrial poles also changes. It does not move parallel to the horizon, but along a spiral path. Since the declination of the Sun will be positive for six months (and the celestial equator for an observer at the North Pole will coincide with the horizon), the center of the Sun will act like a star that never sets at the North Pole from March 21 to September 23, i.e., it will remain above the horizon.

During the other half of the year, when the Sun has a negative declination, it will be below the horizon as seen from the North Pole. Therefore, there are six months of day and six months of night at the terrestrial poles.

The Sun reaches its maximum altitude above the horizon at the North Pole on the date of the summer solstice, i.e., on June 22. The altitude of the Sun on that date is equal to its maximum declination, i.e., 23°27′. At the South Pole, the Sun reaches its maximum altitude on the date of our winter solstice, i.e., December 22.

Motion of the Sun between the North Pole and the Arctic Circle

If an observer is beyond the Arctic Circle i.e., farther north /418 than the latitude of 66°33′ (the latitude of the Arctic Circle), the Sun during its diurnal motion will be a never-setting star for part of the year, a rising and setting star part of the year and a never-rising star for part of the year. In order to understand this phenomenon, let us recall the conditions for the nonrising, setting, rising and nonsetting of heavenly bodies.

Heavenly bodies do not set if their declination is equal to or more than 90° minus the latitude of the observer, i.e. if $\delta \geq 90° - \phi$. This situation also applies to the diurnal motion of the Sun. If, for example, the observer is standing at a latitude of 76°N (between the North Pole and the Arctic Circle), then according to the condition set forth above the Sun will not set after the date when its declination is equal to or more than $90° - \phi$, i.e. more than $90-76° = +14°$. This phenomenon (to cite a specific example) begins on April 26. After April 26 the Sun will rise higher and higher above the horizon.

The Sun reaches a maximum altitude above the horizon on the date of the summer solstice. After June 22, the Sun will dip toward the horizon but will still not set. When its declination is again equal to $90° - \phi$, the Sun will touch the horizon.

After August 19, the Sun's declination will be less than $90° - \phi$, i.e. $\delta < 90° - \phi$ and for a fixed time it will appear to an observer located at this latitude as a rising and setting star.

This phenomenon will continue until the declination of the Sun is not equal to or more than $90° - \phi$, i.e. $\delta \geq 90° - \phi$ and will have a sign opposite to that of the latitude (i.e. the latitude is positive and the declination is negative). For an observer located at 76°N this phenomenon begins on November 3. Beginning on November 3, for an observer located at a latitude of 76°, the Sun will not set, since $-\delta \geq 90° - \phi$.

The Sun will not rise until its negative declination is equal to or less than $90° - \phi(-\delta \leq 90° - \phi)$, i.e., beginning on the date the Sun's declination has a value of 14° and less. In our example, this phenomenon starts on February 9. After this date, the Sun will rise and set every day and the period of daylight will gradually increase. Finally, when the Sun's declination is equal to or more than $90° - \phi$, i.e., beginning on April 26,[2] the Sun again will not set.

TABLE 5.2 /419

Latitude of the position in degrees	Spring		Summer		Autumn		Winter	
	Beg. date	Duration in days	Beg. date	Duration in days	Beg. date	Duration in days	Beg. date	Duration in days
68	4.I	143	27.V	53	19.VII	144	10.XII	25
70	17.I	120	17.V	72	28.VII	121	26.XI	52
72	26.I	103	9.V	88	5.VIII	104	17.XI	70
74	3.II	88	2.V	102	12.VIII	90	10.XI	85
76	9.II	76	26.IV	115	19.VIII	76	3.XI	98
78	15.II	64	20.IV	127	25.VIII	63	28.X	111
80	22.II	51	14.IV	139	31.VIII	52	22.X	123
82	27.II	41	9.IV	150	6.IX	41	17.X	133
84	4.III	31	4.IV	159	10.IX	31	11.X	144
86	9.III	21	30.III	169	15.IX	21	5.X	155
88	14.III	11	25.III	179	20.IX	10	30.IX	165
90	19.III	0	19.III	189	25.IX	0	25.IX	176

[2]The dates for nonsetting, nonrising or rising and setting of the Sun are given, taking into account the phenomenon of refraction.

Thus, let us sum up the diurnal motion of the Sun in the course of a year for an observer located at a latitude of 76° (between the North Pole and the Arctic Circle).

(1) From April 26 to August 19, i.e., for 115 days, the Sun does not set. This period of time is called the *polar summer*.

(2) From August 19 to November 3, i.e., for 76 days, the Sun will rise and set daily and the period of daylight will diminish each day. This period of time is called the *polar autumn*.

(3) From November 3 to February 9, i.e., for 98 days, the Sun will not rise for the observer. This period is called the *polar winter*.

(4) From February 9 to April 26 i.e., for 76 days the Sun will rise and set daily and the period of daylight will increase each day. This period of time is called the *polar spring*.

The dates for the start of the seasons depend on the latitude of the observer. The dates given in our example are for 76°N. We have provided a table for the seasons as a function of the latitude of the observer (Table 5.2).

Motion of the Sun above the Arctic Circle

As we already know, the latitude of the Arctic Circle is equal to $\phi = 66°33'$, or the complement of its latitude to 90° is 23°27'.

Therefore, on the date of the summer solstice (June 22), $\delta = 90° - \phi$, and on the date of the winter solstice (December 22), $\delta = \phi - 90°$. /420

On these dates at the Arctic Circle, the center of the Sun touches the horizon on June 22 at the north point at the moment of lower culmination (point N, Fig. 5.17), and on December 22 at the south point at the moment of upper culmination (point S). During the rest of the year, the Sun will rise and set daily. The period of daylight will increase daily from December 22 to June 22; after June 22, it will decrease.

The Sun reaches its maximum altitude above the horizon on June 22. It will be: $h = 90 - 66°33' + 23°27' \approx 46°54'$.

Motion of the Sun at Middle Latitudes

Knowing the maximum declination of the Sun (equal to 23°27'), it is not difficult to calculate the latitudes of the observer on the Earth's surface where the Sun will be a rising and a setting heavenly body during the year.

From the conditions for the rising and setting of heavenly bodies, it follows that the declination must be less than the complement of the latitude to 90°, or the complement of the latitude to 90° must be more than the declination of the Sun, i.e., $90 - \phi > \delta$. This means that between 66°33'N and 66°33'S, the Sun will rise and set every day of the year (see Fig. 5.17).

Motion of the Sun at the Terrestrial Equator.

In order to better understand the nature of the diurnal motion of the Sun at the Equator, let us analyze Figure 5.18. Here we have sketched schematically the diurnal trajectories (circles) of the Sun at the equator. As we see from the drawing, all the diurnal circles are divided in two by the horizon line. This is understandable, since we already know that the true horizon of an observer located at one of the poles is situated at an angle of 90° to the equator. Since the daily circles of the Sun are divided in half by the horizon, the Sun will be located above the horizon half the day and below the horizon half the day, i.e. during the whole year the Sun rises and sets, and the days and nights are equal in length. /421

Twice a year, on the dates of the equinoxes, when the Sun moves along the celestial equator (the equator itself is the diurnal circle), it passes through the zenith of the observer and its height is 90°. During the rest of the year, the Sun culminates to the north or south of the zenith and is at its greatest distance from it (23°27') on the dates of the solstices.

4. Motion of the Moon

Intrinsic Motion of the Moon

The Moon, participating with all the heavenly bodies in the sky in the diurnal rotation of the celestial sphere, has its own intrinsic motion.

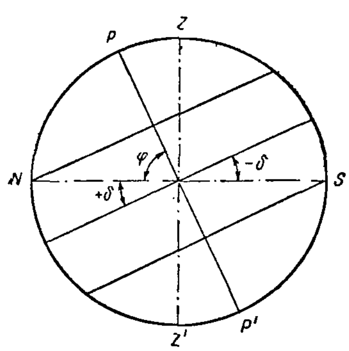


Fig. 5.17. Diurnal Motion of the Sun Above the Arctic Circle.

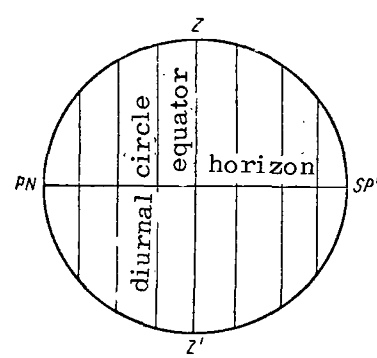


Fig. 5.18. Diurnal Motion of the Sun at the Equator.

If we observe the Moon for one night, we can easily see that it travels like the Sun in the sky (relative to the stars). The apparent motion of the Sun is the result of the Earth's motion around the Sun; the Moon actually moves around the Earth.

Direction and Rate of the Moon's Motion

The Moon moves along the celestial sphere from west to east, i.e., in a direction opposite the diurnal motion of the celestial sphere.

The great circle along which the Moon completes its motion around the Earth has the shape of an ellipse and is called the Moon's orbit. The Moon's orbit is intersected by the solar ecliptic at an angle of 5°08′ (Fig. 5.19). The two diametrically opposed points at which the Moon's orbit is intersected by the solar ecliptic are called the *nodes of the orbit.*

The Moon completes a full revolution along its orbit relative to the stars in 27.32 days. This time interval is called the *sidereal (stellar) month.* Thus, it is easy to calculate that during one day it moves 13.2°. Its hourly shift relative to the stars is approximately 0.5°.

The motion of the Moon in its orbit may be studied in relation to the Sun (which has, as we know, its own motion).

The period of revolution required for the Moon to return to a previous position in relation to the Sun is called the synodic month. It is approximately 29.5 mean solar days. /422

The Moon, revolving around the Earth, accompanies it in its motion around the Sun. The Moon is 356,000 km from the Earth at perigee (the closest point to the Earth) and 407,100 km from the Earth at apogee (the most distant point from the Earth).

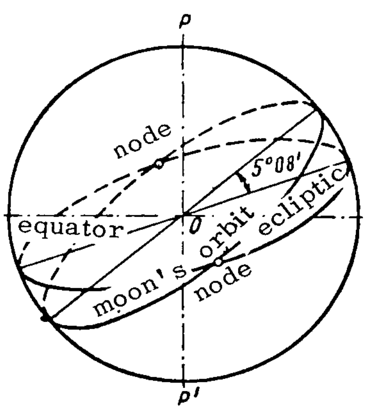


Fig. 5.19. Orbit of the Moon.

Phases of the Moon

The Moon, like our Earth, is an opaque body which illuminates the Earth's surface with reflected sunlight. The illumination of the Earth's surface by the Moon, as we can see, is not always the same. At different times the Moon is visible in the form of a luminous disk, in the form of a luminous half-disk, or a crescent. There is a time when the Moon is entirely invisible. The Moon has various phases. The periodically repeated change in the shape of the Moon is called the *change in lunar phases*.

In order to explain the cause of lunar phases, let us look at Figure 5.20. At Point 0 (in the orbit's center) is our Earth; at Points 1-8 we show the Moon in various positions in its revolution around the Earth. Below the figure we show the shape of the Moon in those eight positions for an observer located on the Earth's surface.

When the Moon is in Position 1, its phase is *new moon*. An observer on the Earth's surface during this phase does not see the Moon, since its nonilluminated side is turned to the Earth and is located at an angular distance of not more than 5° from the Sun. In Position 2, the Moon is visible on the Earth in the form of a narrow crescent only during evening hours. When the Moon is in Position 3, its phase is called the first quarter and an observer sees it in the shape of half an illuminated disk. This phase is called the first quarter because at this time a quarter of the entire lunar surface is visible.

During first quarter, the Moon is visible in the east at noon, in the south about 1800, and in the west at midnight. After first quarter, the illuminated part of the lunar disk begins to increase (Position 4); in Position 5, the entire lunar disk is illuminated. This is the half-moon phase. In this phase, the Moon is visible all night.

After half moon, the illuminated part of the lunar disk begins /423
to decrease from the right side of the disk (in Position 6), and in

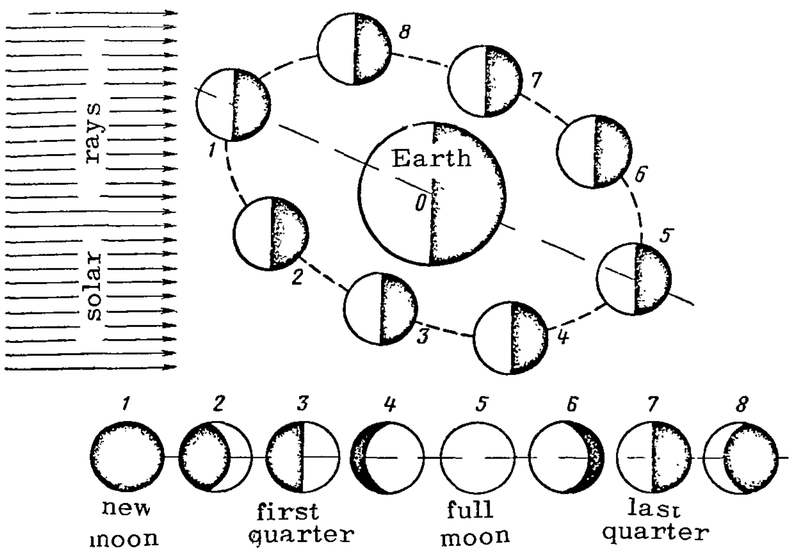


Fig. 5.20. Phases of the Moon.

Position 7 it reaches the phase of last quarter. In last quarter, the Moon is visible in the east at midnight, in the south about 0600, and in the west at noon. After last quarter, the illuminated part of the lunar disk diminishes even more and in Position 8 it is already visible in the form of a narrow crescent (on the left side of the disk); it then becomes invisible again, i.e., the phase of new moon again sets in.

Nature of the Motion of the Moon around the Earth

In view of the fact that the Moon, in its motion around the Earth, is sometimes closer to the Sun than the Earth is and sometimes farther away, it receives acceleration from the Sun which is sometimes more and sometimes less than that of the Earth. As a result, the motion of the Moon around the Earth is complex, since the above factors not only change the shape and dimensions of the lunar orbit, but its position in space.

As a result of the change in position of the plane of the lunar orbit, the angle of inclination of this plane to the plane of the celestial equator changes continuously from 28.5 to 18.5°. Because of this, the declination of the Moon also changes in the range from -28.5 to +28.5° and in the range from -18.5° to +18.5°. In all cases, however, the plane of the lunar orbit is inclined to the plane of the ecliptic at 5°08′. As a result of the change in the /424 position of the plane of the lunar orbit, the nodes of the lunar orbit also travel along the ecliptic against the annual motion of the Sun (or Moon) by 19.3° every year. Thus, the nodes of the lunar orbit complete one revolution along it in 18.6 years.

The Moon, completing one revolution in its orbit, intersects the plane of the celestial equator twice; in the course of one sidereal month, its declination changes from a maximum positive value to a maximum negative one.

The maximum declination of the Moon in a period of one month may be +28°27′ and the minimum may be -28 27′.

Location of the Moon Above the Horizon

The location of the Moon above the horizon depends on the so-called waxing of the Moon (time elapsed after new moon) and the season.

In the Northern Hemisphere, during the summer and the phase of full moon, the Moon is located comparatively low and for a short time above the horizon; in the winter, during the full moon, it is visible all night and rises rather high above the horizon. This depends on the declination of the Moon. In winter in the Northern Hemisphere, the full moons occur with a positive declination, while the full moons in summer occur with negative declination.

5. Measurement of Time

Essence of Calculating Time

Measurement of time is the result of the presence of motion in universal space. Time and motion are synonymous. If motion ceased in Nature and in the Universe, there could be no discussion of time. It is entirely understandable that to measure time, some constant and uniform motion must be used.

The rotation of the Earth on its axis (or, as a result of this, the apparent rotation of the celestial sphere around the world axis) could be such a motion.

Special observations over a very long time interval have shown that the duration of the Earth's rotation around its axis has not changed even by a fraction of a second. This characterizes very well the constancy and uniformity of the Earth's rotation.

In the practice of aviation, the following kinds of time must be studied and used: sidereal, true solar, mean solar, Greenwich, local, zone and standard time.

Sidereal Time /425

Time measured on the basis of the apparent motion of heavenly bodies (stars) is *called sidereal time*. This time may be measured by the hour angle of some heavenly body with respect to the meridian of the observer. However, for convenience, it is advantageous to take the hour angle not of some star but of the point of the vernal equinox from which the right ascension is read.

The western hour angle of the point of the vernal equinox t_γ is called *sidereal time* and is represented by the letter S ($S = t_\gamma$) in Figure 5.21. Sidereal time is equal to the sum of the hour angle and the right ascension of a star: $S = \alpha_\gamma + t$.

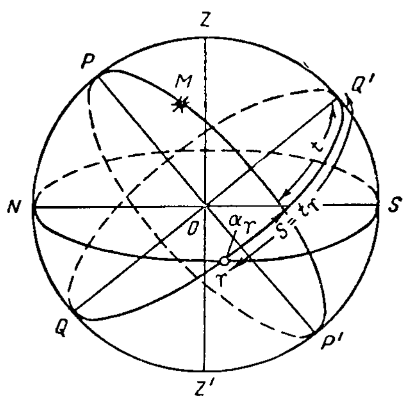


Fig. 5.21. Hour Angle of the Point of the Vernal Equinox.

Sidereal days. The time interval between two successive upper culminations of the point of the vernal equinox is called a *sidereal day*. The moment of the upper culmination of the vernal equinox point was taken as the beginning of sidereal days. At this moment, sidereal time is equal to 00:00:00. Sidereal days are divided into 24 sidereal hours, each hour is divided into 60 sidereal minutes, and each minute into 60 sidereal seconds.

Sidereal time does not have a date and therefore in calculations when the sidereal time is more than 24:00, there is only a surplus of time above 24:00.

Example. In calculations, there is a sidereal time of S = 29:20. Discarding 24:00, we obtain a sidereal time of 5:20.

The practical application of sidereal time. Sidereal time is not used in everyday life, but as the basis of time signals. In every astronomical observatory, there are special clocks which run according to sidereal time. In aviation, sidereal time must be used in observing stars for determining the position of the aircraft or the position line of the aircraft.

In the aviation astronomical yearbook, the sidereal time is given for each date and each hour of Greenwich mean time. Therefore, the sidereal time for any moment at any point on Earth may be determined by means of the aviation astronomical yearbook on the basis /426 of Greenwich time. In everyday life, solar time rather than sidereal time is used, since on the whole man's activity occurs in the daytime hours.

True Solar Time

True solar time $t_{\odot}$ is the time measured on the basis of the diurnal motion of the true Sun.

True solar time is measured by the western hour angle (t_W) of the center of the true Sun.

True solar days are the time intervals between two successive upper culminations of the center of the true Sun. The moment of the upper culmination of the center of the true Sun is taken as the beginning of true solar days.

At the moment of upper culmination, when the hour angle of the Sun is zero, the true solar time is 00:00:00.

In proportion to the diurnal motion of the Sun, its hour angle increases; the true solar time also increases.

At the moment of the lower culmination of the Sun, the true solar time is 12:00; when the center of the Sun is again in the position of upper culmination, the true solar time is 24:00. After this, new days begin.

The duration of true solar days changes in the course of a year. This occurs because the true Sun moves during the year along the ecliptic, which is inclined to the celestial equator at an angle of 23°27′ and is not a circle but an ellipse. For this reason, the daily shift of the Sun to the east is different on different days of the year. This shift is at a maximum near the solstices, when the Sun moves parallel to the equator. On the other hand, near the equinoxes the shift to the east is smallest.

The Sun lags behind the motion of the stars by either a very great or a very small value; the duration of true days changes all the time.

In using true solar time in everyday life, it would almost be necessary (as a result of its nonuniformity) to regulate clocks every day, moving them ahead and back. This would be extremely inconvenient.

In view of the inconvenience of calculating time on the basis of the true Sun, time in everyday life is calculated on the basis of the so-called mean Sun.

The *mean Sun* is the imaginary or real Sun moving uniformly along the celestial equator in the same direction that the true Sun moves along the ecliptic, i.e., in a direction opposite to the diurnal motion of the celestial sphere.

Mean Solar Time /427

Time calculated on the basis of the diurnal motion of the mean (imaginary) Sun is called the *mean solar time* (m).

Mean solar time is measured by the western hour angle (t_W) of the mean (imaginary) Sun. Mean solar days are the basic unit of mean solar time.

Mean solar days are the time intervals between two successive upper culminations of the mean Sun. They are divided into 24 mean hours, each hour is divided into 60 minutes and each minute into 60 seconds. The duration of mean solar days is constant.

Mean civil time (m_C). Since at the moment of the upper culmination of the mean Sun its hour angle will be zero, the mean solar time at this moment (mean noon) will also be zero.

In everyday life, with a 24-hour reckoning of time, this is very inconvenient since in this case the beginning of mean days comes at noon.

Therefore, in everyday life, so-called *civil time* (which is different from mean solar time by 12 hrs.) is used as a variety of mean solar time.

Mean midnight, i.e. the mean time equal to the western hour angle of the mean Sun plus 12 hrs, is taken as the beginning of mean civil days:

$$T_M = T_{GR} \pm \lambda^E_W,$$

where m_c is the mean civil time, and m is the mean solar time.

The plus sign is used when the mean time is less than 12 hrs and the minus sign is used if the mean time is more than 12 hrs.

Example. Determine the mean civil time if the western hour angle of the mean Sun (mean time) is 6:00.

Solution. $m_c = m + 12{:}00 = 6{:}00 + 12{:}00 = 18{:}00$

Local Civil Time

Sidereal time, true solar and mean solar time are measured by the hour angle of a heavenly body or the point of the vernal equinox. But the hour angle (t) of a heavenly body calculated at one physical moment for one heavenly body from the meridians of various points on the Earth's surface varies in value. For some points on the Earth's surface it will be large and for others it will be small. In Figure 5.22 we show a celestial sphere whose equator lies in the plane of the drawing. Let point M_{av} be the position of the mean Sun at a certain moment: Radius P_nA is the meridian of one point /428
on the Earth's surface and radius P_nB is the meridian of a second point on the Earth's surface. From the figure, it is apparent that the hour angle of the mean Sun for the first point (t_1) is expressed by the angle AP_nM_{av} and the hour angle for the second point (t_2) at this moment is expressed by the angle BP_nM_{av}. From this figure it is obvious that the angle AP_nM_{av} is greater than the angle BP_nM_{av}. Therefore, at the same physical moment the time at different meridians will be different.

Time calculated relative to the meridian of midnight of some point on the Earth's surface is called the *local civil time* and is represented by T_l. Local time may be sidereal or solar. It will be the same for all points lying on one meridian (i.e. having the same geographic longitude).

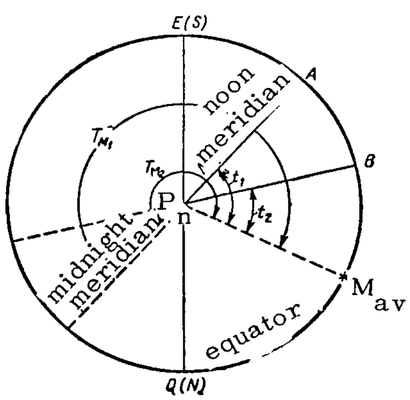


Fig. 5.22. Determining Local Time on the Basis of the Mean Sun.

Greenwich Time

Local civil time calculated from the Greenwich meridian is called *Greenwich time* and is represented by T_{Gr}.

In astronomical yearbooks, the times of some celestial phenomena are given as well as the astronomical values necessary for practical calculations on the basis of Greenwich time.

The relation between local

civil time and Greenwich time. With a knowledge of the Greenwich time and longitude of a place expressed in time units, it is simple to determine the local civil time and vice versa.

The local civil time is equal to the Greenwich time plus or minus the longitude of the place, i.e.

$$T_M = T_{GR} \pm \lambda^E_W,$$

Here the plus sign is used if the longitude of the place is east and the minus sign is used if the longitude is west.

Example. 1. Greenwich time is 14:15. Find the local time for Moscow (λ_E = 2:28).

Solution. Substituting the values for T_{Gr} and λ_E in the formula $T_l = T_{Gr} + \lambda^E$, we obtain the local time T_l = 14:15 + 2:28 = 16.43.

Example. 2. The Greenwich time is 10:42. Find the local time for Washington (λ_W = 05:08).

Solution. $T_l = T_{Gr} - \lambda_W$ = 10:42 - 5:08 = 5:34.

When solving practical problems in astronomy, it is often necessary to change local time to Greenwich time. /429

$$T_{GR} = T_M \pm {}^W_E.$$

The plus sign is used if the longitude of the place is west (λ_W) and minus if the longitude of the place is east (λ_E).

Example. 1. The local time in Ryazan (λ_E = 2:39) is 1430. Determine the Greenwich time.

Solution. Substituting the values for T_l and λ_E into the formula $T_{Gr} = T_l - \lambda_E$, we obtain T_{Gr} = 14:30 - 2:39 = 11:51.

Example. 2. The local time in San Francisco (λ_W = 8:09:44) is 1520. Determine the Greenwich time.

Solution. Substituting the values for T_l and λ_W into the formula $T_{Gr} = T_l + \lambda_W$, we obtain T_{Gr} = 1520 + 8:09:44 = 23:09:44.

Time difference on two meridians. It is easy to imagine that the difference in local time on any two meridians is equal to the difference in their longitudes.

In Figure 5.23 the diameter EQ is the Greenwich meridian, the diameter BA is the meridian of a point on the Earth's surface, the diameter DC is the meridian of a second point on the Earth's surface, M_{av} is the location of the mean Sun at a certain moment and the diameter KM_{av} is the circle of the Sun's declination.

In Figure 5.23 it is evident that the hour angles of the mean Sun ($t_{1\odot av}$ and $t_{2\odot av}$) measured from both local meridians are different from one another by a value equal to the difference in the longitudes of these meridians, since $t_1 - t_2 = \lambda_2 - \lambda_1$.

Hence it follows that the local time on these meridians will differ by the difference in the longitudes of these two meridians.

Example. Let the local time be T_1 = 1204 at a point having an east longitude of λ_E = 2:35. What is the local time at this moment at a point having an east longitude λ_E = 4:35?

Solution. Let us find the difference in the longitudes of these two points $\Delta\lambda$ = 4:35 - 2:35 = 2:00.

Let us find the local time for the point having an east longitude λ_E = 4:35. T_1 = 12:04 + 2:00 = 14:04.

In solving such problems, it is important to remember that the /430 larger the east longitude of the point, the larger will be the local time at this point; the smaller the east longitude of the point, the smaller the local time.

Zone Time

We have already seen that each meridian of the Earth's surface has its own time. If we take Khabarovsk, whose longitude is 135°5′ (9:00:20) its local time is 6:29:48 different from the local time of Moscow, which has a longitude of 37°38′ (2:30:32).

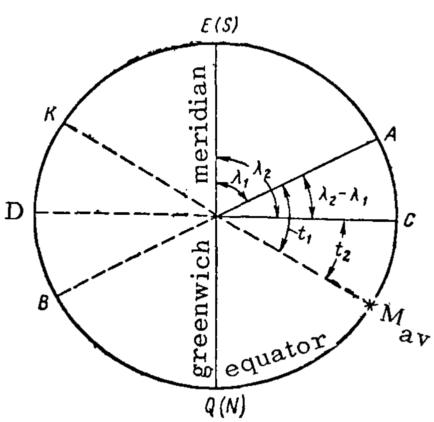


Fig. 5.23. Time Difference on Two Meridians

Since each point on the Earth's surface has its own (local) time, it is too inconvenient to use local time in everyday life. For example, when moving from west to east, it would be necessary to continuously move the hour hands 4 min ahead for every degree of longitude or 1 hr for every 15° of longitude. On the other hand, in moving from east to west, the hour hands would have to be moved back constantly. Therefore, since the middle of the last century, the countries of Europe have begun to introduce a single time in their territories. This time is measured from the meridians of the principal observatories of these countries.

In France so-called "Paris time", was introduced, in Italy "Rome time", and in England "Greenwich time", etc.

The introduction of a single conventional time in these countries did not create great difficulties, since the local time of any meridian of these countries (in view of their small area relative to the meridian of the conventional time introduced) differed insignificantly (only several minutes in all). If we take England and France for example, their outermost populated points (to the east and west) are situated in a range of 7-8° from their respective meridians (Greenwich and Paris): the time difference amounts to a total of 30 min. If we take such a country as the USSR, we know that the difference in the longitudes of its eastern and western boundaries amounts to more than 10 hrs in time. However, in pre-Revolutionary Russia, a common Petersburg time (the local time of the Pulkovo Observatory meridian) was introduced only for railroads. This time was 00:28:58 behind Moscow local time (local time of the Moscow University Observatory meridian).

The introduction of a common time in individual countries partially facilitated its calculation within each country, but it did not solve the problem on an international scale. The problem of calculating time was solved most successfully after the introduction of a zone time system.

In some countries, this system was introduced at the end of the 19th and beginning of the 20th century. In Russia, the zone time system was introduced only after the Revolution, on July 1, 1919 by a special decree of the Soviet Government. /431

Essence of the zone time system. The entire Earth is divided into 24 hour zones. The outer meridians (boundaries) of each band are 15° of longitude (1 hr in time) apart from one another.

The zones are numbered from west to east from the zero zone to the 23rd zone, inclusive. The zone included between the meridians 7°30′N and 7°30′E is taken as the zero zone, i.e. the initial zone. The Greenwich meridian, which has a longitude of 0°, is the mean meridian of this zone. Obviously, the first zone will be located between the meridians λ = 7°30′E and λ = 22°30′E and the mean meridian of this zone has a longitude of 15°; the second zone is located between the meridians λ = 22°30′E and λ = 37°30′E and the mean meridian of this zone has a longitude of 30°, etc.

At all the points located within the limits of the same zone, the common time (time of the given zone) which represents the local time of the mean meridian of the given zone is taken. Such a conventional time is called the *zone time* (T_Z).

In the zero zone, the time is calculated on the basis of the local time of the zero (Greenwich) meridian. In the first zone, the time is calculated on the basis of the local time of the meridian having a longitude of 15°E. In the second zone, it is calculated on the basis of the local time of the meridian having a longitude of 30°E, etc. Since the mean meridians of two adjacent zones are 15° of longitude apart from one another, the difference between the zone time of adjacent zones is one hour.

The number of each zone shown by how many hours the time in this zone is ahead of Greenwich time. For example, in the case of the time of the fourth zone, this means that its time is 4 hrs ahead of Greenwich time (Supplement 4).

The introduction of zone time greatly facilitated the calculation of time on an international scale, since the minute and second hands in all the zones indicate the same number of minutes and seconds. The hour hands must be moved a whole hour only when moving from one zone to another. When the boundaries of the hour zones were determined, the boundaries of states, regions and cities as well as natural boundaries (e.g. rivers, etc) were taken into account. If the boundaries of the hour zones had been determined strictly according to the meridians, calculating the zone time (e.g. in Moscow, which is located on the boundary between the second and third zones) would have to be done on the basis of two zones: in the western part of the city on the basis of the second zone, and in the eastern part of the city on the basis of the third zone. I.e. the time difference in the two parts of the city would be one hour and in crossing the boundary the hour hands would have to be /432
moved one hour.

In view of this, the difference between the local time of the outer points of this zone may be somewhat more or less than 30 min relative to the zone time.

Standard Time

In the Soviet Union, on the basis of a government decree, clocks were moved ahead one hour beginning with the summer of 1930. Since this time, the entire USSR reckons time on the basis of so-called *standard time*. Thus, the zone time was shifted ahead 1 hr, i.e. each zone lives not on the basis of its zone time but rather on the basis of the time of the adjacent eastern zone. For example, Moscow, which is located in the second zone, lives according to the time of the third zone.

The time running 3 hrs ahead of Greenwich time (time of the 3rd zone) is called Moscow time.

All the railroad, water, and air routes of communication in the Soviet Union operate according to Moscow time.

Relation between Greenwich, Local and Zone (Standard) Time

When solving practical problems of aircraft navigation on the ground and in the air, it is very often necessary to convert from one form of time to another.

These problems may be solved correctly only if the crew conscientiously masters the essence of calculating time. To facilitate the work of the crew in solving such problems, there are formulas for converting time from one form to another.

Converting Greenwich time to Mean time. Zone time (T_z) is equal to Greenwich time (T_{Gr}) plus the number of the zone:

$$T_Z = T_{GR} + N.$$

In solving problems for the USSR, the number of the zone is used, taking into account the standard time, i.e. 1 hr. later.

Example. What is the zone (standard) time in Novosibirsk when clocks in Greenwich show 12:00?

Solution. On the basis of a map of hour zones or on the basis of a list of the most important populated points, let us find the number of the zone in which Novosibirsk is located. Taking into account that the standard hour in Novosibirsk, the clocks run according to the time of the 7th zone (6th zone + 1 hr), let us find the zone time on the basis of the formula

$$T_Z = T_{GR} + N = 12^h + 7^h = 19^h$$

Converting Zone Time to Greenwich Time. Greenwich time is equal to the zone (standard) time minus the number of the zone (taking into account the standard time): /433

$$T_{GR} = T_Z - N.$$

Example. What is the Greenwich time when the clocks show 17:00 in Krasnoyarsk?

Solution. On the basis of a map of hour zones or on the basis of a list of populated points, let us find the number of the zone where Krasnoyarsk is located. Taking into account the standard time in Krasnoyarsk, the clocks run on the basis of the time of the 7th zone (6th zone + 1 hr).

Let us find the Greenwich time. $T_{Gr} = T_z - N = 17{:}00 - 7{:}00 = 10{:}00$.

Converting Zone (standard time) time to local time. Local time is equal to the zone time minus the number of the zone plus or minus the longitude of the place (plus is used when the longitude is east, minus when the longitude is west):

$$T_{M} = T_{Z} - N \pm \lambda^{E}_{W}.$$

This formula assumes that the hour zones are counted from 0 to 24 eastward from Greenwich.

Example. What is the local time in Omsk when the zone (standard) time there is 16:00?

Solution. Using a map of hour zones or a list of the most important populated points, let us find the number of the zone (taking into account the standard time) where Omsk is located and its longitude. Taking into account the standard time in Omsk, the clocks run according to the time of the 6th zone (5th zone + 1 hr); the longitude of Omsk λ = 73°24′E (4:53:36).

Let us find the local time $T_l = T_z - N + \lambda_E$ = 16:00 - 6:00 + 4:53:36 = 14:53:36.

Converting local time to zone time (standard time). Zone (standard) time is equal to the local time plus the number of the zone (taking into account the standard time) plus or minus the longitude of the place (plus is used when the longitude is west and minus when the longitude is east).

$$T_{Z} = T_{M} + N \pm \lambda^{W}_{E}.$$

Example. What is the zone (standard) time in Irkutsk when the local time there is 18:00?

Solution. Using a list of the most important populated points, let us find the number of the zone (taking into account the standard time) and the longitude of Irkutsk.

Taking into account the standard time, Irkutsk is located in the 8th zone (7th zone + 1 hr); the longitude of Irkutsk λ = 104° 18′E (6:57:12).

Let us find the zone standard time $T_z = T_l + N - \lambda_E$ = 18:00 + 8:00 - 6:57:12 = 19:02:48.

Measuring Angles in Time Units

Since the values of hour angles and right ascensions are used for measuring time, it is often more convenient to express these values in time units rather than in degrees. Also it is often necessary to express the longitude of a place in time units.

To convert hour angles and right ascensions as well as longitude from degrees to hours and back again, the following equations must be used: /434

24 hr = 360°; 1 hr = 15° or 1° = 4 min; 1 min = 15′ or 1′ = 4 sec; 1 sec = 15″ or 1″ = 1/15 sec.

These equations are based on the fact that the celestial sphere (or the Earth on its axis) makes a complete revolution in 24 hrs, which corresponds to 360°.

To convert hour angles, right ascensions and the longitude of a place from degrees to hours, the following must be done:

(1) Divide the number of degrees by 15 and obtain whole hours.

(2) Multiply the remainder from dividing the degrees and obtain minutes of time.

(3) Divide the number of minutes of arc by 15 to obtain whole minutes of time, which must be added to the minutes of time already obtained, and obtain the total number of minutes of time.

(4) Multiply the remainder from dividing the minutes by 4 and obtain seconds of time.

(5) Divide the seconds of arc by 15 and obtain an additional number of seconds. Add these seconds to the preceding ones and obtain a total number of seconds.

(6) Discard the remainder of seconds of arc when it is less than 8; if it is greater than 8, consider it as 1 sec of time.

Example. 1. Express the hour angle 163°57′35″ in hours.

Solution.

150° =	10:00
13° =	0:52
45′ =	0:03
12′ =	0:00:48
00′30″ =	0:00:02
Total:	10:55:50

To convert hour angles, right ascensions and longitude from hours to degrees, the following must be done:

(1) Multiply the hours by 15 and obtain the radii.

(2) Divide the minutes by 4 and separate out the whole degrees.

(3) Multiply the remainder of the minutes of time by 15 and obtain minutes of arc.

(4) Divide the seconds of time by 4 and separate out minutes of arc.

(5) Multiply the remainder of the seconds of time by 15 and obtain seconds of arc.

Example. 2. Express an hour angle of 11:27:15 in degrees.

Solution.

11 hr	=	165°
24 min	=	6°
3 min	=	45′
12 sec	=	3′
3 sec	=	45′
Total:		171°48′45″

Time Signals

/435

Accurate time in astronomical observatories is determined by means of astronomical observations of the culmination of heavenly bodies.

Special transit instruments are used for this purpose. A transit is mounted in such a way that its main part (the terrestrial telescope) is always located in the plane of the meridian. Such a location of the terrestrial telescope permits the observation of a heavenly body only at the moment when it crosses the meridian, i.e. at the moment of culmination. Since the Sun is at the point of upper culmination (crosses the southern part of the meridian) at true noon, it can be observed only when the hour angle of the true Sun is zero. Therefore, when the Sun passes through the terrestrial telescope of the transit instrument, the moment of true noon on the given meridian will be recorded.

Knowing the precise time of the moment of true noon and comparing it with the actual indication of the clock at the moment of observation, it is possible to check the clock and determine its error. In the field of vision of a transit, vertical lines are drawn which permit more accurate determination of the moment that stars cross the meridian. In astronomical observatories, the accurate time is determined on the basis of observing the passage of stars is based on the fact that at the moment that any star is at the meridian (we already know), the sidereal time will be equal to the right ascension of the star.

Thus, these observations make it possible to determine the exact sidereal time. The time obtained is set on special clocks, which run according to sidereal time. They differ from normal clocks in that (by the means of a special control) they run 3 min and 56 sec ahead of normal clocks per day.

Once the exact sidereal time is obtained, the mean solar time is calculated at astronomical observatories and set on mean solar astronomical clocks. The time obtained will be the local mean solar time on the meridian of the observatory.

When necessary it is always possible to convert this time to zone and standard time. The necessary accuracy in determining the true time at astronomical observatories is computed in hundredths /436 of a second, and therefore the process of determining accurate time is much more complex than we have described here.

For accurately determining the moments of the passage of the stars through the meridian, the moments of the culmination of the stars are automatically recorded at astronomical observatories. With these methods, the determination of the true time is accurate to 2 or 3 hundredths of a second. At every observatory, there are accurate astronomical clocks which are manufactured on special order. Special care is required for these clocks, since the continuous changes in temperature and atmospheric pressure strongly affect the steadiness of the oscillation period of the clock balance wheels. Therefore, astronomical clocks are kept in a special room where a constant temperature is maintained, and they are placed under a hermetically sealed bell jar where a constant atmospheric pressure is maintained.

In order that the astronomical clocks will run smoothly, they are protected from vibrations and their hands are very rarely moved. The clock corrections obtained after calculation are recorded in a special log, but the clock's hands are not moved. This makes it possible to know the accurate clock correction at any moment of time. At astronomical observatories, the clocks are kept running fast or slow to the same degree, i.e. uniformities in the operation of the clocks are obtained. In modern astronomical clocks, the change in the readings of such clocks amounts to hundredths of a second per day. On the basis of the precise time data available at astronomical observatories, radio stations supply special radio signals for checking the time.

Organization of Time Signals in Aviation

Time signals in aviation are organized to ensure accuracy of aircraft navigation. The basic task of the time signals in aviation is the systematic checking of clocks and the guarantee of knowing the accurate time at any time of day. The presence of accurate time is especially important when using astronomical means of aircraft navigation. This necessitates knowing the accurate time not only on the ground, but in flight.

In order for the crew to know the correct time at any time of day there must be master clocks with a constant daily speed. They are installed in the cockpit, at the weather station, or in other special places.

Master clocks are used to determine the correction of other clocks in the periods between the transmissions of accurate time signals.

Master clocks are checked and set according to the correct time on the basis of accurate time radio signals transmitted by broadcasting stations of the USSR. The correction of the clocks and its verification on the basis of signals is recorded in a special log. /437

A Brief History of Time Reckoning.

Sometimes we hear the terms "Old Style" and "New Style".

Systems for measuring and calculating large time intervals are called *calendars*.

The basis for time reckoning is the tropical year, i.e., the time interval between two successive passages of the Sun through the point of the vernal equinox. The length of the tropical year is approximately equal to 365 days, 5 hours, 48 minutes and 46 seconds, but it is very inconvenient to use the tropical year for time reckoning, since it does not contain a whole number of days.

Thus, for example, if we take midnight on January 1 as the beginning of one year, the second year will begin not at midnight of January 1 but on January 1 at 5:48:46 AM, the third year at 11:37:32 AM, etc. We may conclude that each year the beginning of the new year would be shifted by 5:48:46.

Old Style (Julian) Calendar. For Romans, the year was originally lunar and consisted of 12 lunar months. The length of the lunar year was 355 days, i.e., their year was 10 days shorter than the accepted year at the present time. With such a time reckoning, the beginning of the new year shifted rather quickly from one month to another. If, for example, we take a time interval of 10 years, then 100 days were accumulated during this period, i.e., the beginning of the new year shifted by more than three months.

To eliminate these inconveniences, approximately every three years the year was lengthened by one month, i.e., this year had 13 rather than 12 months.

The Roman dictator, Julius Caesar, introduced a calendar reform in 46 B.C. This calendar was called the Julian calendar and we now call it the "Old Style". The essence of it was that the duration of one year was considered to be 365 days rather than 355 days. In addition, in February (which was then considered the last month) an extra day was included every fourth year, i.e., that year had 366 days. The addition of the extra day once every 4 years nearly compensated for the difference accumulated in 4 years (about 6 hr per year) and thus the constancy of the date of the vernal equinox (March 21) was preserved. This principle has been retained until the present time.

The extra day as we know, is now added in February: instead of 28 days there are 29 days in all the years which are divisible by 4, e.g. 1956, 1960, 1964, etc. These years have been called *leap years* up until the present time.

New Style (Gregorian) Calendar. The length of the so-called tropical year is, as we know, 365 days, 5 hours, 48 minutes and 46 seconds. The Julian year (on the average) is equal to 365 days and 6 hours. Thus, the Julian year is 11 min and 14 sec longer than the tropical year. Although this difference is small, over a large interval of time it may also cause the beginning of the year to shift. It is not difficult to calculate that in order to shift the date of the vernal equinox one calendar date (one day), almost 128 years (24 hr or 1440 min, divided by 11 min 14 sec) are required. The gradually accumulating difference in the second half of the 16th century amounted to 10 days. As a result, the date of the vernal equinox came (according to the calendar) not on the 21st, but on the 11th of March.

As a result of the shift of the beginning of spring from 21 to 11 March, the holiday of Easter (which must be close to spring) graduallly moved toward the summer. This greatly disturbed the clery, who did not want to depart from their rules.

The Roman Pope Gregory XIII introduced a new calendar reform in 1582 by decree. The essence of this reform, i.e., the transfer to a new style of calendar, was the following: after October 4, 1582, he ordered everyone to consider the date to be October 15, rather than October 5, i.e., he ordered the 10 days accumulated over 1200 /438
years to be dropped so as to return the date of the vernal equinox to March 21. In order to avoid the accumulation of an error in future, it was decided that every 400 years those three days which differentiate the Julian year from the tropical year be dropped. To do this, it was decided that three leap years in every 400 years be considered regular years, i.e, not on the basis of 366 days but rather on the basis of 365 days. In order to remember this more easily, the centurial years in which the numbers of the century were divisible by 4 were taken as leap years (for example, of the centurial years 1600, 1700, 1800 and 1900, only the year 1600 remained a leap year. The others became regular years, since only 16 is divisible by 4). Of the centurial years, the next leap year will be the year 2000.

As regards the other years (besides the centurial years), the calculation of the leap years remained the same as in the Julian calendar.

The Gregorian calendar was gradually introduced in all civilized countries. In Tsarist Russia, the introduction of the new calendar

(New Style) met with great opposition. Only after the Great October Socialist Revolution on February 1, 1918 was the new style quickly introduced. Then the difference between the Old and New Styles has already 13 days. Neither the Old nor the New Style is absolutely accurate, but the Gregorian (New Style) calendar has less error (1 day in 3000 years).

6. Use of Astronomical Devices

Astronomical means of aircraft navigation permit the determination of flight direction and the position line of the aircraft on the basis of the stars. The advantage of astronomical devices is their autonomy. Their use in flight is not related to any ground equipment and their accuracy is not a function of flight distance. The use of astronomical devices is based on the principle of measuring the azimuths or the altitudes of heavenly bodies.

The position of the aircraft is determined by astronomical instruments from the intersection of two astronomical position lines (APL) or (as they are still called) *lines of equal altitude.*

The astronomical position line (line of equal altitude) is the straightened arc of the circle of equal altitude whose center is the geographic position of the star.

The *geographic position of a star* (GPS) is the point on the Earth's surface at which the given star is observed at the zenith, or the projection of the star onto the surface of the Earth. The coordinates of the geographic position of the star represent the equatorial coordinates of the given star, i.e., the latitude is equal to the declination of the star ($\phi = \delta$) and the longitude is equal to the Greenwich hour angle of the star ($\lambda = t_{Gr}$). This is evident in Figure 5.24.

Circle of equal altitude. Let an observer at point A on the Earth's surface (Fig. 5.25) at any moment of time measure the altitude of the star M. On the celestial sphere, if we draw a circle from point M with a spherical radius equal to the zenith distance, it will be called the *circle of equal zenith distances,* since the distance from any point on this circle to the star M is equal to the zenith distance (radius). /439

The projection of the circle of equal zenith distances (each point on the circle) onto the Earth is called the *circle of equal altitude* (AA′) and the center of this circle is the geographic position of the star M on the Earth (GPS). It is called the circle of equal altitude because at any point on this circle, the star M will have the same altitude, i.e. it is observed at an angle for the same altitude of the star. This may be proved rather simply if we recall the relation between the altitude of a star (h) and its zenith distance (Z): $h = 90° - Z$. But since the zenith distance for any observer located on the circle of equal altitude is the same

(R of the circle = Z) then the height (h) for all observers will be the same.

Knowing this principle, we may make the reverse conclusion: if an observer, by measuring, determines the altitude of a star at a certain moment of time, then by plotting the geographic position of the star (GPS) with a radius equal to the zenith distance of the star ($Z = 90° - h$), the circle of equal altitude may be drawn. Obviously, at the moment of measuring the altitude of the star the observer (aircraft) is located on the circumference of the circle of equal altitude. Therefore, the circle of equal altitude is the circle of the position of the aircraft.

In practice, the altitude (h) and azimuth (A) of a star are determined on the basis of tables at a moment of time planned beforehand for the point being calculated whose coordinates approximately coincide with the location of the aircraft at the given /440
moment. On the map, a straight-line segment equal to Δh is drawn from the point being calculated in the direction of the azimuth of the star. The segment APL is drawn perpendicular to the straight lines through its end.

At the moment for which the calculation of the APL of the point being calculated was made, the altitude of the star h is measured by means of a sextant. In general, the measured altitude does not coincide with the altitude of the star at the point being calculated. The difference between these altitudes is equal to the difference between the radii of the circles of equal altitude of the calculated and actual points of the aircraft's position. Since all the circles of

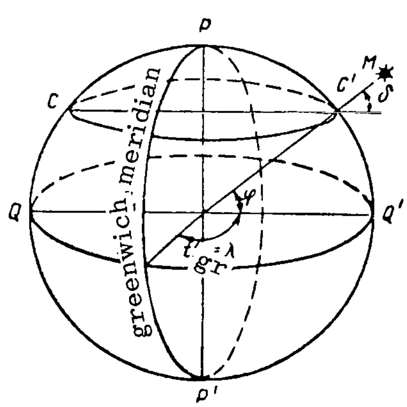


Fig. 5.24. Geographic Position of a Star.

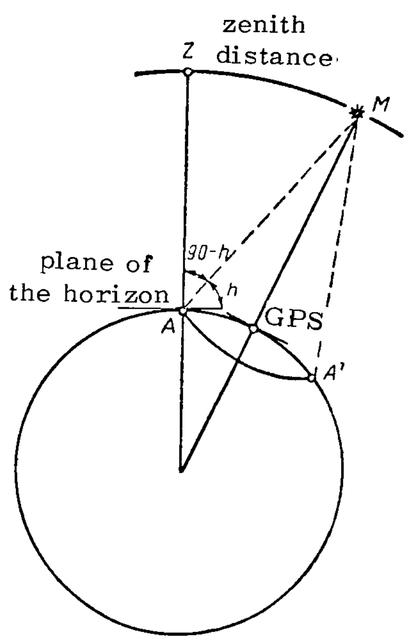


Fig. 5.25. Circle of Equal Altitude.

equal altitude of the same star at the same moment of time are concentric, the APL representing various altitudes of the star are parallel to one another.

Astronomical position lines which represent high altitudes are situated closer to the geographic position of the star, and vice versa. Therefore, if the measured altitude (h_m) is greater than the calculated altitude (h_c), this means that the aircraft at the moment of measuring was located not on the APL of the point being calculated but on the APL parallel to it moved in the direction of the star by a value $\Delta h = h_m - h_c$.

If the measured altitude is less than the calculated altitude, the APL is moved in a direction opposite the direction of the star.

To construct the APL on a map, it is necessary to know:

(a) The approximate coordinates of the aircraft's position (the point being calculated) ϕ, λ.

(b) The azimuth of the star (A) for the point being calculated:

(c) The distance between the measured and calculated altitudes of the star (Δh).

Determining the astronomical position lines. Before beginning the measurements, the electrical supply and the sextant light are switched on, the averaging mechanism is wound up, and the repeaters of the chronometer are matched with the indicator of the current time.

On the basis of a map of the stellar sky or tables of altitudes and azimuths, the azimuth and then the course angle of the star are determined roughly. By rotating the course-angle drum, the sextant is set to the course angle of the star.

By rotating the altitude drum, we bring the star into the sextant's field of vision. The sextant is set on a level base; by rotating the course-angle drum, the star is lined up with the bubble level. By rotating the altitude drum, the star is made to coincide with the bubble level and the averaging mechanism is connected. Once the pilot has been notified beforehand about the beginning of measurement, and the star has been accurately superposed on the bubble level, the averaging mechanism is switched on. When the averager has completed its work, a reading is taken.

On the basis of the measured altitudes of the stars, problems in determining the following are solved:

(a) One astronomical position line on the basis of the Sun /441
to check the path with respect to distance and direction.

(b) Two position lines on the basis of two navigational stars or on the basis of one navigational star and Polaris to determine the position of the aircraft

To check the path with respect to distance by means of one APL, stars are used whose directions are similar to the direction of the line of the given path. To check the path with respect to direction, stars are used which are situated at right angles to the line of the given path.

In determining the position of an aircraft on the basis of two APL's, stars must be chosen so that the directions to them differ by an angle close to 90°.

In calculating the astronomical position lines, the following auxiliary tables are used:

(a) Detached pages from an aviation astronomical yearbook (AAY) for the flight date;

(b) Tables of the altitudes and azimuths of the Sun, Moon and planets (TAA):

(c) Tables of the altitudes and azimuths of the stars (TAAS).

The astronomical position line is calculated on a form like the following:

Calculating the APL

Order of Operation	Date GPA, W	Name of the star	Name of the star
1	2	3	4
1	T_{Moscow}		
5	T_{Gr}		
6	t_{Gr}		
9	$\Delta t_{Gr} (\Delta S_{Gr})$		
4	λ		
10	$t_1 (S_1)$		
3	ϕ		
7	δ		
13	FB		
2	h_m		
8	P		
14	S		
15	-r		
16	σ		
17	h		
11	h_c		
18	Δh		
19	Δh_{KM}		
12	A		

Calculation of the astronomical position line with respect to the Sun, Moon or some planet is done in the order indicated in the left-hand column of the form: /442

(1) The Moscow time of measuring the altitude of the star is recorded (T_{Moscow}).

(2) The measured altitude of the star is recorded (h_m).

(3) and (4). The latitude and longitude of the calculated point (ϕ and λ) are recorded.

(5) The Greenwich time of measuring is determined on the basis of the formula

$$T_{GR} = T_z - (N_z + 1),$$

where (N_z + 1) is the number of the hour zone plus the standard hour and is recorded on the form.

(6) and (7) The declination of the star (δ) and its hour angle (t_{Gr}) for the whole hour corresponding to the time of measurement are copied out of the AAY. (When using the Moon, the t_{Gr} is written for whole tens of minutes.)

(8) In measuring the altitude of the Moon, the parallax (P) is copied from the AAY.

(9). On the basis of the interpolation table available in the TAA or the AAY, the correction (Δt_{Gr}) for T_{Gr} in minutes and seconds is found and recorded.

(10) The local hour angle of the star (t_1) is determined by adding t_{Gr}, Δt_{Gr} and λ. Increasing or decreasing λ, it is necessary that t_1 be expressed by a whole even number of degrees. If the western hour angle is more than 180°, its complement to 360° is taken. The value found for t_1 is considered the eastern hour angle and is recorded in the form.

The value for the longitude of the calculated point λ, written earlier on the form, is refined in accordance with the change introduced with the selection of t_1.

(11) and (12) From the TAA, the value for the altitude of the star at the calculated point (h_c) is written, taking into account the correction for minutes of declination, and the azimuth A is recorded. If the hour angle is western, then the complement of its tabular value up to 360° is taken as the azimuth.

(13) The path bearing (PB) of the star (path angle) is determined on the basis of the formula PB = A - GPA and is written on

the form.

(14) (15) and (16) Corrections are written on the form from the pertinent tables: sextant (S), for the refraction (-r) and for the Earth's rotation (δ).

(17) The measured altitude of the star (h) is adjusted for corrections Nos. 8, 14, 15, 16.

(18) The difference between the corrected value of the measured altitude (h_m) and the altitude of the star at the calculated point (h_c) is calculated on the basis of the formula $\Delta h = h_m - h_c$.

(19) Another value for Δh is recalculated in kilometers. After the calculations, APL is plotted on the chart as shown above. /443

The APL on the basis of stars is calculated in the same order as on the basis of the Sun, Moon and planets. Tables of the TAAS and AAY are used. In addition, instead of the Greenwich and local hour angles, the Greenwich and local sidereal time are determined.

The sidereal Greenwich time (S_{Gr}) is taken from the table entitled "Stars" in the AAY for the moment T_{Gr}. The local sidereal time is determined on the basis of the formula

$$S_{м} = S_{GR} + \lambda,$$

where λ is the longitude of the calculated point refined with this calculation so that S_1 is equal to a whole number of degrees.

When determining the position of the aircraft on the basis of the intersection of the APL from two navigational stars, the measuring and recording of the time of the readings are done successively with the shortest possible time interval. When plotting on the chart, the first APL is shifted parallel to itself in the direction of the vector of the flight speed for the distance traversed in this interval of time.

The correction for the movement of the aircraft between the moments of the first and second measurements is also determined by means of a special table applied to the tables of the altitudes and azimuths of stars.

The correction for the rotation of the Earth (σ) is introduced in the altitude of the star measured first. It is not necessary to shift the first APL determined, taking into account the correction of δ by the movement of the aircraft in this case.

When determining the position of an aircraft on the basis of stars in the Northern Hemisphere, Polaris and one of the navigational stars situated in a westerly or easterly direction are used. Polaris is approximately 1° from the north celestial pole and therefore its height above the horizon is always roughly equal to the latitude of the position. This simplifies the calculating and plotting of APL's.

The accurate latitude of the position of an aircraft on the basis of Polaris is determined by simple addition:

Its measured altitude h_m; the correction of the sextant S; the correction for refraction -r; the correction for the Earth's rotation σ and the correction for the altitude of Polaris $\Delta\phi$. The correction $\Delta\phi$ is given in TAAS on the basis of the value of the local sidereal time S_1.

The altitude of Polaris is measured later than the altitude of the navigational star and therefore the parallel corresponding to the latitude found is shifted in the direction of the flight-speed vector for the segment of the path traversed during the time interval between the first and second measurements. /444

The correction due to the travel of the aircraft is also introduced directly in the calculated latitude by means of a table of corrections "D" in the TAAS.

Astronomical Compasses

Modern astronomical compasses are automatic devices for determining the true course of the aircraft by the direction-finding of the Sun or other stars.

Astronomical compasses of the type DAK-DB are used on aircraft.

These astrocompasses are mainly intended for:

(a) Incidental determination of the true course on the basis of the Sun;

(b) Continuous measurement of the course in flight along the orthodrome on the basis of the Sun.

Astrocompasses of the DAK-DB type can transmit the values of the true course to course system indicators, and they can also permit the true course to be determined on the basis of stars at night by means of a periscope sextant.

Astrocompasses of DAK-DB type may be used in the range of latitudes from the North Pole to 10°S. Astrocompasses of a special type are intended for use in the Southern Hemisphere as well. They can operate when the Sun is not more than 70° above the horizon. Here the permissible error in determining the true course must not exceed ±2°.

An astrocompass automatically solves problems of determining the true course of an aircraft according to the equation:

$$TC = A - CA$$

where A is the azimuth of the heavenly body and CA is the course angle of the heavenly body.

The course angle of the Sun is determined automatically by means of a course-angle data transmitter (CAD).

The photoelectric head is situated in a transparent case in the fuselage of the aircraft; by means of an electronic system, it is automatically oriented in the direction of the Sun and supplies an electrical signal representing the course angle (CA) to a computer device.

The azimuth of the star is determined by a special computer whose basis is a spatial computer mechanism (spherant). When establishing the equatorial coordinates on computers, the hour angle and declination of the star as well as the latitude and longitude of the position, the azimuth of the star, i.e. the horizontal coordinate, is given at the output in the form of electrical signals. /445

The table for the Greenwich hour angles of the Sun is given in Supplement 5.

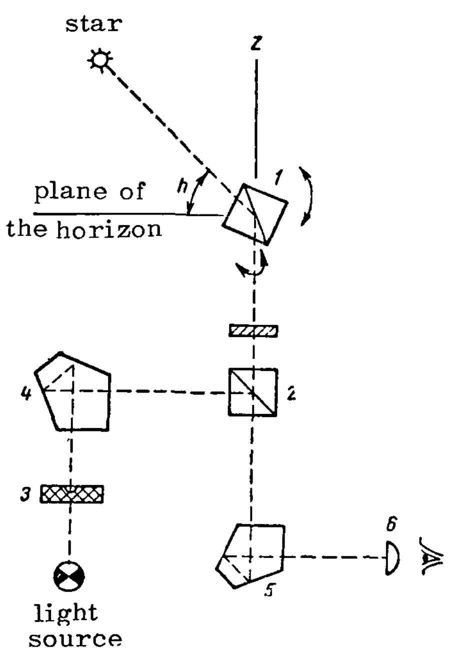


Fig. 5.26. Optical Diagram of an Aviational Sextant.

A signal representing the difference in the azimuth and course angle, i.e. the value of the true course, is fed to the indicator of the astrocompass.

When using the astrocompass to determine and retain the orthodrome course, coordinates pertaining to the initial point of the orthodrome path line are fed into the astrocompass. During flight along the orthodrome, the course angles at the initial point of the route. To preserve a constant value of the true course relative to the reference meridian of the beginning of the path, a correction on the basis of the flight correction method is automatically fed in. This method entails the following: The axis of rotation of the head of the CAD is vertical at the beginning of the path. Later with movement of the aircraft along the orthodrome, it slopes back toward the tail of the aircraft by an angle equal to the arc of the traversed part of the orthodrome at the same time remaining parallel to the original position. The automatic calculation of the angle proportional to the arc of the traversed segment of the orthodrome is performed by the flight corrector, with a manual setting of the airspeed of the aircraft.

Astronomical Sextants

Aviational astronomical sextants are intended for measuring the altitudes of stars to determine the astronomical position lines and the position of the aircraft, as well as for measuring the course angles of stars.

At the present time, periscope sextants (PS) which are adapted for mounting on aircraft with hermetic fuselages are the most common variety.

The optical system of the PS sextant (Fig. 5.26) includes a cubic prism 1 for sighting stars. The cubic prism turns in a vertical plane from 0 to 85°, with a goniometer drum to indicate altitude of a star.

In the horizontal plane, it turns through 360° with the goniometer drum for course angles. The cubic prism projects an image of the star along the optical axis of the sextant. Between the nodes of the transformation system, a special prism 2 is situated on the optical axis. In this prism, the image of the star is matched with the bubble level. The image of the bubble is projected onto prism 2 by means of rotating prism 4. The combined images of the star and the bubble, by means of rotating prism 5, are directed to the ocular 6, where they are observed by the eye. /446

The sextant has a chronometer with two independent repeaters, the clock mechanism of the averager of the readings and the course-angle transmitting selsyn.

CHAPTER SIX

ACCURACY IN AIRCRAFT NAVIGATION

1. Accuracy in Measuring Navigational Elements and in Aircraft Navigation as a Whole

The process of aircraft navigation is directed toward a crew's maintaining given trajectories of aircraft movement with respect to direction, altitude, distance, and time. /447

Since the coordinates of an aircraft and the parameters of its speed along the axes of coordinates of a chosen frame of reference are measured with definite errors, it is natural that a given trajectory of aircraft movement will likewise be maintained with some errors.

By *accuracy of aircraft navigation* is meant the limits within which the errors of any flight-trajectory parameter are included with a definite probability.

In contrast to the accuracy of navigational devices, which characterizes (in the majority of cases) the errors in measuring one coordinate or two aircraft coordinates simultaneously, the accuracy of aircraft navigation depends on the conditions of implementing indicated measurements and, in some cases, on the dynamics of aircraft flight.

Let us assume that an aircraft is moving in a field of constant wind or under conditions of calm. The direction of flight is maintained on the basis of results of measuring the lateral deviation of the aircraft (Z) from the line of the given path at designated points (Fig. 6.1).

Points A and B in the figure correspond to the actual coordinates of the aircraft, while points A_1 and B_1 correspond to measured coordinates.

It is obvious that on the basis of results of measurements (A_1 and B_1), the aircraft crew does not obtain an accurate notion concerning the direction of movement, i.e. there is an error in determining the actual angle of flight $\Delta\psi$.

In general, errors in measuring the Z-coordinate (and, therefore, ψ) will exert the same influence on the accuracy of aircraft navigation with respect to direction, independently of whether the actual trajectory of aircraft movement will coincide with the given trajectory or whether it is situated at some slight angle to it. /448

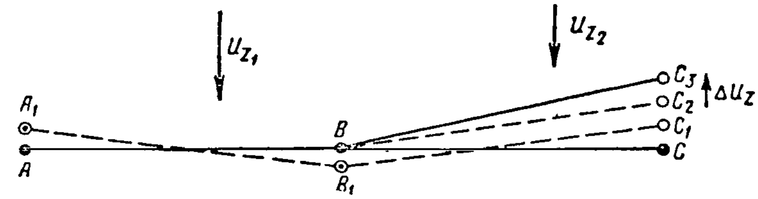


Fig. 6.1 Diagram of the Occurrence of Errors in Aircraft Navigation with Respect to Direction.

However, for simplicity of argument, we will consider that on segment AB the actual path line of the aircraft accidentally turned out to correspond strictly with the given line. In this case, angle $\Delta\psi$ and coordinate B_1 will be magnitudes of misinformation for the crew which, in their graphic form, determine errors in the crew's actions in the flight segment BC.

Actually, a crew located at point B precisely on the given path will assume that the aircraft is located at point B_1. Therefore, for an approach to point C it will be obliged to make an advance in the course:

$$\Delta\gamma = \operatorname{arctg} \frac{BB_1}{BC}.$$

In addition, the crew will assume that an aircraft on segment AB did not travel parallel to the given path line, but at an angle $\Delta\psi$, equal to

$$\Delta\psi_1 = \operatorname{arctg} \frac{AA_1 + BB_1}{AB}.$$

Therefore, the total incorrect advance in the course

$$\Delta\gamma_{total} = \Delta\psi_2 = \Delta\psi_1 + \Delta\gamma.$$

Therefore, if the distance BC is approximately equal to AB, the aircraft must go not to point C but to point C_2, situated the following distance from point C:

$$CC_2 = AA_1 + 2BB_1,$$

where $AA_1 = \Delta Z_1$; $BB_1 = \Delta Z_2$.

Under actual flight conditions, it is difficult to expect that the wind in segment *BC* will be the same as in segment *AB*. Therefore, if a flight is made over *BC* by maintaining the condition selected in segment *AB*, the aircraft will not appear at point C_2, but at point C_3, displaced from point C_2 by the value of the change in the wind vector in segment *BC* with respect to segment *AB* relative to the flying time *BC*. /449

For aircraft navigation with respect to direction, only the lateral component of the wind change vector $\Delta \overline{U}_z$ will have any significance. Thus, the general error in aircraft navigation with respect to direction in segment *BC* is:

$$\Delta Z_{BC} = \Delta Z_1 + 2\Delta Z_2 + \Delta \overline{H}_z t. \tag{6.1}$$

It is possible to come to an analogous conclusion by examining the accuracy of aircraft navigation with respect to distance if the condition of speed is chosen on the basis of results of measuring the *X*-coordinate at points *A* and *B*:

$$\Delta X_{BC} = \Delta X_1 + 2\Delta X_2 + \Delta \overline{H}_x t. \tag{6.2}$$

Formulas (6.1) and (6.2) determine the *absolute errors in aircraft navigation* with respection to direction and distance. In these formulas, only the third term on the right-hand side ($\Delta \overline{U}_z t$ and $\Delta \overline{U}_x t$) is a value which depends on the length of the stage of the path and therefore, on flight time. Therefore, the absolute error grows smoothly with an increase in the length of the stage in the path between the control points.

The ratio of the absolute error of a given parameter to the length of the stage in the path of the aircraft in which this error arises is called the *relative error of aircraft navigation*. Therefore, the relative error exerts an influence on the stability of the flight conditions of the aircraft. Let us illustrate this with a specific example.

Let us assume that at the control stage in the path of an aircraft, with a length of 200 km, an error of aircraft navigation of 5 km in distance and 4 km in direction has accumulated.

The relative error in aircraft navigation with respect to distance and direction will be:

$$\frac{\Delta X}{X} = \frac{5}{200} = \frac{1}{40} = 2{,}5\%;$$

$$\frac{\Delta Z}{X} = \frac{4}{200} = \frac{1}{50} = 2\%.$$

The relative error with respect to direction characterizes the conditional errors of aircraft navigation:

$$\Delta\psi = \mathrm{arctg}\,\frac{\Delta Z}{X}.$$

In the following stage of flight of equal length (200 km) in order to balance the errors of aircraft navigation which were accumulated in the preceding stage, it is necessary:

(a) to introduce a correction in the aircraft course equal to the error $\Delta\psi$, in our case arctg 1/50 $\approx 1°$, and /450

(b) to change the airspeed, in our example by 2.5%.

Let us assume now that the same error in aircraft navigation arose at a stage in the path about 50 km long. Then

$$\frac{\Delta X}{X} = \frac{5}{50} = 10\%;$$

$$\frac{\Delta Z}{X} = \frac{4}{50} \approx 4°.$$

In this case it would be necessary for us to change the aircraft course by 4° and the airspeed by 10% for every 50 km of the path, i.e. in modern aircraft, every 3-4 min of flight.

Considering that the error in aircraft navigation increases with respect to time only as a result of a change in the wind vector, it becomes entirely obvious that it is advantageous to choose control stages of flight which are very long, both from the point of view of the frequency of introducing corrections in the aircraft flight condition and in the values of the corrections being introduced.

However, with excessively long flight stages between control points, an absolute error can be accumulated in aircraft navigation, (with respect to both distance and direction) which exceeds the permissible limit as a result of the third terms of (6.1) and (6.2). Therefore there are conditions for choosing an optimum length of the control flight stage at which the results of aircraft navigation are optimum, both from the point of view of maintaining a given trajectory with allowable limits of deviation and stability of aircraft flight conditions.

The necessary accuracy of aircraft navigation with respect to direction of the flight path is determined by the set width of air routes and approach paths to airports, as well as national boundaries.

However, it is necessary to consider that at turning points on the paths, with significant turn angles for the route, the errors of aircraft navigation with respect to distance become errors with respect to direction, and vice versa.

The accuracy of aircraft navigation during the approach of an aircraft landing on instruments acquires a special significance. The necessary length of the path of an aircraft's approach to a given trajectory, after changing to visual flight, depends on the magnitude of the aircraft's deviation from the given descent trajectory during an instrument approach for landing, and therefore on the weather conditions during which a landing can be made. /451

With automatic or semiautomatic approach to landing by aircraft up to low altitudes (for example, up to leveling off or landing) the accuracy of aircraft navigation must be such that the landing of the aircraft in all cases will be ensured with the execution of safe deviation norms with respect to the landing position and direction of the aircraft vector in the path.

2. Methods of Evaluating the Accuracy of Aircraft Navigation

In special books on the study of the accuracy of aircraft navigation with the application of navigational systems, the methods of probability theory (Laws of the distribution of random variables) are used.

To evaluate the accuracy of aircraft navigation under practical conditions, it is sufficient to use only the basic conclusions of probability theory. Since the study of probability theory as a science is not the purpose of this textbook, in the majority of cases these conclusions will be given without proofs.

In probability theory, variables which cannot be determined in advance by classical methods of mathematics, or are determined by methods so complex that they cannot be used for practical purposes, are considered to be random variables.

In connection with problems of the accuracy of aircraft navigation or the accuracy of measuring aircraft coordinates by means of navigational systems, the errors in measuring or maintaining some of the navigational parameters will be random variables.

Let us assume that the value of some navigational flight parameter (on the basis of some especially precise control device) is known exactly. However, in carrying out a number of measurements by the usual means, we always obtain new values for the parameter which differ from its precise value.

The precise value of a measured parameter will be called its mathematical expectation. If a series of measurements is sufficiently great, then in all probability we will obtain many values for the measured parameter, with both positive and negative errors. Here the mean arithmetic value of all the measurements will approach (depending on the increase in their number) the mathematical expectation of the measured value. Therefore, to raise the accuracy of aircraft navigation, in many cases measurements are carried out repeatedly and the arithmetic mean of the series of measurements is found.

The arithmetic mean of a measured parameter cannot characterize /452 the probable accuracy of carrying out individual measurements. Therefore, probability theory includes a concept of mean square deviation from the precise value.

Let us designate the precise value of a measured quantity by a, and its measured values by x_i, where $i = 1, 2, 3 \ldots$

Let us call the value $(x_i - a)$ the *measurement error.*

The value obtained by extracting the square root from the sum of the squares of the errors divided by the number of measurements is considered the mean square error of measurement:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{i=n} (x_i - a)^2}{n}}. \tag{6.3}$$

According to (6.3), the mean square error of measurement is determined when the precise value of magnitude a is known.

If the value of the measured magnitude is determined as an arithmetic mean from a series of observations, it is considered that one of the measured magnitudes coincides with or very closely approaches the arithmetic mean. The error of this measurement is considered to be zero, resulting in an increase in the sum in the numerator under the root of (6.3) equal to zero. Therefore, in order to avoid decreasing the value of the mean square error, especially with a short series of measurements, the denominator of (6.3) reduces to 1. Then this formula assumes the form:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{i=n} (x_1 - a)^2}{n - 1}}. \tag{6.4}$$

The mean square error characterizes the accuracy of the measurements in a rather definite way. With the raising of each of the errors to a square, its sign always becomes positive. Therefore, in determining mean square errors, only the absolute value of each plays a role.

It is considered that the mean square error does not have a sign.

If we examine only one of a series of measurements, with a probability equal to 1 (complete probability), it is possible to say that the magnitude being measured will undoubtedly have some value. However the probability that the magnitude being measured will have a strict and absolutely precise value is practically equal to zero, except in cases when it can assume only a discrete value. Therefore, in determining the probability of an error of measurement it is not the precise value of the error, but the limits /453 in which it must be found, which are given (for example, the probability of error in the range from 500-600 m or from 2 to 2.5 km, etc.).

All the measured navigational magnitudes are (to a certain degree) calibrated magnitudes, i.e., they have errors limited by certain boundaries. These boundaries depend on the allowances in the regulation of the measuring apparatus and on the maximum possible distortions of the measured magnitudes as a result of the influence of external factors (electromagnetic wave propagation, the physical composition of the airspace, variations in the Earth's magnetic field, etc.).

Allowances in the regulation of measuring apparatus are known quantities. Century-old observations permit the determination of the limit of change in the parameters of the environment. There are ways of evaluating the maximum influence and other factors on the accuracy of measurements. Therefore, it is always possible to predetermine the maximum errors of some kind of measurements.

The quantitative characteristics of the distribution of errors from their zero to maximum values, in the majority of cases, are subject to the normal law of random variable distribution.

If in some cases the law of error distribution is not normal, it will be close in any case.

Considering that devices of probability theory are used not in calculating measurement errors, but only in evaluating limits and the probability of possible measurement errors within these limits, it is considered permissible in all cases to use the normal law of distribution of random variables.

The normal law of random variable distribution (Gauss formula) characterizes the probability density of a random variable, in our case of the measurement errors $(x - a)$, depending on its value:

$$\varphi(x-a) = \frac{1}{2\sqrt{2\pi\sigma}} e^{\frac{-(x-a)^2}{2\sigma^2}} \qquad (6.5)$$

where $\phi(x - a)$ is the probability density of errors of a given magnitude, σ is the mean square error of a series of measurements, e is a Napier number equal to 2.71828, and a is the precise value of the magnitude being measured.

It is obvious that the probability of finding the result of measuring (x) in the range of values from a to x can be determined by integrating (6.5) over x:

$$\Phi(x-a)' = \frac{1}{\sqrt{2\pi\sigma}} \int_a^x e^{\frac{-(x-a)^2}{2\sigma^2}} dx. \qquad (6.6)$$

The graph of the probability of random variables subordinate to the normal distribution law is shown in Figure 6.2. /454

The curve on the graph shows the probability density of random variable deviations from zero to maximum positive and negative values. The left side of the graph corresponds to errors with a negative sign, the right to errors with a positive sign.

Since the absolute probability of obtaining any value of the measured magnitude is equal to one, the probability that the value of the magnitude will be negative or positive is 0.5.

Let us note that on the abcissa of the graph there are two values of a random variable, x_1 and x_2. The area bounded by the segment $x_1 x_2$, by the ordinates P_{x1}, P_{x2}, and by the curve is the probability of finding the result of measurement in the limits between x_1 and x_2.

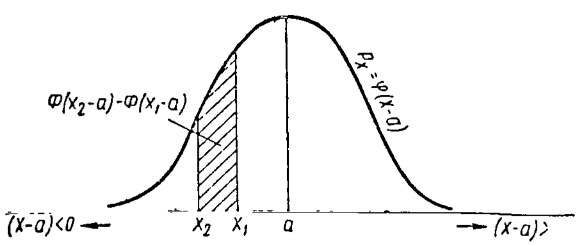


Fig. 6.2 Graph of the Probability of Random Variables Under the Normal Law of Distribution.

With the convergence of points x_1 and x_2 at one point, the probability of finding an error of measurement between these points will diminish and converge to zero.

The analogous problem for determining probability can be solved on the basis of the right side of the graph for errors of measurement which have a positive sign.

Without stopping at the methods of solving an integral (6.6), let us indicate that the overall probability of finding positive and negative errors of measurements is 68.3% in the range from 0 to σ, 95% from 0 to 2σ, and 99.7% from 0 to 3σ. A table of values of the function $\Phi(x - a)$ for $(x - a)$ from 0 to 5σ is given in Supplement 6.

For example, if the mean square error of measuring the drift angle with a Doppler meter is equal to 15′, then with a probability of 95% it is possible to expect that the measurement error will not exceed 30′, and with a practically complete probability (99.7%), 45′.

The value of the mean square error of measuring a given kind of parameter permits evaluation of the accuracy of other parameters which have a functional dependence on the first.

Example: The mean square error of the direction-finding of an aircraft by means of a ground direction finder $\sigma = 1°$. Determine the limits of linear error in determining the lateral deviation of an aircraft from the line of a given path with a probability of 95% if the aircraft is located at a distance of 300 km from the direction finder.

Solution: With a probability of 95%, the angular error of a direction finder in measuring does not exceed 2°.

Therefore, $\Delta Z(P = 95\%) = 300 \text{ tg } 2° \approx 10$ km. Solving the same problem for a practical probability of 100% (more precisely, 99.7%), $\Delta Z(P = 100\%) = 300 \text{ tg } 3° \approx 15$ km. /455

Let us now assume that we must solve the reverse problem, i.e. determine the necessary accuracy of a direction finder which ensures the given accuracy of measurements of lateral deviations.

Example: $\Delta Z_{max}(P = 100\%) = 10$ km. Determine the necessary accuracy of a direction finder for distances up to 300 km.

Solution: $3\sigma_d = \text{arctg } \frac{10}{300} \approx 2°$. Therefore, $\sigma_d = 0.7°$.

3. Linear and Two-Dimensional Problems of Probability Theory

The normal law of random variable distribution examined in the preceding paragraph includes the linear (one-dimensional) problem of probability theory for one parameter of measurement.

In aircraft navigation, it is often necessary to deal with several measurement parameters. For example, in calculating the path of an aircraft with respect to direction by automatic navigational devices, on the basis of results of measuring the drift angle and groundspeed of the aircraft with a Doppler meter, the following errors exert an influence on the accuracy of calculating this parameter: errors in calculating the given flight angle;

errors in measuring the course, drift angle, and groundspeed; errors in the operation of an integrating device.

Each of these factors separately will create the following error components in calculating the path with respect to direction:

$$\Delta Z_{\psi} = x \sin \Delta\psi = Wt \sin \Delta\psi;$$
$$\Delta Z_{\gamma} = Wt \sin \Delta\gamma;$$
$$\Delta Z_{a} = Wt \sin \Delta a;$$
$$\Delta Z_{w} = \Delta Wt \sin (\psi_{\text{ф}} - \psi_{\text{з}});$$
$$\Delta Z_{\int_z} = Wt\Delta \int_z .$$

If the indicated components had the same sign and had a maximum value within the calibration limits of each of the parameters, the general error would be equal to the arithmetic sum of these components. However, according to the law of normal random variable distribution, even when measuring one parameter, the maximum error is encountered rather rarely. The probability that all the errors /456 will take on a maximum value, and even one sign, will be extraordinarily low.

In spite of the fact that we must deal simultaneously with many measured parameters, the solution of the above example includes a linear problem of probability theory, since the random variables are summed along one axis of the chosen frame of reference of their coordinates.

To solve similar problems, the concept of the dispersion of random variables σ^2 is introduced into probability theory.

It is known that the law of random variable distribution, obtained by adding other random variables which are subject to the normal distribution law, is also a normal distribution law. Here, the scatter of an overall random variable is the sum of the scatters of the values being added.

In our example of calculating the path of an aircraft by means of automatic navigational devices, the value σ^2_z is the scatter of the sum.

Here,

$$\sigma_z^2 = \sigma^2 Z_{\psi} + \sigma^2 Z_{\gamma} + \sigma^2 Z_{a} + \sigma^2 Z_{\int_z} + \sigma^2 Z_{w}. \qquad (6.7)$$

The mean square error of the measurement is equal to the square root of the scatter: $\sigma = \sqrt{\sigma^2}$. Therefore, the mean square error of the total value will equal the square root of the sum of the scatters. For our example,

$$\sigma_z = \sqrt{\sigma^2 Z_\psi + \sigma^2 Z_\gamma + \sigma^2 Z_a + \sigma^2 Z_{\int_z} + \sigma^2 Z_w}. \qquad (6.8)$$

The value σ^2_{zw} in (6.8) is a small second-order value:

$$\Delta Z_n = \Delta W \sin \Delta\psi.$$

Therefore, it is necessary to disregard this value.

Let us assume that the remaining values included in (6.8) have been mean square errors as follows:

$$\sigma_\psi = 20'; \quad \sigma_\gamma = 20'; \quad \sigma_a = 15'; \quad \sigma_{\int_z} = 0{,}5\% \text{ of } X.$$

Since the first three values are small, their sinces can be replaced by angle values. Then, considering 1° equal to 0.017 by 1.7% X, their value can be expressed in percent of the distance traversed:

$$\sigma_\psi = 0{,}56\% X; \quad \sigma_\gamma = 0{,}56\% X; \quad \sigma_a = 0{,}42\% X; \quad \sigma_{\int_z} = 0{,}5\% X,$$

where $X = Wt$.

Therefore, the mean square error in calculating the path with respect to direction is /457

$$\sigma_z = Wt\sqrt{0{,}56^2 + 0{,}56^2 + 0{,}42^2 + 0{,}5^2} = Wt\sqrt{1{,}05} = 1{,}02\% X.$$

Hence, it is possible to consider that the mean square error in calculating the path with respect to direction amounts to ~ 1% of the distance traversed.

Let us assume that we have set ourselves the goal of maintaining an aircraft within the limits of an air route with a width of 20 km (up to 10 km from LGP) with a probability of 95%. Here the mean square error in determining the initial coordinates of the aircraft equals 2 km.

For a probability of 95%, the error in the initial formulation of the aircraft's coordinates must be taken as 4 km, while the accuracy of calculating the path with respect to direction must be taken as 2%. The maximum error in calculating the path with respect to distance must not exceed

$$\Delta Z_{max} = \sqrt{10^2 - 4^2} = \sqrt{84} \approx 9 \text{ km}$$

The value 9 km must amount to 2% of the distance covered.

Therefore, the allowable length of the stage of the path between the control points (S) must be not more than

$$S = \frac{9}{0{,}02} = 450 \text{ km}$$

If we set ourselves the goal of maintaining an aircraft within the limits of a route with a probability of 99.7%, the accuracy of the initial display of coordinates and the calculation of the path of the aircraft would have to be taken as 3σ or a reading accuracy equal to 6 km and an accuracy for calculating the path equal to 3% x. Then

$$\Delta Z_{max} = \sqrt{10^2 - 6^2} = \sqrt{64} = 8 \text{ km}$$

$$8 \text{ km} = 3\%\ X; \quad X = \frac{8}{0{,}03} = 266 \text{ km}$$

If we take the limits of calibrating each parameter as 3σ, then by adding the errors on the basis of the calibration rules we would obtain the value

$$\Delta Z_{max} = 3\sigma_{\psi} + 2\sigma_{\gamma} + 3\sigma_{a} + 3\sigma_{\int z}$$

or, in our example,

$$\Delta Z_{max} = 1{,}7 + 1{,}7 + 1{,}5 + 1{,}2 = 6{,}1\%,$$

i.e., in the case when all errors have a maximum value and the same sign, the error of calculating the path can reach 6%. Since it reaches 2% with a probability of 95%, with a practically complete probability of 99.7% it reaches 3%.

The probability that calculating the path will occur with /458
errors within the range of an overall calibration of the system is expressed by in hundred millionths of a percent. Therefore, when there is no threat of disturbing the safety of a flight, it is not necessary for practical purposes to take the limits of overall calibration into consideration.

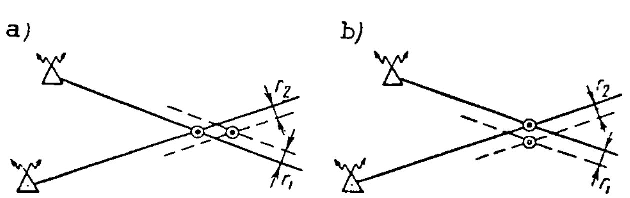


Fig. 6.3. Diagram of the Occurrence of Errors in Determining the Position of an Aircraft: (a) with Multilateral Errors in Rearings; (b) With Unilateral Errors.

In the majority of cases, there is sufficient error in calculating the path to calculate with a probability of 95%, and only in especially responsible cases, with 99.7%.

The majority of problems in determining the accuracy of aircraft navigation or measuring its separate parameters with the use of some method reduces directly to linear problems of probability theory. The final result of solving all the problems of aircraft navigation must be one-dimensional, since the goals and requirements for accuracy of aircraft navigation with respect to distance, direction, and flight altitude are different.

At the present time, there are no navigational systems which determine the position of an aircraft in three-dimensional space. Therefore, the necessity for solving volumetric problems in probability theory is superfluous. However, a number of navigational systems such as a hyperbolic, two-pole goniometer, or goniometric rangefinder (if only the location of a ground beacon is known for the LGP of a given segment), permit the solution of a problem in determining an aircraft's coordinates in two-dimensional space.

An evaluation of the accuracy of aircraft navigation along each of the axes of the coordinate system chosen for aircraft navigation, in this case, can be carried out only after solving a one-dimensional problem in probability theory.

Let us assume that we have an oblique-angled surface system of aircraft position lines, each of which does not coincide with the given flight path (Fig. 6.3, a,b).

The linear error in determining the first (r_1) and second (r_2) aircraft position lines depends on both the accuracy of measuring the navigational parameter and its gradient.

The gradient of a navigational parameter is the ratio of its increase to the movement of an aircraft in a direction perpendicular to the position lines of the operating region of the system /459

$$g = \frac{da}{dr}, \qquad (6.9)$$

where g is the gradient of the navigational parameter, and a is the navigational parameter being measured.

For example, if the navigational system is a goniometer, then

$$g = \frac{da}{dr} = \frac{dA}{dr} = \frac{1}{S},$$

where A is the azimuth of the aircraft and S is the distance from the ground beacon to point PA. In this case,

$$dr = \frac{da}{g} = daS \text{ or } r = \Delta AS.$$

With the introduction of the concept of the gradient of a navigational parameter, all the existing coordinate systems reduce to a generalized system, i.e., the problems of determining the accuracy of navigational measurements are solved on the basis of a general scheme, independent of the geometry of application of the navigational device.

In Fig. 6.3 a,b two possible cases of the appearance of errors in determining the position of an aircraft on the basis of the intersection of position lines are shown:

(a) Errors in r_1 and r_2 have different signs; in this case, the measured position of the aircraft lies in an acute angle between the actual position lines. This leads to larger errors of determination.

(b) Errors in r_1 and r_2 have identical signs; the measured position of the aircraft lies in an obtuse angle between the position lines. The errors in determining the position of the aircraft in this case are close to the linear errors of one bearing.

It is necessary to note that the probability of errors in r_1 and r_2 with the same sign in the majority of cases is more than the probability of errors with different signs. For example, in taking bearings with a radio compass, the error component as a result of an error in measuring the aircraft course will be general for two measurements. If the angle between the bearings is sufficiently acute, the radio deviation will have either one sign or different signs, but a small value in any case.

A similar relationship between measurement errors in probability theory is called *correlation* (ρ).

The general error in determining the position of the aircraft in our case will be (Fig. 6.4):

$$r^2 = \frac{r_1^2}{\sin^2 \omega} + \frac{r_2^2}{\sin^2 \omega} - \frac{2r_1 r_2 \cos \omega}{\sin^2 \omega}$$

or

$$r = \frac{\sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos \omega}}{\sin \omega}. \qquad (6.10)$$

Formula (6.10) characterizes only the magnitude of error in /460 determining the position of an aircraft on the basis of two position lines with known errors in measurement for each of them. However, it does not give us an idea of the nature of the distribution of the indicated errors around the point of the actual position of the aircraft (center of scatter).

In contrast to a linear problem, where the probability of an overall error in several measurements is examined, in a two-dimensional problem it is necessary to examine the products of the probabilities of these errors.

For simplicity of argument, let us assume that we have a rectangular coordinate system (Fig. 6.5); let us set ourselves the goal of limiting the area within which the aircraft is located with a probability of 95%. Here, the mean square errors of measuring the two position lines will be considered identical.

Let us examine a certain large number of measurements (e.g., 10,000) and let us see what will be the probability that the measured position of the aircraft will be in an exterior angle at a distance from the center of the area of scatter which exceeds the diagonal of a square constructed with errors 2σ, and $2\sigma_2$.

Since the probability of an error in the first measurement exceeding 2σ equals 5%, then 500 of 10,000 measurements must be beyond the limits of a side of the indicated square. The remaining 9,500 measurements lie in the range from zero to 2σ, and there is no need to calculate them during common measurements with the second position line.

It is obvious that of the 500 remaining measurements, where an aircraft will be located a distance more than 2σ, from the first position line, the errors in the second bearing will exceed the value $2\sigma_2$ only in 5% of the cases, and thus there will be only 25 cases (or 0.25%) simultaneously exceeding the errors of the values /461
2σ, and $2\sigma_2$.

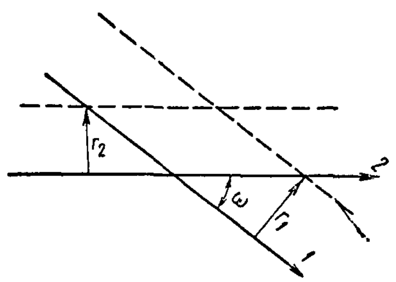


Fig. 6.4. Total Error in Determining the Position of an Aircraft.

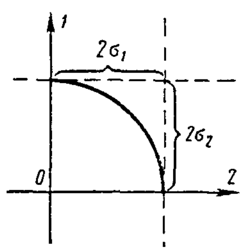


Fig. 6.5. Probability of the Simultaneous Yield of Errors Beyond the Limits of the Given Values.

The example examined shows clearly that the probability density of errors directed toward the indicated angle diminishes sharply. It is practically possible to consider that a large number of the common measurements of the first and second position lines lie in a circle, the radius of which equals $2\sigma_1 = 2\sigma_2$, while the limit of equal probability of deviations from the center of scattering will be a circle. In general, when the errors in the first bearing are not equal to the errors in the second bearing, this boundary has the shape of an ellipse.

If we examine a number of cases in which an aircraft is within the limits of an ellipse with axes equal to 2σ, it turns out to be significantly less than 95%, since even in rectangles constructed with sides equal to 2σ there will be 95% of 95%, or 90.025%.

However, this is of value only when the probability of an aircraft's entering a given area is examined. From the point of view of aircraft navigation, it is not the location of an aircraft in a given area, but the deviation from the given path trajectory and the retaining of flight distance with respect to time which play a role. Therefore, the results of adding the probabilities are again distributed according to direction. This again raises the probability of each of them to 95% (Fig. 6.6).

In the figure, an ellipse of measurement errors, located in a certain position relative to the path line of an aircraft, is shown. In the above case, the possible errors in measuring the coordinates of an aircraft with a given probability are determined by tangents, parallels, and perpendiculars to the line of the given path (the orthodrome coordinates are kept in mind). The distance of the tangents from point PA, as well as the maintenance of correlation dependence between the errors in the maintenance of the path with respect to distance and direction, will depend on the shape dimensions, and orientation of the ellipse of errors. The correlation dependence plays a role only at turning points in the route, when errors of aircraft navigation with respect to distance definitely become errors of aircraft navigation with respect to direction, and vice versa.

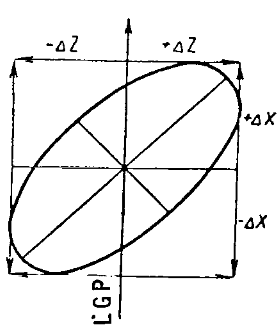


Fig. 6.6. Ellipse of Errors in the Aircraft Position.

The physical sense of the above arguments becomes clear if we assume that the ellipse of errors constructed from mean square measurement errors can be arranged with the major axis both in the direction of the path line and /462
perpendicular to it. Then the accuracy of aircraft navigation with respect to distance and direction can be determined by mean square values of the axis of the ellipse. Obviously the

orientation of the axis of the ellipse at an angle to the line of the given path occupies an intermediate position between those in the figures. The accuracy of maintenance of the path along the axes of the coordinates in this case is determined in the same way as for the case shown in (Fig. 6.7 a,b). However, a correlation dependence between these measurements will be seen.

In the above examples of a circle and ellipse of errors, we assumed that the aircraft position lines were situated at right angles to one another, although an angle to the given path line is possible, and that the correlation dependence between the measurements of the position lines is absent.

Let us now present, without derivation, the formulas which can be used as a basis for determining the dimensions and orientation of the axes of an ellipse for a situation when the position lines are situated at an angle (not a right angle) with an independent accuracy of measurement for each of them (correlation coefficient equal to zero):

$$a^2 = c^2 \frac{\sigma_{r_1}^2 + \sigma_{r_2}^2}{2\sin^2\omega} + \frac{\sqrt{(\sigma_{r_1}^2 + \sigma_{r_2}^2) - 4\sigma_{r_1}^2 \sigma_{r_2}^2 \sin^2\omega}}{2\sin^2\omega} \qquad (6.11)$$

$$b^2 = c^2 \frac{\sigma_{r_1}^2 + \sigma_{r_2}^2}{2\sin^2\omega} - \frac{\sqrt{(\sigma_{r_1}^2 + \sigma_{r_2}^2)^2 - 4\sigma_{r_1}^2 \sigma_{r_2}^2 \sin^2\omega}}{2\sin^2\omega}$$

$$\mathrm{tg}\, 2\alpha = -\frac{\sigma_{r_1}^2 \sin 2\omega}{\sigma_{r_2}^2 + \sigma_{r_1}^2 \cos 2\omega}, \qquad (6.12)$$

where a is the major semiaxis of the ellipse; b is the minor semiaxis of the ellipse; ω is the angle between the position lines; c is the parameter of the ellipse chosen for a given probability; and α is the angle between the bisector of the angle of intersection of the position lines and the major axis of the ellipse (it is plotted in the direction of the position line with the smallest error).

These formulas are used in the majority of cases, since the accuracy of determining position lines is usually independent or the correlation coefficient is unknown.

Only in individual cases, e.g., in determining position lines with an aircraft radio compass, is the correlation coefficient determined rather simply, since part of the error of measuring the bearing (depending on the aircraft course) will be general. For example: $\sigma_{course} = 3°$; $\sigma_{\gamma} = 2°$.

The general mean square error in measuring a bearing will be:

$$\sigma A = \sqrt{\sigma_{\gamma}^{2} + \sigma^{2}_{\text{course}}} = \sqrt{13} \approx 3.6^{\circ}.$$

In measuring two bearings on one aircraft course, it is necessary to expect that both errors will be shifted in one direction by the mean square error of measuring the aircraft course, in our case by 2°, which amounts to 0.55 of the total error. Therefore, the correlation coefficient ρ = 0.55.

As shown above (see Fig. 6.3), the presence of a total component in the errors of measuring position lines raises the accuracy of determining the position of an aircraft. According to (6.10), since the third term under the radical in the numerator can be given both positive and negative values, for independent measurements it is possible to write /463

$$\sigma_r = \frac{\sqrt{\sigma_{r_1}^2 + \sigma_{r_2}^2}}{\sin \omega}. \tag{6.13}$$

With dependent measurements, the third term under the radical must be multiplied by the correlation coefficient ρ and (6.13) takes the form:

$$\sigma_r = \frac{\sqrt{\sigma_{r_1}^2 + \sigma_{r_2}^2 - 2\rho\sigma_{r_1}\sigma_{r_2}\cos\omega}}{\sin \omega}. \tag{6.14}$$

With a correlation coefficient equal to zero, or with angle ω equal to 90°, (6.14) is transformed into (6.13).

In the presence of correlation, the dimensions and orientation of the axes of the ellipse are determined according to the formulas:

$$\operatorname{tg} 2\alpha = \frac{2\rho\sigma_{r_1}\sigma_{r_2}\sin\omega - \sigma_{r_1}^2 \sin 2\omega}{\sigma_{r_1}^2 + \sigma_{r_2}^2\cos^2\omega - 2\rho\sigma_{r_1}\sigma_{r_2}\cos\omega}; \tag{6.15}$$

$$a^2 = c^2\,\frac{\sigma_{r_1}^2 + \sigma_{r_2}^2 - 2\rho\sigma_{r_1}\sigma_{r_2}\cos\omega}{2\sin^2\omega} + \frac{\sqrt{(\sigma_{r_1}^2 + \sigma_{r_2}^2 - 2\rho\sigma_{r_1}\sigma_{r_2}\cos\omega)^2 - 4(1-\rho^2)\sigma_{r_1}^2\sigma_{r_2}^2\sin^2\omega}}{2\sin^2\omega}. \tag{6.16}$$

The length of the minor axis of an ellipse is also determined on the basis of (6.16), with replacement of the positive sign in front of the radical by a negative sign.

In probability theory, the probability of locating an object within the limits of the indicated ellipse of errors (given definite values of the magnitude c) is examined.

Since we have agreed to examine the accuracy of aircraft navigation separately with respect to distance and direction, this problem will not interest us. We are using the ellipse for an evaluation of the accuracy of aircraft navigation with respect to both distance and direction.

Let us assume that we know the orientation of an aircraft's position lines and a given path line on a map, and have determined, on the basis of (6.11) and (6.12) or (6.15) and (6.16), the lengths of the axes of the ellipse $2a$ and $2b$, as well as the orientation of the major axis of the ellipse with parameter $c = 1$ (mean square ellipse).

In this case, the mean square error in the maintenance of the path with respect to distance and direction is determined by tangents to the ellipse at points X_0 and Z_0, perpendicular to the path line, and at points X_1 and Z_1, parallel to it (Fig. 6.7).

It is possible to show that the mean square errors in maintaining the path (with respect to distance σ_x and direction σ_z) in this case are

$$\left.\begin{aligned} \sigma_x &= \sqrt{\frac{a^4\cos^2\alpha}{a^2+b^2\operatorname{ctg}^2\alpha}} - \sqrt{\frac{b^4\sin^2\alpha}{a^2\operatorname{ctg}^2\alpha+b^2}};\\ \sigma_z &= \sqrt{\frac{a^4\sin^2\alpha}{a^2+b^2\operatorname{ctg}^2\alpha}} + \sqrt{\frac{b^4\cos^2\alpha}{a^2\operatorname{ctg}^2\alpha+b^2}}, \end{aligned}\right\} \qquad (6.17)$$

where α is the angle between the line of the given path and the major axis of the ellipse.

In Fig. 6.7, it is obvious that in a general case, when the /464
major axis of an ellipse does not coincide with the given path line or the line perpendicular to it, there is a correlation dependence between the errors in the maintenance of the path with respect to distance and direction.

Actually, if we have a positive error in determining the X coordinate, the measured position of the aircraft is located in the right-hand side of the ellipse. The mathematical expectation of the value of the Z-coordinate in this case will be found in the middle of the chord of the ellipse, parallel to OZ and intersecting the X-axis at a point corresponding to ΔX.

The diameter of the ellipse, dividing its chords (which are perpendicular to some other diameter) in half, is called the *conjugate diameter*.

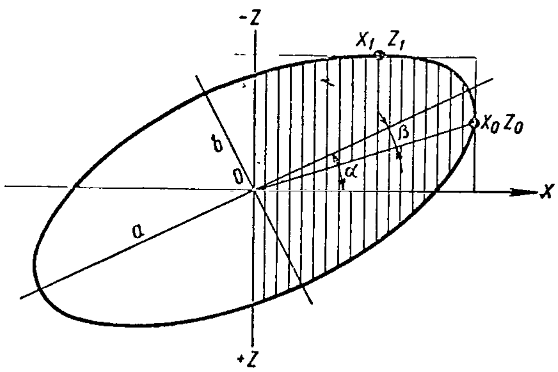


Fig. 6.7. Correlation Dependence of the Errors in the Control Path with Respect to Distance and Direction.

The direction of the conjugate diameter is determined according to the formula

$$\operatorname{tg}\beta = \frac{b^2}{a(\operatorname{tg}90-\alpha)}. \qquad (6.18)$$

In probability theory, lines which determine the dependence between random variables are similar to the conjugate diameters of an ellipse of errors. In the case described by us, they are called *regression lines*, while the angular coefficients of these lines (tangents of the angles to the axes of the frame of reference) are called *angular coefficients of regression*.

In our case, the direction of the conjugate diameter connecting the errors of measurement of the *Z*-coordinate with errors for the *X*-coordinate is determined on the basis of (6.18), where α is the rotation angle of the major axis of the ellipse relative to the *X*-axis.

The direction of the second conjugate diameter, which connects the measurement errors of the *X*-coordinate with the errors of the *Z*-coordinate, is determined according to the formula

$$\operatorname{tg}\beta_1 = \frac{b^2}{a^2 \operatorname{tg}\alpha}. \qquad (6.19)$$

Here, the angular coefficients of regression will be:

$$R_z \text{ of } X = tg(\alpha+\beta);$$
$$R_x \text{ of } Z = \operatorname{tg}(\alpha+\beta_1). \qquad (6.20)$$

4. Combination of Methods of Mathematical Analysis and Mathematical Statistics in Evaluating the Accuracy of Navigational Measurements /465

In the preceding paragraphs, we examined methods of mathematical statistics (probability theory) used to evaluate the accuracy of navigational measurements. As a group, these devices permit the solution of any two-dimensional or linear problem encountered in aircraft navigation.

However, in examining the methods of probability theory, we assumed that the accuracy (in general, mean square error) of measurements of separate parameters was known.

Actually, the accuracy of measurements of navigational parameters has a functional dependence on other physical or geometric values connected with the principles of measuring or determining a navigational parameter.

Since this functional dependence is always known, the simplest (and presently most universal) method of determining the accuracy of a navigational parameter is that of the variation of independent variables included in the equations of formulas which determine a navigational parameter within the limits in which the indicated variations are encountered in the practice of aircraft navigation.

For example, the basic equation for an orthodrome which determines its shift in a geographic coordinate system has the form:

$$\mathrm{ctg}\,\lambda_{01} = \mathrm{tg}\,\varphi_2\,\mathrm{ctg}\,\varphi_1\,\mathrm{cosec}\,\Delta\lambda - \mathrm{ctg}\,\Delta\lambda.$$

Let us determine the accuracy of solving the above equation, assuming that the measurement accuracy of each of the parameters included in the equation is known.

The dependence of the accuracy of the solution of the equation on the accuracy of measuring the coordinates of ϕ_2 is expressed by the equation:

$$\frac{d\,\mathrm{ctg}\,\lambda_{01}}{d\varphi_2} = \frac{d\,(\mathrm{tg}\,\varphi^2\,\mathrm{ctg}\,\varphi_1\,\mathrm{cosec}\,\Delta\lambda - \mathrm{ctg}\,\Delta\lambda)}{d\varphi_2} = \mathrm{ctg}\,\varphi_1\,\mathrm{cosec}\,\Delta\lambda\,\sec^2\varphi^2. \quad (6.21)$$

The final result of solving (6.21) would have to be viewed as an arc tangent of the right-hand side. However, from the point of view of a mathematical solution, this would lead to a significant complication of the given problem. It is advisable to use the following method:

$$\frac{d\,\mathrm{ctg}\,\lambda_{01}}{d\varphi_2} = \frac{d\,\mathrm{ctg}\,\lambda_{01}}{d\lambda_{01}} \cdot \frac{d\lambda_{01}}{d\varphi_2}$$

or
$$\frac{d\,\mathrm{ctg}\,\lambda_{01}}{d\lambda_1} = -\,\mathrm{cosec}^2\,\lambda_{01}.$$

Therefore,
$$\frac{d\lambda_{01}}{d\varphi_2} = -\frac{\mathrm{ctg}\,\varphi_1\,\mathrm{cosec}\,\Delta\lambda\,\sec^2\varphi_2}{\mathrm{cosec}^2\,\lambda_{01}}. \qquad (6.22)$$

Thus, if the mean square error in measuring the ϕ_2 coordinate equals $\sigma\phi_2$, it causes an error in determining λ_{01}:

$$\sigma\lambda_{01_{\varphi_2}} = -\,\sigma_{\varphi_2}\frac{\mathrm{ctg}\,\varphi_1\,\mathrm{cosec}\,\Delta\lambda\,\sec^2\varphi_2}{\mathrm{cosec}^2\,\lambda_{01}} \qquad (6.23)$$

The dependence of the accuracy in determing λ_{01} on the accuracy /466 of coordinate ϕ_1 can be obtained analogously:

$$\sigma\lambda_{01_{\varphi_1}} = \sigma\varphi_1\frac{\mathrm{tg}\,\varphi_2\,\mathrm{cosec}\,\Delta\lambda\,\mathrm{cosec}^2\,\varphi_1}{\mathrm{cosec}^2\,\lambda_{01}}; \qquad (6.24)$$

For the parameter $\Delta\lambda$

$$\sigma\lambda_{01_{\Delta\lambda}} = \sigma\Delta\lambda\frac{\mathrm{tg}\,\varphi_2\,\mathrm{ctg}\,\varphi_1\,\mathrm{ctg}\,\Delta\lambda\,\mathrm{cosec}\,\Delta\lambda - \mathrm{cosec}^2\,\Delta\lambda}{\mathrm{cosec}^2\,\lambda_{01}} \qquad (6.25)$$

The total error in solving the equation will be:

$$\sigma\lambda_0 = \sqrt{\sigma^2\lambda_{01_{\varphi_2}} + \sigma^2\lambda_{01_{\varphi_1}} + \sigma^2\lambda_{01_{\Delta\lambda}}}.$$

Since we have examined as an example the accuracy of solving the basic equation for an orthodrome, it is appropriate to examine the accuracy of solving all the special equations which determine its parameters:

(a) Initial azimuth of an orthodrome:

$$\mathrm{tg}\,\alpha_0 = \frac{\sin\lambda_{0i}}{\mathrm{tg}\,\varphi_i} = \sin\lambda_{0i}\,\mathrm{ctg}\,\varphi_i;$$

Using this method of transition from the arc tangent of the angle to its value, as in the preceding example, we obtain:

$$\sigma\alpha_{0_{\lambda_{0i}}} = (\cos\lambda_{0i}\,\mathrm{ctg}\,\varphi_i\cos^2\alpha_0)\,\sigma\lambda_{0i};$$
$$\sigma\alpha_{0_{\varphi_i}} = (\sin\lambda_{0i}\,\mathrm{cosec}^2\varphi_i\cos^2\alpha_0)\,\sigma\varphi_i; \quad (6.26)$$
$$\sigma\alpha_0 = \sqrt{\sigma^2\alpha_{0_{\lambda_{0i}}} + \sigma^2\alpha_{0_{\varphi_i}}};$$

(b) The moving azimuth of an orthodrome:

$$\mathrm{tg}\,\alpha_i = \frac{\mathrm{tg}\,\lambda_{0i}}{\sin\varphi_i};$$
$$\sigma\alpha_{i_{\varphi_i}} = -(\mathrm{ctg}\,\varphi_i\,\mathrm{cosec}\,\varphi_i\,\mathrm{tg}\,\lambda_{0i}\cos^2\alpha_i)\,\sigma\varphi_i; \quad (6.27)$$
$$\sigma\alpha_{i_{\lambda_{0i}}} = (\sec^2\lambda_{0i}\,\mathrm{cosec}\,\varphi_i\cos^2\alpha_i)\,\sigma\lambda_{0i};$$
$$\sigma\alpha_i = \sqrt{\sigma^2\alpha_{i_{\varphi_i}} + \sigma^2\alpha_{i_{\lambda_{0i}}}};$$

(c) Coordinates of intermediate points:

$$\mathrm{tg}\,\varphi = \frac{\sin\lambda_{0i}}{\mathrm{tg}\,\alpha_0};$$
$$\sigma\varphi_{\lambda_{0i}} = \cos\lambda_{0i}\,\mathrm{ctg}\,\alpha_0\cos^2\varphi\,\sigma\lambda_{0i}; \quad (6.28)$$
$$\sigma\varphi_{\alpha_0} = \sin\lambda_{0i}\,\mathrm{cosec}^2\alpha_0\cos^2\varphi\,\sigma\alpha_0;$$
$$\sigma_\varphi = \sqrt{\sigma^2\varphi_{\lambda_{0i}} + \sigma^2\varphi_{\alpha_0}};$$

(d) Distance along the orthodrome from its source: /467

$$\cos S_i = \cos\lambda_{0i}\cos\varphi_i;$$
$$\sigma S_{i_{\lambda_{0i}}} = \sin\lambda_{0i}\cos\varphi_i\,\mathrm{cosec}\,S_i\,\sigma\lambda_{0i};$$
$$\sigma S_{i_{\varphi_i}} = \cos\lambda_{0i}\sin\varphi_i\,\mathrm{cosec}\,S_i\,\sigma\varphi_i; \quad (6.29)$$
$$\sigma S_i = \sqrt{\sigma^2 S_{i_{\lambda_{0i}}} + \sigma^2 S_{i_{\varphi_i}}}.$$

The above formulas (6.23) to (6.29) have the following practical significance.

Let us assume that in solving the basic and special equations of an orthodrome, we use trigonometric tables to five decimal places or a computer with 18 binary digit bits (which are also equivalent to 5 decimal places). In the first case, the error in the value of each independent variable will have a magnitude of from 0 - 5 units of the sixth sign, and (in the second case) from 0 - 10 units of the sixth sign. Substituting the values of the possible errors of each independent variable into these formulas, we will obtain the possible errors in the solution of the equations.

Thus, it is possible to determine the necessary accuracy of the tables (number of signs) or the computers (number of orders) for obtaining a satisfactory result in solving equations within the given value limits of the independent variables.

Calculations show that for geographic latitudes from 0 to 80°, while solving equations for an orthodrome, it is necessary to use tables with 6 decimal points or computers with 21-22 binary digit bits.

5. Influence of the Geometry of a Navigational System on the Accuracy of Determining Aircraft Coordinates

The accuracy of determining the coordinates of an aircraft by means of navigational systems depends both on the accuracy of measuring a navigational parameter and on the geometry of the navigational system being used.

Means of solving one-dimensional problems of probability theory for a generalized oblique-angled coordinate system were examined above. The azimuth coordinate system was given as an example for determining the gradient of a navigational parameter.

Since it is necessary to know the value and direction of the gradient vector (g) of the navigational parameter to solve problems in a generalized coordinate system, only the reduction of different coordinate systems to a generalized system is examined in this section.

Two-pole goniometric, two-pole circular and one-pole range-finding are most simply reduced to a generalized coordinate system.

As has already been indicated, for an azimuthal system at distances on the order of up to 3,000 km, we can consider

$$g = \frac{1}{S}. \tag{6.30}$$

where S is the distance from the aircraft to the ground radio beacon.

For greater distances, we must consider the convergence of the position lines as a result of the sphericity of the Earth's surface, and (6.30) assumes the form: /468

$$g = \frac{1}{R \sin S}, \tag{6.31}$$

where R is the radius of the Earth and

$$dr = dAR \sin S,$$

where dr is the increase in linear error; dA is the increase in azimuth.

The directions of the position lines in this case can be determined as the moving azimuths of the orthodromes at a given point M, which intersect foci of the systems A_1 and A_2.

We must take point M as the starting point of both orthodromes; in this case,

$$\operatorname{ctg} \lambda_{OM_1} = \operatorname{tg} \varphi_{A_1} \operatorname{ctg} \varphi_M \operatorname{cosec} \Delta\lambda_1 - \operatorname{ctg} \Delta\lambda_1;$$
$$\operatorname{ctg} \lambda_{OM_2} = \operatorname{tg} \varphi_{A_2} \operatorname{ctg} \varphi_M \operatorname{cosec} \Delta\lambda_2 - \operatorname{ctg} \Delta\lambda_2;$$
$$\operatorname{tg} \alpha_1 = \frac{\operatorname{tg} \lambda_{OM_1}}{\sin \varphi_M}; \quad \operatorname{tg} \alpha_2 = \frac{\operatorname{tg} \lambda_{OM_2}}{\sin \varphi_M};$$
$$S_1 = \arccos (\cos \lambda_{OM_1} \cos \varphi_M) - \arccos (\cos \lambda_{0A_1} \cos \varphi_{A_1});$$
$$S_2 = \arccos (\cos \lambda_{OM_2} \cos \varphi_M) - \arccos (\cos \lambda_{0A_2} \cos \varphi_{A_2}).$$

The problem of finding azimuths of the position lines for a two-pole circular system is solved analogously, with the sole difference being that the vector-gradient will not be directed perpendicular to the azimuths of the orthodromes, but along these orthodromes; accordingly, the formulas for determining the azimuths of the position lines take the form:

$$\operatorname{ctg} \alpha_1 = \frac{\operatorname{tg} \lambda_{OM_1}}{\sin \varphi_M};$$
$$\operatorname{ctg} \alpha_2 = \frac{\operatorname{tg} \lambda_{OM_2}}{\sin \varphi_M}.$$

Since the density of circular position lines $1/g$ does not depend on distance,

$$g = 1: \quad dr = dR \text{ и } \Delta r = \Delta R,$$

where ΔR is the error in measuring distance.

In goniometric range-finding systems, the task of finding the density and position of the azimuthal position lines is solved in the same way as for goniometric systems. In the case of circular position lines at point M, their direction will differ from the azimuthal lines by 90°.

The axes of the ellipses of errors in this case will coincide with the position lines. Here, at short distances from the focus of the system (without taking into account the convergence of the azimuthal position lines), the minor axis of the ellipse coincides with the position line which is determined most accurately (usually the circular line). At great distances, we must also consider the convergence of the azimuthal position lines according to (6.31).

The problem of conversion to the generalized coordinate system from the hyperbolic or hyperbolic-elliptical system is somewhat more complicated. Let us use (1.74) for this purpose: /469

$$\cos\lambda_1 = \frac{\cos S_1 \cos 2c - \cos(S_1 - 2a)}{\sin S_1 \sin 2c}.$$

Developing $\cos(S_1 - 2a)$, we can present this formula in the form (Fig. 6.8a):

$$\cos\lambda_1 = \frac{\cos S_1 \cos 2c - \cos S_1 \cos 2a - \sin S_1 \sin 2a}{\sin S_1 \sin 2c}.$$

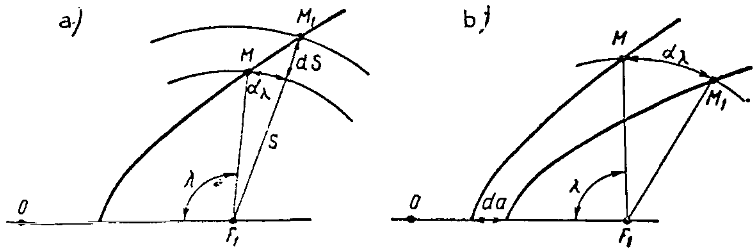


Fig. 6.8. Determining Hyperbolic Position Lines: (a) Direction; (b) Distance.

The direction of the position lines at point M can be determined after differentiating (1.74) on the basis of S:

$$\frac{d\lambda_1}{dS_1} = \frac{\cos S_1 \sin 2c\,(\cos S_1 \cos 2c - \sin S_1 \cos 2a - \sin S_1 \sin 2a)}{\sin^2 S_1 \sin^2 2c \sin\lambda_1} - \frac{\sin S_1 \sin 2c\,(\sin S_1 \cos 2c - \sin S_1 \cos 2a - \cos S_1 \sin 2a)}{\sin^2 S_1 \sin^2 2c \sin\lambda_1}. \quad (6.32)$$

The hyperbolic position line intersects the azimuth line of point M, drawn from the focus F_1, at an angle

$$\alpha = \operatorname{arctg} \frac{d\lambda_1}{dS} S. \qquad (6.33)$$

Let us determine the density of the hyperbolic position lines after differentiating (1.74) on the basis of the parameter and with a constant S (Fig. 6.8, b):

$$\frac{d\lambda_1}{da} = \frac{\sin S_1 \sin 2c (\sin S_1 \cos 2a - \cos S_1 \sin 2a)}{\sin^2 S_1 \sin^2 2c \sin \lambda_1}, \qquad (6.34)$$

Then,

$$\frac{dr}{da} = \frac{d\lambda}{da} S \cos \alpha. \qquad (6.35)$$

For conversion a generalized coordinate system from a hyperbolic-elliptical system, it is sufficient to solve the problem for hyperbolic position lines.

The directions of the elliptical position lines are then easily determined as being normal to the hyperbolic ones. The density of the elliptical lines is constant for the whole area of the activity of the system, since these lines do not diverge. /470

The reduction of special coordinate systems of radio-engineering devices to a generalized system permits the construction of ellipses of errors in determining the coordinates of an aircraft by means of these devices and the estimation of the maintenance of the aircraft path with respect to distance and direction.

In some cases, the boundaries of the operational area of the system are designated, within the limits of which the dimensions of the axes of the ellipses of errors do not exceed the given values. The operational areas of the system can be constructed on the basis of other factors (for example, on the basis of the accepted accuracy of the maintenance of the path with respect to direction alone or with respect to distance alone.

The examined means of evaluating the accuracy of the navigational measurements include, on the whole, radio-engineering devices. However, the accuracy of the calculationof an aircraft path by means of geotechnical means of aircraft navigation can be examined with an azimuthal range-finding system. The accuracy of determining aircraft coordinates by astronomical means can be examined by a circular system.

6. Evaluation of the Accuracy of Measuring a Navigational Parameter

In the preceding paragraph, the effect of the geometry of a navigational system on the accuracy of determining aircraft coordinates (assuming that the accuracy of measuring a navigational parameter is known) was examined.

By measured navigational parameter of a system, we mean the value being measured at the output of a navigational device: azimuth, course angle, distance, difference in distances, or sum of distances to objects on the ground.

In addition to measured navigational parameters, there are parameters which are determined by calculating the path of an aircraft on the basis of its speed and time components; for example, the orthodrome or geographic coordinates of the aircraft.

Let us examine briefly the reasons for errors and the methods of evaluating the accuracy of measurements or determinations of the indicated parameters.

During visual aircraft navigation with the use of geotechnical devices and with the use of astronomical and nonautonomous radio devices, the calculation of the aircraft path at each succeeding stage is carried out on the basis of the results of measuring the parameters of aircraft movement in the preceding stage. In this case, two factors will influence the accuracy of aircraft navigation:

(a) The accuracy of juncture (determination of the location of the aircraft) at the beginning and end of the control stage of the path.

(b) Wind variation at flight altitude from stage to stage.

The accuracy of visual junctures depends on flight altitude and on methods of measuring both the vertical and course angles of reference points.

Visual methods of aircraft navigation are usually used at low flight altitudes, in conjunction with closely spaced reference points. Therefore, the errors of juncture are very small, and do /471
not have values comparable to the wind variation at flight altitude.

The accuracy of measuring a navigational parameter with a radio-navigational system depends on the principle of operation, the frequency range used, the distance of the measurements, the effect of a relief or the ionizing layers of the atmosphere, and also (partially) on the physical state of the atmosphere (optical density). In a number of cases, the accuracy of measuring a navigational parameter of a radio-navigational system is evaluated statistically on the basis of the results of tests of the system under different operating conditions.

The accuracy of navigational junctures of an aircraft by means of navigational systems is evaluated by the means set forth in the preceding section, proceeding from the mean square error in measuring a navigational parameter, considering the geometry of the system in this area of application.

The accuracy of measuring a navigational parameter by astronomical means (as a rule, the altitude of a star) is determined, in the first place, by the accuracy of the installation at the level from which the measurements are carried out.

The bubble levels of aviation sextants, which are subject to constant acceleration under the effect of Coriolis forces, which can be taken into account in flight if the groundspeed is known, to a still greater degree, they can be subjected to varying accelerations by unstable flight conditions.

Gyroscopic levels are also subject to Coriolis accelerations and also to long-period fluctuations in the shape of a precession cone, connected with the errors of balancing a gyroscope, where calculation is practically impossible.

However, gyroscopic levels are free from the short-period interferences which are connected with disturbance of the aircraft's flight condition.

If, as a result of the astronomical observations, we introduce corrections for the instrument error of the sextant, the refraction of the atmosphere and the Coriolis accelerations of the level, then the errors as a result of short-period fluctuations of the bubble level or long-period (gyroscopic) fluctuations will be predominant.

The magnitude of the errors in bubble levels depends on the state of the atmosphere and the flight speed while the errors in the gyroscopic levels depend on the manufacturing accuracy of the level (precession cone).

The accuracy of astronomical measurements is determined by statistical methods. The error of measurements with bubble levels (2σ) is within the limits of 5 - 6' in a clam atmosphere at a low flight speed (up to 15 - 20' for high-speed aircraft), and at low flight speeds with the presence of an atmospheric disturbance.

The accuracy of gyroscopic levels is within the limits of 10-15'.

7. Calculation of the Wind with an Evaluation of the Accuracy of Aircraft Navigation /472

In all cases when the parameters of aircraft movement are determined on the basis of successive indications of PA or by the sighting of ground reference points at separate points, the wind variation at flight altitude with respect to the distance traversed exerts the greatest influence on the accuracy of aircraft navigation.

It is only with the use of Doppler or inertial guidance devices that the wind variation does not exert a direct influence on the accuracy of aircraft navigation, with the exception of cases when the range of cruising speeds of the aircraft does not make it possible to compensate completely for the changes of the groundspeed during flight with respect to a given groundspeed.

The accuracy of aircraft navigation at any stage in flight, with respect to distance and direction, can be expressed by the formulas:

$$\sigma_X = \sqrt{\sigma_{x_1}^2 + \sigma_w^2 t + \sigma^2 u_{x_s} t}\,; \tag{6.36}$$

$$\sigma_Z = \sqrt{\sigma_{z_1}^2 + \sigma_{\psi_1}^2 Wt + \sigma^2 u_{z_s} t}\,, \tag{6.37}$$

where σ_{x_1} and σ_{z_1} are the errors in measuring the X- and Z-coordinates of an aircraft at the beginning of a stage; σ_{w_1} and σ_{ψ_1} are the errors in determining the groundspeed and flight angle in a preceding stage; and $\sigma_{u_{xs}}$ and $\sigma_{u_{zs}}$ are the variations of the longitudinal and lateral components of the wind from one stage of the path to the next.

As has already been said, the accuracy of measuring the groundspeed and the actual flight angle depends on the accuracy of the initial and final "junctures" of the aircraft to the preceding stage; if the length of the chosen control flight stages is the same, then with the same accuracy of the initial and final "junctures" of the aircraft it can be expressed by the formulas:

$$\left.\begin{aligned} \sigma Wt &= \sqrt{2}\,\sigma X; \\ \sigma_\psi Wt &= \sqrt{2}\,\sigma Z. \end{aligned}\right\} \tag{6.38}$$

To evaluate the wind variation at flight altitude with respect to distance, the following formula is used:

$$\sigma u_s = K_s \sqrt{S}\,, \tag{6.39}$$

The coefficient of wind variation with respect to distance K_3 depends on the flight altitude and the time of year (Fig. 6.9a).

The components of the wind variation vector, on the basis of the coordinate axes, are considered equal to its modulus divided by the root of the two:

$$\left.\begin{aligned} \sigma u_{x_s} &= 0{,}7\sigma u_s; \\ \sigma u_{z_s} &= 0{,}7\sigma u_s. \end{aligned}\right\} \qquad (6.40)$$

To characterize the possible errors in aircraft navigation for a following aircraft on the basis of results of wind measurements for the aircraft ahead, the following formula is used: /473

$$\sigma u_t = K_t \sqrt{t}. \qquad (6.41)$$

where K_t is the coefficient of wind variation with respect to time.

The dependence of the coefficient K_t on the flight altitude is shown in Fig. 6.9,b.

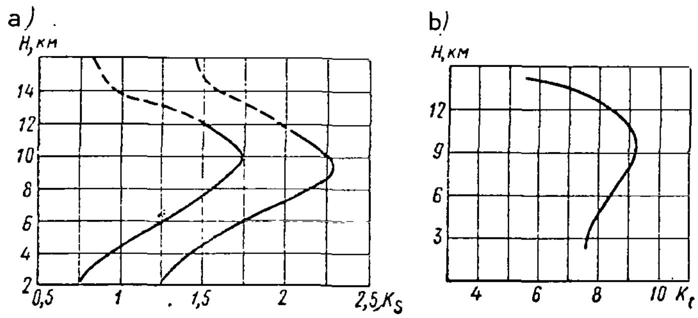


Fig. 6.9. Dependence of Wind Variation on Flight Altitude: (a) With Respect to Distance; (b) With Respect to Time.

For the case of wind variation with respect to distance, the components of wind variation along the coordinate axes are considered equal:

$$\left.\begin{aligned} \sigma u_{xt} &= 0{,}7\sigma u_t; \\ \sigma u_{zt} &= 0{,}7\sigma u_t. \end{aligned}\right\} \qquad (6.42)$$

8. Consideration of the Polar Flattening of the Earth in the Determination of Directions and Distances on the Earth's Surface

The polar flattening of the Earth is explained by its diurnal rotation.

In fact, each point of the Earth's surface is subject to the action of two forces:

(a) A force of gravitation, directed toward the center of the Earth.

(b) A centrifugal force, directed along the radius of the parallel of the point.

In Fig. 6.10, *O* is the point of suspension of a pendulum; *OD* is the vector of the force of gravitation, and DF_c is the vector of the centrifugal force.

As a result, the direction of the pendulum cord (or, in other words, the direction of the vertical of the locus) coincides with the resultant of these forces. It is natural that the plane of the true horizon must be perpendicular to the vertical of the locus, especially since the majority of the Earth's surface is covered with water, while the level of the dry land differs from sea level significantly in only a few places.

Therefore, the Earth has assumed a shape close to an ellipsoid of rotation.

The angle between the plane of the equator and the vertical of the locus is considered the *geographic latitude of the locus*. /474

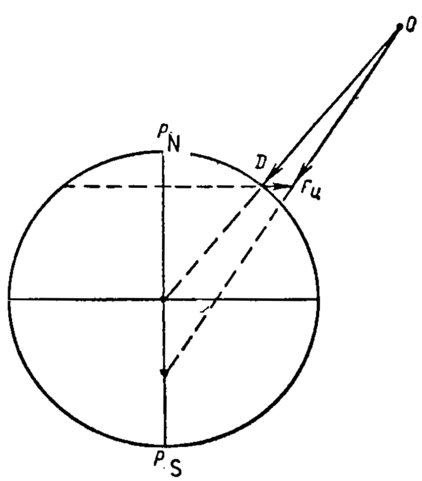


Fig. 6.10. Geographic and Geocentric Verticals.

The angle between the plane of the equator and the geocentric vertical of the locus (line connecting a point at the Earth's center with the center of the Earth) is called the *geocentric latitude of the locus* (ϕ_1).

The angle between the plane of the equator and the geocentric vertical of such a point on the Earth's surface (taken as a sphere), the radius of whose parallel is equal to the radius of the parallel of our point on the ellipsoid of rotation, is called the *corrected latitude* (u).

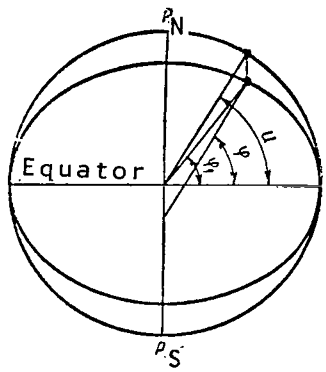


Fig. 6.11. Geographic, Geocentric and Reduced Latitudes.

The indicated latitudes are connected by the following relationships:

$$\operatorname{tg} \varphi_1 = (1 - e^2) \operatorname{tg} \varphi; \qquad (6.43)$$

$$\operatorname{tg} u = \sqrt{1 - e^2} \operatorname{tg} \varphi \qquad (6.44)$$

or

$$\operatorname{tg} u = \frac{\operatorname{tg} \varphi_1}{\sqrt{1 - e^2}}, \qquad (6.45)$$

where e is a parameter equal to $\frac{c}{a}$, c is a parameter equal to $\sqrt{a^2 - b^2}$, and a is the large semiaxis of the ellipse.

From Fig. 6.11, it is obvious that the ratio of the Earth's radius at any point to the major semiaxis of the ellipse can be expressed by the formula

$$\frac{R}{a} = \frac{\cos \operatorname{arc} \operatorname{tg} u}{\cos \varphi_1} = \frac{\cos \operatorname{arc} \operatorname{tg} \frac{\operatorname{tg} \varphi_1}{\sqrt{1 - e^2}}}{\cos \varphi_1},$$

where R is the radius of the Earth.

Since /475

$$\operatorname{arc} \operatorname{tg} \frac{\operatorname{tg} \varphi_1}{\sqrt{1 - e^2}} = \operatorname{arc} \cos \frac{1}{\sqrt{1 + \left(\frac{\operatorname{tg} \varphi_1}{\sqrt{1 - e^2}} \right)^2}}$$

Then,

$$\frac{R}{a} = \frac{1}{\cos \varphi_1 \sqrt{1 + \frac{\operatorname{tg}^2 \varphi_1}{1 - e^2}}} = \frac{\sqrt{1 - e^2}}{\sqrt{1 - e^2 \cos^2 \varphi_1}}. \qquad (6.46)$$

For the Earth's surface, the polar flattening is

$$\frac{a - b}{a} = \frac{1}{298},$$

where b is the minor semiaxis of the ellipse.

If we take the major semiaxis as equal to 1, then the minor semiaxis will be:

$$b = 1 - \frac{1}{298} = 0{,}99665.$$

Therefore, the parameter e has the value:

$$e = \frac{\sqrt{a^2 - b^2}}{a} = 0{,}006689.$$

The length of the arc of the Earth's meridian from latitude ϕ_1 to ϕ_2 will equal:

$$S = \int_{\varphi_1}^{\varphi_2} \frac{R(\varphi_1)\, d\varphi_1}{\cos(\varphi - \varphi_1)}. \tag{6.47}$$

In practice, the maximum difference $\phi - \phi_1$ can not exceed 12', therefore, the cosine in the denominator of (6.47) can be taken as one (first approximation). Then (6.47) has the form:

$$S = \int_{\varphi_1}^{\varphi_2} R(\varphi_1)\, d\varphi_1. \tag{6.47a}$$

However, according to (6.46),

$$R = a\sqrt{\frac{1 - e^1}{1 - e^2 \cos^2 \varphi_1}}.$$

Therefore, (6.47a) can be reduced to the form:

$$S \doteq a \int_{\varphi_1}^{\varphi_2} \sqrt{\frac{1 - e^2}{1 - e^2 \cos^2 \varphi_1}}\, d\varphi_1. \tag{6.47b}$$

The integral (6.47b) is not solved in final form or is solved /476
very complexly. However, considering that the value $e^2 \cos^2\phi_1 \ll 1$, with a very small error we can consider (second approximation) that

$$\frac{1-e^2}{1-e^2\cos^2\varphi_1} = 1-e^2+e^2\cos^2\varphi_1.$$

Then the integral (6.47b) takes the form:

$$S = a\int_{\varphi_1}^{\varphi_2}\sqrt{1-e^2+e^2\cos^2\varphi_1}\,d\varphi_1. \qquad (6.47c)$$

The integral (6.47c) can be further simplified if we take $1 - e^2$ as X and the value $e^2\cos^2\phi_1$ as ε

Since $\varepsilon \ll X$, it is possible to consider approximately (third approximation):

$$\sqrt{X+\varepsilon} = \frac{\varepsilon}{2}\sqrt{X}$$

or in our example

$$\sqrt{1-e^2+e^2\cos^2\varphi_1} = \frac{e^2\cos^2\varphi_1}{2}+\sqrt{1-e^2}. \qquad (6.48)$$

Taking the third approximation into account, the integral (6.47c) takes the form:

$$S = a\sqrt{1-e^2}\int_{\varphi_1}^{\varphi_2}d\varphi_1 + a\frac{e^2}{2}\int_{\varphi_1}^{\varphi_2}\cos^2\varphi_1 d\varphi_1. \qquad (6.47d)$$

The integral (6.47d) has the simple solution:

$$S = \left|a\sqrt{1-e^2}\,\varphi_1+\frac{a}{2}e^2\frac{1}{2}\left(\varphi_1+\frac{1}{2}\sin 2\varphi_1\right)\right|_{\varphi_1}^{\varphi_2}.$$

Considering that $a\sqrt{1-e^2}=b$,

$$S = \left|b\varphi_1 + ae^2\left(\frac{\varphi_1}{4}+\frac{\sin 2\varphi_1}{8}\right)\right|_{\varphi_1}^{\varphi_2}$$

or

$$S = b(\varphi_{1,2}-\varphi_{1,1}) + ae^2\frac{\varphi_{1,2}-\varphi_{1,1}}{4}+\frac{\sin 2\varphi_{1,2}-\sin 2\varphi_{1,1}}{8} \qquad (6.49)$$

Formula (6.49) is the simplest one for finding the distances along the arc of the meridian after reducing the geographic latitudes to geocentric ones.

Since the formula is obtained by three successive approximations, we must indicate the degree of its accuracy.

In the first approximation, we substituted 1 for cos $(\phi - \phi_1)$. On the Earth's surface, the difference $\phi_1 - \phi_2$ changes from zero at the pole or equator to 12' at middle latitudes. Therefore, cos $\phi_1 - \phi_1$ can not be less than 0.9,999,939, while its mean value is 0.9,999,985.

The maximum error as a result of the first approximation is equal to 0.000,006 of the measured value, while its mean error is 0.0,000,015 of its value. This gives a total of 30 m for the distance from the North to the South Pole.

The second approximation with respect to the parameter of the flattening of the Earth also results in an underestimation of the calculated value at a maximum of 0.000,002 of its value. /477

The third approximation, at a maximum of up to 0.000,011 of the measured value, causes the greatest error. However, this error (as opposed to the first two approximations) tends toward an overestimation of the value being calculated.

Thus, the maximum error of the three approximations does not exceed 0.000,001 ot the calculated value, while the mean error is 0.000,005 of its value (around 100 m for the distance from the North to the South Pole).

Let us now examine the means of calculating the flattening of the Earth with measurements of distances along the orthodrome for any flight direction.

Obviously, just as the Earth's meridian, an orthodrome drawn in any direction is an ellipse, the degree of flattening of which depends on the dip angle of the plane of the orthodrome to the plane of the equator (latitude of the vertex).

In a special case, when the latitude of the vertex is 90°, the orthodrome coincides with the meridian and its minor axis is equal to the minor axis of the ellipsoid of rotation of the Earth. When the latitude of the vertex is 0°, the orthodrome coincides with the equator and is converted to a circle.

In the general case, the minor semiaxis of the ellipse of the orthodrome is equal to the radius of the Earth at the point of its vertex, i.e., according to)6.48):

$$b_{orth} = a\left(\frac{e^2 \cos^2 \varphi_{1vert}}{2} + \sqrt{1-e^2}\right)$$

$$b_{orth} = b + \frac{ae^2 \cos^2 \varphi_{1vert}}{2}, \qquad (6.50)$$

where a = 6,378,245; b = 6,356, 863; and e^2 = 0.00669.

Having thus determined the parameter b for the orthodrome, the distance along its arc can be determined according to (6.49), by substitution as follows:

(a) parameter b for b_{orth};

(b) geocentric latitudes for distances along the orthodrome from the point of intersection with the equator expressed in angular measurement;

(c) parameter e for the value

$$e_{orth} = \frac{\sqrt{a^2 - b^2_{orth}}}{a}.$$

Since the flattening of the ellipse of an orthodrome is always less than the flattening of an ellipse of a meridian, except when they coincide, the calculation errors resulting from the three approximations introduced by us for the arc of an ellipse will be significantly less in the general case than for the arc of the meridian.

To determine directions on the Earth's surface, taking into account the flattening of the Earth, it is sufficient to convert the geographic longitudes of the starting and end points of the orthodrome segment of the path into geocentric longitudes on the basis of (6.43). The azimuth of the orthodrome at any point is then determined on the basis of those formulas, just as for the spherical shape of the Earth.

CHAPTER SEVEN

FLIGHT PREPARATION

1. Goals and Problems of Flight Preparation

Flight preparation plays an exceptionally important role in ensuring accuracy and reliability in aircraft navigation. The improvement of navigational equipment for aircraft is making higher demands on flight preparation and is also changing its nature. /478

During preparation for visual flight at low altitudes and for short distances, attention must be paid to the nature of the local relief, both from the point of view of the orientation conditions and the point of view of flight safety. In this case it is permissible to measure the necessary navigational magnitudes (angles and distances) directly on a flight chart with a scale and protractor.

Detailed study of the relief plays a significantly smaller role in automatic or semiautomatic navigation at high altitudes and airspeeds. However, the accurate calculation of navigational course magnitudes, especially flight angles and distances, becomes more important. More attention must be paid to problems of fuel consumption during flight, maneuvering in airport landing areas, etc.

Flight preparation is usually divided into two stages: preliminary and pre-flight.

All the basic and time-consuming problems are solved in the preliminary preparation which can be executed beforehand, the day before flight or earlier. For example:

(a) Selecting and preparing flight charts, plotting the route, calculating the path angles and distances, marking the charts.

(b) Studying the route, calculating a safe flight altitude.

(c) Special preparation of charts and aids for the use of radio-engineering and astronomical devices during flight.

(d) Selecting, studying and refining the maneuvering diagrams in airport areas and the operating rules for radio-engineering devices.

(e) Preliminarily calculating the fuel consumption during flight. /479

(f) Checking navigational equipment and eliminating compass deviations, etc.

If the crew has made repeated flights along the same route, most and perhaps all of the problems of preliminary preparation have been solved during a previous flight. In addition, each preceding flight is the best preparation for the subsequent one. Therefore, in cases of repeated flights the volume of preliminary preparation can be substantially reduced or excluded entirely if only a short time has elapsed since completing the previous flight and if there have been no substantial changes in routes or flight conditions and radio-engineering equipment, etc., along the route.

During pre-flight preparation, only those problems are solved which could not be solved beforehand, i.e.:

(a) Studying the meteorological situation and compiling a navigational flight plan.

(b) Refining the calculation of fuel consumption during flight and the points of closest approach to reserve airports.

(c) Introducing changes in the operating rules for radio-engineering devices and in the maneuvering diagrams, which may occur directly before flight.

(d) Surveying and inspecting the operating condition of the aircraft's navigational equipment.

2. Preparing Flight Charts and Marking the Route

The selection of a scale and the method for preparing flight charts depend on the tactical data of the aircraft (altitude, speed, type of navigational equipment).

Charts with a scale of 1:500,000 or 1:1,000,000 may be used on aircraft with low speeds and flight altitudes (speed up to 150-250 km/hr, altitude up to 2000 - 3000 m).

A scale of 1:1,000,000 is most suitable for flight speeds of 300 - 600 km/hr and in a number of cases is desirable for speeds of 800 - 1000 km/hr. However, charts with a scale of 1:2,000,000 are used mainly at flight speeds of more than 600 km/hr, especially for great flight distances, since a scale of 1:1,000,000 becomes very unwieldy for long-distance flights.

Charts must be selected on the basis of special composite tables. Charts with scales of 1:1,000,000, 1:500,000 and larger which have conventional standard international, symbols may also be selected on the basis of the coordinates of the initial, final and intermediate points of the path and according to the structural principle of the conventional symbols and the division of chart sheets.

Accurate cutting of the sheets must be done from the bottom and right of the sheet (south and east) with allowance of space for splicing from the top and left. This makes it possible to draw lines on the chart, especially with India ink. If the sheets are spliced the other way round, each of the seams will pose an obstacle to a pencil or drawing pen. /480

Charts of large areas must first be spliced by columns and not by rows. After this, the arranged columns are spliced.

After the sheets are spliced on flight charts, the route of the forthcoming flight is constructed, path angles and distances are calculated or changed, and the flight path is drawn.

As we already know (Chapter Two), the reference system of the path angles depends on the flight distance and the navigational equipment used.

For short-distance flights, path angles and distances may be measured directly on a flight chart with the use of magnetic compasses.

In addition, the path angles and distances for flights over a greater distance may be measured via given points on a chart with the use of course instruments of average accuracy, without automatic calculation of the path. However, if the magnetic latitude changes to a significant degree, during flight preparation the deviation of the magnetic compass must be calculated for each straight segment of the path, taking into account the change in the horizontal component of the Earth's magnetic field. If the straight segment of the path is very long, it must be divided into several segments not more than 1000 - 1200 km in length.

The most time-consuming and complicated process is calculating the path angles, distances, intermediate points of the path and orthodrome azimuths for long-distance flights by means of accurate course devices and automatic means of aircraft navigation.

Below we give a series of the most comprehensive systematic calculation of the orthodrome elements for a flight with the use of the above means.

It is advantageous to calculate the orthodrome elements in two stages with the use of one table which systematizes the calculation sequence at each stage.

In the first stage, the shift of the orthodrome relative to the Greenwich meridian and the azimuth of the origin at its source are determined, using the following formulas and the form of Table 7.1.

$$\operatorname{ctg} \lambda_{01} = \operatorname{tg} \varphi_2 \operatorname{ctg} \varphi_1 \operatorname{cosec} \Delta\lambda - \operatorname{ctg} \Delta\lambda;$$

$$\operatorname{tg} \alpha_0 = \frac{\sin \lambda_{01}}{\operatorname{tg} \varphi_1};$$

$$\lambda_{CM} = \lambda_{01} - \lambda,$$

where ϕ_1 and ϕ_2 represent the geographic latitudes of the initial and final points; $\Delta\lambda$ is the difference in longitudes of the initial and final points; λ_{01} is the longitude of the initial point measured from the point of intersection of the orthodrome with the Equator; λ_{cm} is the shift of the orthodrome from the Greenwich meridian (difference in geographic and orthodromic longitude); and α_0 is the azimuth of the orthodrome at its origin. /481

TABLE 7.1

Value of the arguement or function	Argument or function					
	φ_1	φ_2	$\Delta\lambda$	λ_{01}	α_0	λ_{CM}
1	2	3	4	5	6	7
Angular magnitude					—	
sin	—	—	—		—	—
cos			—	—	—	—
tg			—	—	—	—
ctg		—			—	—
cosec	—	—		—	—	—

Columns 1 through 4 of Table 7.1. are filled directly on the basis of the values of the trigonometric functions of the coordinates of the initial and final points of the orthodrome, with accuracy to the sixth decimal place.

Then the figures from the table are transferred to the formula to determine λ_{01}. The result of the solution is given in Column 5.

After this, the formula for determining α_0 is solved and the result of the solution is given in Column 6.

Finally the value of λ_{cm} is determined and written in Column 7.

It is convenient to apply the above order of solution when using a computer with a keyboard.

When using tables instead of trigonometric functions, it is more convenient to record their logarithms with the exception of the magnitudes ctg $\Delta\lambda$ and ctg λ_{01} which (with respect to the characteristics of the solution) must be represented by trigonometric values.

In the second stage, all the other special values of the orthodrome elements are determined on the basis of the formulas:

$$\mathrm{tg}\,\varphi_i = \frac{\sin\lambda_{0i}}{\mathrm{tg}\,\alpha_0}; \quad \mathrm{tg}\,\alpha_i = \frac{\mathrm{tg}\,\lambda_{0i}}{\sin\varphi_i}; \quad \cos S_{0i} = \cos\lambda_{0i}\cos\varphi_i;$$

where ϕ_i, λ_{0i}, S_{0i} are the coordinates of any point on the orthodrome (λ_{0i} and S_{0i} are measured from the point of intersection of the orthodrome with the Equator).

λ_{cm}=79′49° /482

λ_i	λ_{oi}	$\sin\lambda_{oi}$	$\cos\lambda_{oi}$	$\mathrm{tg}\,\lambda_{oi}$	$\mathrm{tg}\varphi_i$	φ_i
—70°00′	9°49′	0,17049	0,98536	0,17303	0,37057	20°20′
—60°00′	19°49′	0,33901	0,94078	0,36035	0,73683	36°23′
—51°00′	28°49′	0,48201	0,87617	0,55013	1,04764	46°20′

The initial data for solving these problems and the results of the solution are written in Table 7.2 (see the example of the solution) using data from Table 7.1

Example. Determine the orthodrome parameters for a flight from points $\lambda_1 = -70°00'$; $\phi_1 = 20°20'$to point $\lambda_2 = -51°$; $\phi_2 = 46°20'$. Calculate the intermediate point of the orthodrome for the given longitude 60°.

Solution. Proceeding from coordinates of the initial and final points of the orthodrome, let us fill in Columns 1-4 in Table 7.1.

Substituting the above values in the formula for λ_{01} and α_0, let us find the values to fill Columns 5-7.

Using the data from Table 7.1 and the formulas for Table 7.2, let us fill in the columns of this table in succession.

After filling in Table 7.2, let us write out the results from it which are necessary for executing the flight.

Knowing the elements of the orthodrome is necessary with any frame of reference of path angles and aircraft courses. If course instruments of high accuracy are used, the most advantageous frame of reference for flight angles is the system of the summation of the turn angles of the path line. In this case, the path angle of the first section is assumed to be equal to the azimuth of the orthodrome of the initial point of the route (for example, at the point of the aircraft's take-off). The path angle of each successive segment will be

$$\psi_i = \psi_{(i-1)} + TA_{(i-1)}\,.$$

The turn angles of the route are determined as the difference of the azimuths of the intersecting orthodromes at the intermediate points of the route.

The intermediate points of the orthodromes are drawn on the flight chart. On the basis of these points, the orthodrome lines of a path of great length are drawn on the flight chart.

All the information about the orthodrome route must be tabulated in a special table (Table 7.3). Some of the information must be drawn directly on the flight chart. In addition to information about the orthodrome, Table 7.3 must include data on the control reference points or other correcting points (CP) along the segments of the route.

TABLE 7.2 /483

$tg\alpha_0 = 0,460091$

$\sin\varphi_i$	$\cos\varphi_i$	$tg\alpha_1$	α_1	$\cos S$	$S°$	S км
0,34748	0,93768	0,49796	26°28′	0,92395	22°29′	—
0,59318	0,80507	0,60749	31°17′	0,75739	46°46′	2032
0,72337	0,69046	0,76050	37°15′	0,60496	52°46′	1333
					total	3365

TABLE 7.3

no. in order	TA	ψ_λ	initial point $\lambda\varphi$	final point $\lambda\varphi$	λ_{cm}	tgd_0	α init	α fin	S	control points		
										X_{CP}	Z_{CP}	$\psi_{\lambda CP}$

The orthodrome coordinates of the correcting points may be determined accurately on the basis of (1.64 and 1.65). However, taking into account the fact that the aircraft's coordinates with respect to these points are corrected at distances of not more than 300-400 km, the orthodrome coordinates of the correcting points may also be measured on the flight chart. To do this, the correcting point must be projected accurately to the path line by means of a protractor (Fig. 7.1).

It is practically always permissible to measure the $Z_{C.P.}$ coordinate on the map by means of a scale. As for the $X_{C.P.}$ coordinate, it can be measured only when it is situated not far from the turning point of the route. In other cases, it must be calculated on the basis of the formula as the distance along the orthodrome to the traverse of the correcting point.

The azimuth of the orthodrome relative to the meridian of the correcting point is determined when this point is a goniometric or goniometric-rangefinding device and the azimuth of the aircraft is measured from it.

The above angle is calculated on the basis of the coordinates of the point of intersection of the orthodrome with the meridian of the correcting point. If the flight is executed parallel to the meridian of the correcting point or at a small angle to the meridian, the azimuth of the orthodrome is first determined on the basis of the coordinates of the point of the orthodrome on the traverse of /484
the correcting point, and the convergence of the meridians between the point of the orthodrome and the correcting point is taken into account.

With long orthodrome sections of the route, it may be necessary to correct the aircraft's coordinates with respect to more than one correcting point. In this case, the section of the route is divided into two or three and sometimes more sections, on the basis of the intermediate points of the orthodrome being calculated; a separate line of Table 7.3 is filled for each section. Therefore, the turning angle of the path line at the beginning of these sections will be zero and the flight angle is common for all the sections.

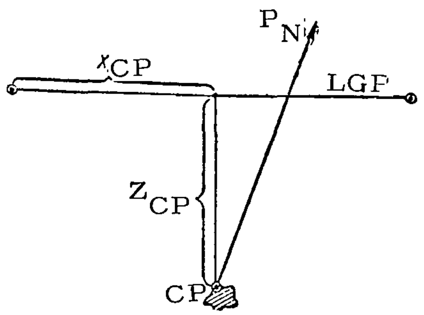


Fig. 7.1. Orthodromic Coordinates of the Correcting Point

From the information in Table 7.3, it is advisable to draw on the chart the numbers of the sections of the route, and the orthodrome flight angles and distances. In addition, it is desirable to draw on the chart the loxodromic magnetic flight angles with respect to the sections of the route and to mark the path line with dashes: normal ones with numbering every 100 km and short ones every 50 km.

Since reading coordinates in flight is done from the initial point of each straight section of the path, the marking with respect to distance must also begin from the turning point of the route independently for each section of the path in the forward and reverse directions.

3. Studying the Route and Calculating a Safe Flight Altitude

Just as in the preparation of flight charts, the procedure for studying the flight route depends on the nature of the flight to be executed and the aircraft navigation means being used.

For short-distance visual flights, the following must be carefully studied: the relief, the presence of linear and areal reference points and their additional characteristics, the procedure for re-establishing orientation when it is lost, the equipment and conditions for approaching the airports and landing areas, and sites which are suitable for a forced landing outside an airport.

The safe flight altitude in this case is calculated relative to the level of the take-off airport, separately for each section of the route. In addition, safe flight altitudes must be calculated along the sections of the route for cases when deviation from the route is necessary in order to re-establish orientation. /485

In calculating the safe flight altitude, temperature corrections for altimeter responses and the necessary altitude allowances above obstacles (established for a given type of relief by the instructions on executing flights) are taken into account.

In determining the safe flight altitude on the basis of an instrument, the following formula may be used

$$H_{\text{s instr}} = H_{\text{r}} + H_{\text{i}} - H_{\text{air}} + \Delta H_t$$

where $H_{\text{s instr}}$ is the safe flight altitude on the basis of an instrument; H_{r} is the maximum height of a relief on the flight path; H_{i} is the permissible flight altitude above the relief in accordance with the requirement of the instructions for executing flights; H_{air} is the altitude of the take-off airport above sea level; and ΔH_t represents the methodological error in measuring the altitude as a result of the discrepancy between the actual air temperature and the standard conditions.

The reduced atmospheric pressure along the sections of the route is not taken into account in this case since visual flights at low altitudes are usually executed over a small distance and the pressure changes within small limits. As regards the reduced pressure in the vicinity of the take-off airport, it is automatically taken into account by setting the pointers of the altimeters to zero before the aircraft's take-off.

The methodological errors in measuring altitude as a result of the discrepancy between the actual air temperature and the standard conditions are taken into account by means of a navigational slide rule, for example, the NS-10M.

Example. The height of the take-off airport above sea level is 270 m. The air temperature at the airport is +18°; the maximum height of the relief on the flight path is 680 m; the permissible flight altitude above the relief is 300 m and the temperature at flight altitude is +12°. Determine the safe flight altitude on the basis of an instrument.

Solution. The safe flight altitude relative to the take-off airport is

$$H_{s\ instr} = 680 + 300 - 270 + \Delta H_t = 710\ m + \Delta H_t$$

The total air temperature $t_0 + t_H = 18 + 12 = 30°$.

On the NS-10M ruler let us find the safe flight altitude on the basis of an instrument (Fig. 7.2). Answer: $H_{s\ instr} = 660$ m.

The nature of studying the flight path for aircraft navigation with instruments at average flight altitudes changes significantly. In this case, most of the attention is devoted to the use of aircraft navigational devices on the path and also for the aircraft's approach to the airport region and the approach for landing. The relief and the visual reference point conditions on the flight path are also studied in great detail.

The safe flight altitude in this case is calculated with /486
reference to standard conditions with an atmospheric pressure of 760 mm Hg, taking into account the reduced atmospheric pressure and the air temperature along sections of the path:

$$H_{s760} = H_i + H_r + (760 - p_{min})\cdot 11 + \Delta H_t,$$

where H_{s760} is the safe flight altitude on the basis of an instrument, established for a pressure of 760 mm Hg; H_i is the permissible

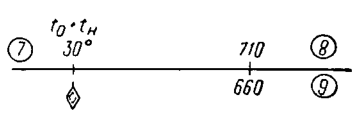


Fig. 7.2. Determining the Safe Flight Altitude by Instrument on the NS-10M.

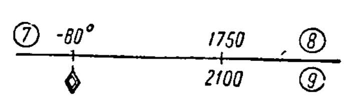


Fig. 7.3. Determining the Safe Flight Echelon on the NS-10M.

safe flight altitude with instruments in accordance with the requirements of the instructions for executing flights; and H_{min} is the minimum atmospheric pressure on a section of the flight path.

In this case, moreover, the altitude of the lowest safe flight echelon is determined, taking into account the established system of flight echeloning with respect to altitudes, i.e., the reverse problem is solved.

Example. The minimum atmospheric pressure reduced to sea level on the flight path is 750 mm Hg. The height of the relief on the path is 1300 m; the air temperature on the ground is 35°, and the air temperature at flight altitude is 45°.

Determine whether an altitude of 2100 m is a safe altitude for the lowest flight echelon if the true flight altitude above the relief must be not less than 600 m.

Solution. 1. Let us determine on the navigational slide rule, the flight altitude relative to the standard level of 760 mm Hg at H_{instr} = 2100 m, taking into account the air temperature (Fig. 7.3). Answer: 1750 m.

2. Let us determine the true flight altitude above the relief taking into account the minimum reduced atmospheric pressure $(p_{min} - 760) \cdot 11 = -110$ m and the height of the relief along the path.

$$H_{tr} = 1750 - 110 - 1300 = 340 \text{ m}.$$

Thus, H_{instr} = 2100 m will not be a safe altitude for the lowest flight echelon, since the true altitude above the relief is equal to only 340 m. The safe altitude of the lowest flight echelon in this case will be H_{instr} = 2400 m.

The study of the routes for flights on high-speed jet aircraft is of an entirely different nature.

In this case, the flight altitude is chosen exclusively for reasons of flight economy and the relative location of aircraft /487 moving in the other direction, taking into account the meteorological situation with respect to altitude.

The safe flight altitude above the relief is significant only under conditions of the aircraft's climb and descent.

For example, the possibility of overcoming obstacles during climb is determined, taking into account the airspeed, the climbing rate of the aircraft, and the counter or incidental component of the wind speed. If the reserve altitude above the relief is insufficient, part of the flight altitude must be gained after takeoff by circling above the airport.

The distance and vertical speed of descent from the given flight echelon are determined analogously, taking into account the obstacles on the approaches to the landing airport. When the vertical rate of descent is more than is permissible, the approach to the airport is executed at a certain safe altitude which is later lost in the landing approach maneuver.

In studying the flight path for jet aircraft, special attention must be focused on the possibility of using radio-engineering devices for aircraft navigation, the typical radar reference points, conditions for take-off and climb along the path, conditions of the approaches to the landing airports and maneuvering before landing.

4. Special Preparation of Charts and Aids for Using Various Navigational Devices in Flight.

The special preparation of charts and aids depends on the nature of the navigational devices which are supposed to be used in the flight.

In using goniometric and goniometric-rangefinding devices, it is sufficient to note on the chart the points where these devices are located, and the calculated directions and distances from them to the most typical points on the flight path (turning points of the path, control reference points, points of the beginning of the descent, and points of the entrance and exit corridors, etc.).

In using hyperbolic navigational systems, it is necessary to have special charts with hyperbolic position lines plotted on them. Before flight, the path of the forthcoming flight is drawn.

In using autonomous Doppler aircraft navigational systems with automatic navigational devices, several things must be prepared carefully: the flight angles and distances, as well as the orthodrome coordinates of the visual and radar reference points, the azimuth-rangefinding devices and the azimuths of the orthodromes relative to the meridians of the locations of the orthodromes.

If astronomical devices are supposed to be used in flight, /488
special astronomical tables must be prepared: detached sheets from the astronomical yearbook for the flight date, tables of azimuths and altitudes of stars, special calculating tables for recording and calculating astronomical parameters.

The navigational equipment of the aircraft must be prepared and checked accordingly.

During the inspection of this equipment, attention is focused on its general condition, efficiency, regulating parameters of currents and voltage potentials, as well as the calibration of measured parameters according to instructions on the use of specific types of equipment.

In using astronomical devices, special attention must also be focused on inspecting the aircraft's clocks and special chronometers.

5. Calculating the Distance and Duration of Flight

The distance and duration of the flight are calculated in order to determine the necessary fuel supply, as well as monitoring its consumption in flight and determining the maximum distance of the point of possible return to the take-off airport or the alternate airports when the weather is changing at the landing airport.

Depending on the types of aircraft, and primarily on the type of propulsion and altitude of the aircraft, one of three methods for calculating the distance and duration of flight is used:

(a) For aircraft with low-altitude piston engines, with respect to the hourly fuel consumption at cruising speed.

(b) For aircraft with high-altitude piston engines (with superchargers), with respect to the hourly fuel consumption under the most advantageous selected conditions of supercharging and rpm of the engines at a given altitude and airspeed.

(c) For aircraft with jet and turboprop engines, with respect to the consumption of fuel per km at different values of the flying weight of the aircraft and the altitude and speed of flight.

Calculating the Fuel Supply for Flight on Aircraft with Low-Altitude Piston Engines

On aircraft with low-altitude piston engines and fixed pitch of the propeller in flight, the control of the engines is executed only by the fuel-control lever. Flights on these aircraft are executed at low altitudes, which makes it possible in calculation not to take into account the change in fuel consumption with the /489 flight altitude. The hourly fuel consumption (Q, kg/hr) is approximately proportional to the cube of the airspeed; the fuel consumption per kilometer of airspeed (q, kg/km) is approximately proportional to the square of the airspeed.

Since the fuel consumption per km on these aircraft changes sharply depending on airspeed, taking into account the large ratio of the wind speed to the airspeed, the necessary fuel supply is customarily calculated with respect to the hourly consumption at a given airspeed.

In this case,

$$W = V_{\mathrm{tr}} + u_x; \quad t = \frac{S}{W};$$

$$G_{\mathrm{req}} = Q \cdot t + G_{\mathrm{n.r.}}$$

where W is the ground speed of the aircraft; V_{tr} is the airspeed (as a rule it is assumed to be equal to the speed on the basis of an instrument); u_x is the incidental (encountered with a minus sign) component of the wind speed; S is the distance along the route; t is the flying time; G_{req} is the necessary fuel supply; and $G_{n.r.}$ is the established navigational supply (reserve over the calculated amount).

Combining the above formulas, we obtain:

$$G_{\text{потр}} = Q \frac{S}{V + u_x} + G_{\text{н,з}}.$$

Example. The airspeed is 150 km/hr; the distance along the route is 480 km; the incidental wind component on the route is 30 km/hr; the hourly fuel consumption is 30 kg/hr which is the established navigational supply for 1 hr of flight.

Determine the required amount of fuel to execute the flight.

Solution.

$$G_{\text{потр}} = 30 \frac{480}{150 - 30} + 30 = 150 \text{ кг}.$$

It is characteristic for aircraft of this type that the additional fuel consumption due to climb is not taken into account.

Obviously, proceeding from the energy equations, the additional fuel consumption as a result of climb G_H must be:

$$G_H = qKH$$

where q is the fuel consumption per kilometer of the path in horizontal flight; K is the aerodynamic quality of the aircraft under conditions of climb $K = c_y/c_x$, where c_y is the coefficient of lift and $c_x$is the coefficient of drag); and H is the given flight altitude.

For example, in our case, if the given flight altitude is 300 m and the quality of the aircraft is 15, we will have: /490

$$G_H = \frac{30}{150} \cdot 15 \cdot 0{,}3 = 0{,}9 \text{ kg}$$

which represents the fuel consumption for 4.5 km of the airpath, or in our case, 0.6% of the total fuel supply.

However, taking into account the fact that the aircraft's descent involves a saving of fuel, though somewhat less than the

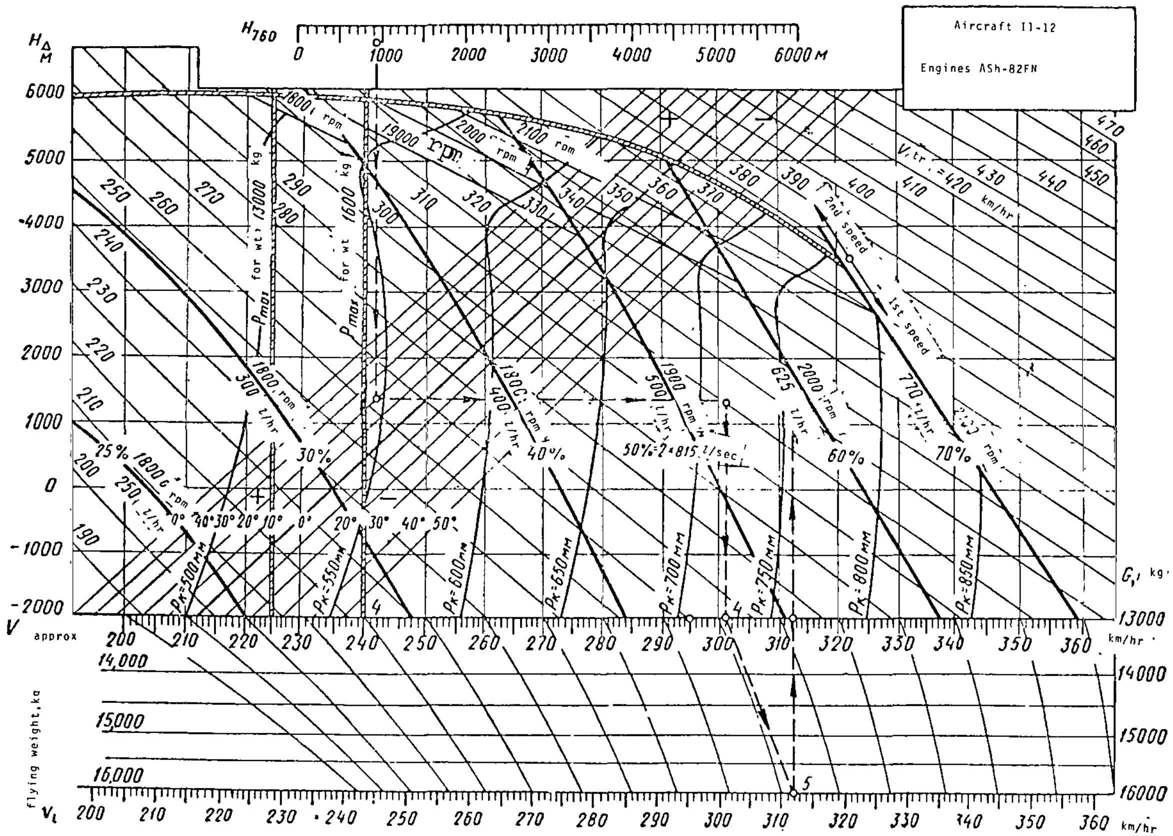


Fig. 7.4. Graph for Determining the Operating Conditions of Engines when Cruising.

loss during climb (as a result of the lower quality of the aircraft under descent conditions), the total overexpenditure of fuel under climb and descent conditions is insignificant on the whole and is disregarded.

Calculating the Fuel Supply for Flight in Aircraft with High-Altitude Piston Engines

The means of calculating the necessary fuel supply for flights on aircraft with high-altitude piston engines coincides in principle with the means for calculating it for aircraft with low-altitude engines. The distinguishing feature of the calculation is the necessity for a preliminary estimate of the most advantageous operating conditions of the engines at a given altitude and flight speed (supercharging, turns) and also a determination of the hourly fuel consumption under these conditions.

In Figure 7.4, we have provided a graph for determining the optimum operating conditions for engines on the Il-12 aircraft. Usually, in addition to graphs for each type of aircraft, tabular data for the optimum operating conditions of engines and for fuel consumption at various altitudes and flight speeds are given.

After determining the hourly fuel consumption for a given altitude and speed on the basis of the graph, the fuel supply is calculated (just as for aircraft with low-altitude engines). The additional total overconsumption of fuel due to climb and descent conditions is disregarded, since it is insignificant in this case.

Calculating the Fuel Supply for Flight on Aircraft with Gas Turbine Engines

The means for calculating the necessary fuel supply for flights on aircraft with gas turbine engines differ somewhat from the means used for aircraft with piston engines.

With an increase in the airspeed in aircraft with gas turbine engines, their operating conditions are improved (the volumetric efficiency increases). In addition, an increase in the flight /492
speed is due to the decrease in the angle of attack of the wing (and therefore the coefficient of drag c_x).

These factors are of great significance on aircraft with gas turbine engines; up to certain limits, they not only compensate for the natural increase in the fuel consumption on the basis of the square of the airspeed, but exceed it. As a result, on these aircraft (with a constant flying weight) the fuel consumption per km over the entire range of cruising speeds remains practically constant (with an accuracy of 2 to 3%).

Beyond the limits of the range of cruising speeds, especially if the maximum speed of the aircraft is close to the speed of sound, the fuel consumption per km increases sharply.

The minimum fuel consumption per km on aircraft with gas turbine engines corresponds to the mean cruising speed and increases slowly with a change in the flight conditions, both in the directions of an increase and a decrease of speed (within the limits of the cruising speed).

If we assume that the fuel consumption per km within the limits of the range of cruising speeds of the aircraft is practically constant, then the hourly fuel consumption will change in proportion to the flight speed.

The comparatively clearly expressed steadiness of the fuel consumption per km on aircraft with gas turbine engines, over the entire range of cruising speeds with a variable hourly fuel consumption, makes it necessary to calculate the fuel supply on the basis of the consumption per hour rather than per kilometer. This must be done on the basis of other considerations.

In contrast to aircraft with piston engines, the fuel supply on gas turbine aircraft represents a large part of the flying weight of the aircraft (on individual types of aircraft, it may be close to half the flying weight). This causes great changes in the flying weight and therefore in the fuel consumption with respect to the aircraft's flying time.

While it is comparatively simple to calculate the fuel on the basis of the variable (depending on flying weight) fuel consumption per km, it is very difficult to calculate it on the basis of the hourly expenditure, which varies both on the basis of the flying weight and the airspeed.

In constructing the graph of the fuel consumption per km on aircraft with gas turbine engines as a function of the altitude and airspeed, the reduced weight of an aircraft (G_{red}) which has the following physical structure is selected as the initial argument.

Let us assume that the flight is executed at speeds at which the aerodynamic drag of the aircraft satisfies the Bernoulli law, then

$$X = c_x S \frac{\rho V^2}{2};$$ /493

$$Y = c_y S \frac{\rho V^2}{2},$$

where X is the aircraft's drag; Y is the lift of the wing; c_x and c_y are the coefficients of drag and lift; S represents the cross-sectional areas of the aircraft which represent planes; and ρ is the density of the air at flight altitude.

Obviously, the horizontal flight of the aircraft at various altitudes, with the same values of flight speed and angle of attack of the aircraft (c_y = const and V = const) will be possible only when the flying weight of the aircraft and the air density are changed in equal proportion.

For example, with the given values of the speed and angle of attack, if horizontal flight near the ground is possible with a flying weight of 60 tons, then at an altitude where $\rho_H = 0.5\rho_0$, horizontal flight is possible with a flying weight of 30 tons.

However, at the flight altitude in our example, the drag is 2 times less than near the ground because the fuel consumption for each ton of flying weight remains constant.

Thus, in order to reduce the flying weight of the aircraft to standard atmospheric conditions in order to determine the fuel consumption per kilometer for each ton of flying weight (actual, not reduced) it is sufficient to multiply the actual flying weight by the proportion ρ_0/ρ_H.

In our example, the actual weight of the aircraft at an altitude is 30 tons; the ratio $\rho_0/\rho_H = 2$; therefore, $G_{red} = 30 \cdot 2 = 60$ tons.

If, with a weight of 60 tons the fuel consumption at a mean cruising speed is 0.2 kg per ton of actual flying weight, this means that near the ground the consumption per km is $0.2 \cdot 60 = 12$ kg/km, and at an altitude where $\rho_H = 0.5\,\rho_0$ it will be 6 kg/km. Therefore, for angles of attack of an aircraft which are smaller than the angle of the greatest quality, i.e., over the entire range of the aircraft's cruising speed with an increase in the reduced weight, the fuel consumption per ton of actual weight will decrease.

Actually, the coefficient c_y, with an increase in the angle of attack within the above limits, will grow much more rapidly than c_x and the significant increase in the weight of the aircraft leads to a relatively smaller increase in the drag and, therefore, in the fuel consumption per kilometer. Therefore, with an increase in flight altitude, when the reduced weight grows without an increase in the actual weight of the aircraft, the fuel consumption per km /494
decreases significantly.

In addition, the Mach number is used in constructing graphs of the fuel consumption per km on aircraft with gas turbine engines.

The method for calculating the fuel consumption on the basis of the reduced flying weight and the Mach number is very advantageous, since it permits all the characteristics of fuel consumption along the air path over the entire range of speeds, altitudes, and flying weights of the aircraft to be represented on a two-dimensional graph or table.

Without the reduced weight, a three-dimensional graph or table would have been the minimum requirement for this. Since such a graph must be volumetric rather than flat, it can not be represented on paper and it must be replaced by a series of graphs which have been calculated, for example, for each constant value of flight altitude with variable flying weights and speeds or on the basis of any other constant value with two other variable magnitudes.

This is exactly what is done in compiling operating tables for calculating the distance and duration of flight on aircraft with gas turbine engines.

Using the general graph constructed on the basis of the reduced flying weight and the Mach number as the initial generalized source /495
tables of the fuel consumption as a function of the flying weight and the flight altitude with a given air speed are compiled.

Here it is often sufficient to calculate the table of fuel consumption for one (mean) flight speed, bearing in mind that the consumption per km over the entire range of cruising speeds is almost the same as the fuel consumption at the mean speed.

In any case, for aircraft with gas turbine engines and with great range, it is sufficient to compile tables of fuel consumption for not more than three flight cruising speeds. For the other

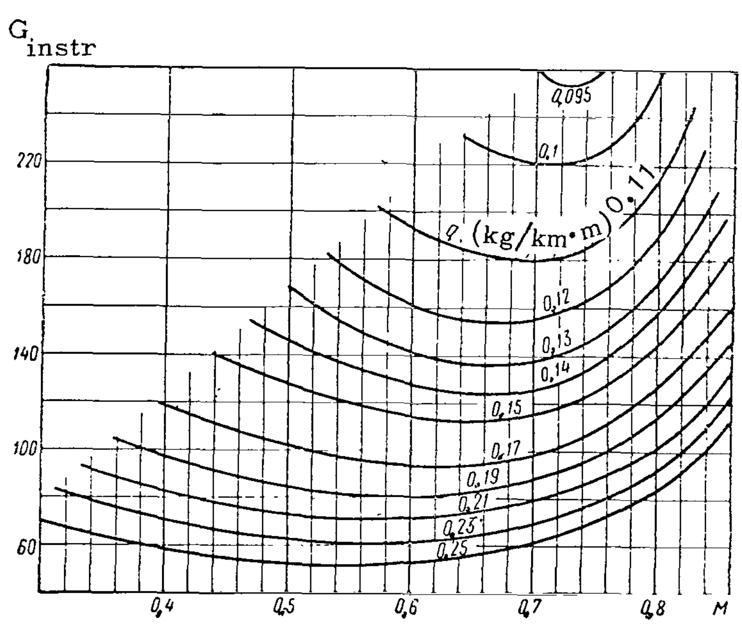


Fig. 7.5. Graph of the Fuel Consumption/km on a Jet Aircraft

speeds, the fuel consumption may be interpolated when necessary.

The fuel consumption for the given airspeed is determined in compiling the tables on the basis of the Mach number for this speed, which corresponds to the standard temperature at flight altitude. In practice, at high flight altitudes the actual air temperature differs sharply from the standard temperature (by more than 10-15°). In the above ranges of temperature change, the hourly fuel consumption changes somewhat (with an increase in temperature, it increases; with a decrease in temperature, it decreases). However, the true airspeed of flight at a constant Mach number changes in approximately the same proportion. This causes a relative steadiness in the fuel consumption per kilometer.

In Figure 7.5, we have plotted a general graph of the fuel consumption per kilometer of a civil aircraft. Tables 7.4 and 7.5 represent the total fuel consumption per flight for the same aircraft and the fuel consumption with respect to stages of the flight. We compiled these tables for a mean cruising speed of 800 km/hr and a mean flying weight of 62 tons, bearing in mind that the fuel consumption per km for an aircraft over the entire range of cruising speeds differs by not more than 2% from the mean and the flying weight at different stages of horizontal flight does not differ by not more than 5 tons.

Example. The distance along the path is 2200 km; the flight altitude is 10,000 m; the adjusted navigational fuel supply is 5.4 tons, and the wind component at the flight altitude is minus 60 km/hr.

Determine the amount of fuel necessary to execute the flight and the calculated fuel surplus at the following stages of flight: at the end of the climb, at distances from the take-off point of 800 and 1600 km, at the beginning of descent, and during the landing of the aircraft.

Solution. On the basis of Table 7.4, for a distance of 2200 km, a flight altitude of 10,000 m and a head wind, let us find the amount of fuel required (G_{req}) to execute the flight, adding to it the adjusted navigational supply of 5.4 tons: 15.5 + (0.62.2) + 5.4 = 22.14 tons.

On the basis of Table 7.5 for the same conditions, let us find:

(a) The aircraft's path during the climb (S_{cl} = 275 - 22 = 253 km).

(b) The fuel consumption during the climb (Q_{cl} = 3.95 tons).

TABLE 7.4 /496

path length km	Flight altitude, m									
	7000		8000		9000		10000		11000	
	Fuel consumption for the executing flight									
	Greq T	Δ, T	Greq Δ	Δ, T	Greq T	Δ, T	Greq T	Δ, T	Greq T	Δ, T
800	7,9	0,27	7,7	0,25	7,5	0,23	7,5	0,21	7,4	0,20
900	8,6	0,30	8,3	0,28	8,1	0,26	8,0	0,24	8,0	0,23
1000	9,3	0,34	9,0	0,31	8,7	0,29	8,6	0,27	8,5	0,26
1100	10,0	0,37	9,6	0,34	9,3	0,32	9,2	0,30	9,1	0,29
1200	10,7	0,41	10,3	0,37	9,9	0,35	9,8	0,33	9,6	0,32
1300	11,4	0,44	10,9	0,41	10,5	0,38	10,3	0,36	10,2	0,34
1400	12,1	0,47	11,5	0,44	11,1	0,41	10,9	0,39	10,7	0,37
1500	12,8	0,51	12,2	0,47	11,7	0,44	11,5	0,42	11,3	0,40
1600	13,5	0,54	12,8	0,50	12,3	0,47	12,0	0,45	11,8	0,43
1700	14,2	0,58	13,5	0,53	12,9	0,50	12,6	0,48	12,4	0,46
1800	14,8	0,61	14,1	0,57	13,5	0,53	13,2	0,50	12,9	0,48
1900	15,5	0,64	14,7	0,60	14,1	0,56	13,8	0,53	13,5	0,51
2000	16,2	0,68	15,4	0,63	14,7	0,59	14,3	0,56	14,0	0,54
2100	16,9	0,71	16,0	0,66	15,3	0,62	14,9	0,59	14,6	0,57
2200	17,6	0,75	16,7	0,69	15,9	0,65	15,5	0,62	15,1	0,60
2300	18,3	0,78	17,3	0,73	16,5	0,68	16,0	0,65	15,7	0,62
2400	19,0	0,81	17,9	0,76	17,1	0,71	16,6	0,68	16,2	0,65
2500	19,7	0,85	18,6	0,79	17,7	0,74	17,2	0,71	16,8	0,68
2600	20,4	0,88	19,2	0,82	18,3	0,77	17,8	0,74	17,3	0,71
2700	21,1	0,92	19,9	0,85	18,9	0,80	18,3	0,77	17,9	0,74
2800	21,7	0,95	20,5	0,89	19,5	0,83	18,9	0,79	18,4	0,76
2900	22,4	0,98	21,1	0,92	20,1	0,86	19,5	0,82	19,0	0,79
3000	23,1	1,02	21,8	0,95	20,7	0,89	20,0	0,85	19,5	0,82

Note. In the Table, Δ is the indicated correction for the total fuel consumption for every 30 km/hr of the wind component: favorable with a minus sign and contrary with a plus sign.

(c) Path of the aircraft during descent S_{des} = 220 - 20 = 200 km;

(d) Fuel consumption for descent Q_{des} = 0.8 tons.

The fuel consumption on the sections of horizontal flight is:

(a) From 253 to 800 km, Q = 553 (5.72 + 0.58) = 3.44 tons.

(b) From 800 to 1600 km, Q = 800 (5.72 + 0.58) = 5.04 tons;

(c) From 1600 to the beginning of descent Q = 400 (5.72 + 0.58) = 2.52 tons.

The fuel consumption for the landing approach maneuver Q_{app} = 1 ton.

Let us tabulate the result of the solution in Table 7.6.

The above calculation procedure is acceptable for aircraft with a flight duration of 3 - 3.5 hr when the fuel consumption in horizontal flight does not exceed 10 - 12 tons.

Note. In the Table, *N* is the indicated fuel consumption, path /497
traversed and flight time for a stage. In the Table, Δ is the increase in the path and the fuel consumption per km for every 30 km/hr of a head or tail wind.

For the path, the plus sign indicates a tail wind; for the fuel consumption, the plus sign indicates a head wind.

TABLE 7.5

Flight stages	Flight altitude, m									
	7000		8000		9000		10000		11000	
	N	Δ	*N*	Δ	*N*	Δ	*N*	Δ	*N*	Δ
Take-off and climb fuel consumption, tons	2,7	—	3,0	—	3,4	—	3,95	—	4,5	—
time, min	14	—	16,5	—	19	—	23	—	30	—
path, km	145	7	180	—	220	10	275	11	375	12
Horizontal flight fuel consumption kg/km	6,9	0,34	6,4	0,32	6	0,3	5,72	0,29	5,5	0,28
Descent fuel consumption, tons	0,42	—	0,55	—	0,68	—	0,8	—	0,95	—
time, min	11	—	14	—	17	—	20	—	23	—
path, km	110	5	140	6	180	8	220	10	250	12
Landing approach maneuver fuel consumption, tons	1	—	1	—	1	—	1	—	1	—

TABLE 7.6

Sections of the route	S, km	Q_{req}, T	Calculated Surplus in tons
Before take-off	—	—	22,14
Take-off and Climb	253	3,95	18,19
253 - 800 km	547	3,44	14,75
800 - 1600 km	800	5,04	9,71
1600 - beginning of descent	400	2,52	7,19
Descent	200	0,80	6,39
Landing	—	1,00	5,39

PILOT NAVIGATOR GRAPH

Flight Route Moscow-Khabarovsk, 196 g

Aircraft No: Navigator

Crew Commander Aircraft Engineer

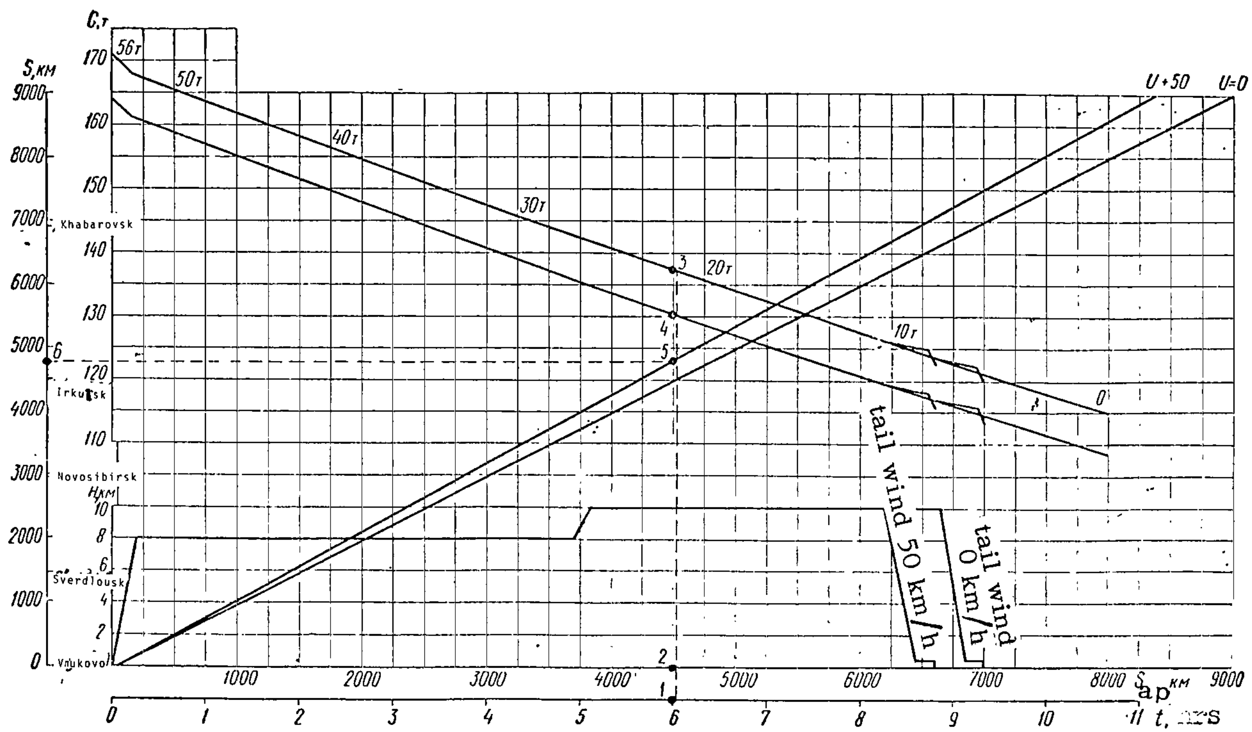


Fig. 7.6. Pilot-Navigator Graph

For aircraft operating over long distances, calculating the fuel consumption on the basis of one mean flying weight of the aircraft becomes entirely insufficient.

For such aircraft, it is advantageous to compile a pilot-navigator table for checking the fuel consumption in flight. To do this, a form of Table 7.7 is used. Data for the fuel consumption at various altitudes and airspeeds as a function of the aircraft's flying weight are entered into the table. /499

TABLE 7.7

Flight altitude, km	Limits of flying weights, T	V_{tr} 750 km/hr		V_{tr} 800 km/hr		V_{tr} 850 km/hr	
		q, kg/km	path for ΔG 10 tons	q, kg/km	path for ΔG 10 tons	q, kg/km	path for ΔG 10 tons
1	2	3	4	5	6	7	8
6	160—150 150—140 140—130 130—120 120—110						
7	160—150 150—140 140—130 130—120 120—110 etc						

Usually it is sufficient to have a table with the altitude indicated for every 1 km, and the flying weight is indicated for every 10 tons for the mean and for two other cruising speeds of the flight. This is given in the table when the weight of the aircraft changes in horizontal flight within the limits from 160 to 110 tons and the cruising speed changes from 750 - 850 km/hr.

In addition to Table 7.7, to compile a pilot-navigator table, data for the aircraft's path and the fuel consumption at stages of the climb, descent and pre-landing maneuvering are necessary.

The most convenient form of the pilot-navigator table for compilation and usage is given in Figure 7.6.

First, the curve of the change in flying weight of the aircraft along the flight path is plotted on the graph:

(a) During climb.

(b) From the moment of approaching the given altitude to the closest flying weight given in Column 2 of Table 7.7.

(c) Later, after every 10 tons of flying weight along the distance traversed, with their development in accordance with Columns 4, 6 and 8 in Table 7.7;

(d) During the time of descent and pre-landing maneuvering.

After this, a curve parallel to the curve of the change in flying weight along the flight path is plotted on the graph in such /500 a way that the fuel surplus (in multiples of 10 tons) coincides with the thickened horizontal lines of the graph grid. This creates conditions for conveniently measuring the fuel surplus, with its measurement at 10 ton intervals directly on the basis of the curve.

For example, if the flying weight of the aircraft of 140 tons corresponds to a fuel surplus equal to 36 tons, it is convenient to place the curve of fuel consumption 6 tons higher or 4 tons lower than the flying weight curve. In this case, all the points for the fuel surplus in multiples of 10 tons, coincide with the thickened lines on the graph given every 5 ot 10 tons.

In addition to the curves of the change in flying weight and fuel consumption along the flight path of the aircraft, the following have been plotted on the graph: the profile of the flight and the sloping grid of the wind at flight altitude for transferring the air path of the aircraft to the actual path, as we have shown in Figure 7.6.

For checking the fuel consumption in flight, Point 1 is marked on the time scale; sighting upward, the following can be obtained in succession: the air path traversed (2), the calculated fuel surplus (3), the calculated weight of the aircraft (4) and the point of intersection with the slope line of the mean wind (5). This point may be sighted along the horizontal to the left to point 6, showing the calculated path of the aircraft relative to the ground.

The above calculated data are compared with the actual data for the aircraft's path and for the fuel surplus. Checking the fuel consumption in flight properly includes this procedure.

Calculating the Greatest Distance of the Aircraft's Point of Closest Approach to a Reserve Airport

In order to guarantee the regularity of flights, aircraft are sometimes permitted to fly without complete assurance that they will land at the designated airport. However, such a decision is accepted only with the full guarantee that (depending on the weather conditions) the possibility of the aircraft's landing at one of the alternate airports (including the take-off airport) is reliably ensured.

In these instances, the distance of the flight to a point where it is still possible to make a final decision to land at the designated airport or to return to an alternate airport is calculated. This point is called the *point of closest approach.*

The point of closest approach must be located at the takeoff airport, at such a distance from it that the air path of the aircraft to this point and back to the takeoff airport does not exceed the maximum flight distance during a calm with the surplus adjusted navigational fuel supply on board.

Let us represent the maximum air path of the aircraft by S_{ap}, and let us assume that the aircraft will travel to the point of closest approach with a tail wind, and back again with a head wind. Without taking into account the aircraft's path during a turn to the reverse course, for calm flight conditions the following equation will hold: /501

$$S_{ap} = Vt_1 + Vt_2,$$

where V is the air speed, t_1 is the flight time up to the point of closest approach, and t_2 is the flight time in the reverse direction.

Since

$$t_1 = \frac{S_{pca}}{V + u_x}; \quad t_2 = \frac{S_{pca}}{V - u_x},$$

where $S_{p.c.a.}$ is the distance of the point of closest approach; and u_x is the wind component along the path line.
We can write the formula for S_{ap} in the form:

$$S_{ap} = S_{pca}\left(\frac{V}{V + u_x} + \frac{V}{V - u_x}\right) =$$
$$= S_{pca}\left(\frac{V^2 - u_x V + V^2 + u_x V}{V^2 - u_x^2}\right) = S_{pca}\left(\frac{2V^2}{V^2 - u_x^2}\right),$$

whence

$$S_{pca} = \frac{S_{ap}}{2}\left(\frac{V^2 - u_x^2}{V^2}\right) = \frac{S_{ap}}{2}\left(1 - \frac{u_x^2}{V^2}\right).$$

Taking into account the path of the aircraft in the turn to the reverse course (L_r), we finally obtain:

$$S_{pca} = \frac{S_{ap} - L_p}{2}\left(1 - \frac{u_x^2}{V^2}\right).$$

Obviously, the coefficient $1 - \frac{u^2_x}{V^2}$ and, therefore, the distance of the point of closest approach will be maximum at $u_x = 0$ and will diminish with an increase in the wind component along the path line, independent of its sign.

It is important to note that the distance of the point of closest approach for aircraft with gas turbine engines depends little on the airspeed within the limits of the range of cruising speeds of the aircraft, since the distance of the flight in a calm changes insignificantly. In addition, with an increase in the airspeed and with some increase in the fuel consumption per kilometer of air path, the coefficient $1 - \frac{u^2_x}{V^2}$ increases somewhat. This also stabilizes the distance of the point of closest approach.

For a specific airspeed, the coefficient $1 - \frac{u^2_x}{V^2}$ may be expressed in percent. This simplifies the calculations significantly. /502

For example, at V_{av} = 800 km/hr it will be:

u_x, km/hr	$1-\frac{u_x^2}{V^2}$, %	Multiplication factor
0	100,0	1,000
50	99,6	0,996
100	98,5	0,985
150	96,5	0,965
200	94,0	0,940
250	91,0	0,910

If the point of closest approach is not calculated for the takeoff airport but rather for an alternate airport situated on the flight path, the calculation is performed on the basis of the same formula. Instead of a completely calm distance, in this case the calm distance from the alternate airport is taken, taking into account the amount of fuel remaining until the alternate airport is reached.

6. Pre-flight Preparation and Flight Calculation

Pre-flight preparation in practice is always necessary. However, its volume will be less as more problems are solved in the preliminary preparation.

Pre-flight preparation is caused by the need to:

(a) Analyze the meteorological situation prior to takeoff;

(b) Introduce possible changes in the operating rules for radio-engineering devices or flight conditions along the air route and in the vicinity of airports.

In addition, before take-off the navigational equipment on board the aircraft must be switched on and inspected.

In studying the meteorological situation along the flight path and at the landing airport, attention must be focused on the correspondence of the actual or predicted weather with the established minimum, the distribution of the winds at flight altitude, the location of zones with dangerous weather phenomena and conditions for avoiding them, as well as the state of the weather at the alternate airports.

Calculating the flight consists of taking into account the wind distribution at flight altitude, the location of zones with dangerous meteorological phenomena, as well as taking into account the situation of the air on that day.

In calculating the flight, it is necessary to average the equivalent wind in order to determine the necessary airspeed to maintain the flight according to schedule.

The precise value of the equivalent wind (u_e) may be determined on the basis of the formula /503

$$u_{\text{э}} = u \cos \text{УВ} + \sqrt{V^2 - u^2 \sin^2 \text{УВ}} - V$$

or, on the basis of the rough formula,

$$u_{\text{э}} = u \cos \text{УВ} - \frac{u^2}{2V} \sin^2 \text{УВ}.$$

In practice, however, with a sufficient degree of accuracy, the head-wind or tail-wind component of the equivalent wind may be taken as the equivalent wind:

$$u_e \approx u \cos \text{WA}$$

Averaging the equivalent wind along the entire route is done on the basis of the formula:

$$u_{e\,av} = \frac{\sum_{i=0}^{i=n} u_{e_i} S_i}{\sum_{i=0}^{i=n} S_i},$$

The necessary constant airspeed for traveling according to schedule is then easily determined:

$$V_{\text{sched}} = W_{\text{sched}} - u_{\text{e. av.}}$$

After the necessary airspeed for flight along the path has been obtained tentatively, the elements of the flight along sections of the flight path are included in the calculation.

For each section of the path, the following must be determined:

(a) The path of travel and the airspeed, proceeding from the flight angle of a section of the path, the airspeed determined for the flight according to the schedule, and the wind at flight altitude.

(b) The flying time proceeding from the length of the section and the flight speed.

(c) The fuel surplus at the end of the section.

The results of the calculation are entered in a preliminary flight calculation table in a log on the aircraft, which has the following form:

Section of the route	Time at the beginning of the section	Flight angle	Calculated course	V	W	S	T	Fuel Surplus at the end of the stage

The flight angles and calculated travel paths are determined and recorded in accordance with a selected frame of reference (orthodromic, true or magnetic). /504

In some cases, during flight preparation the time and place of the aircraft's encountering darkness must be determined, as well as the location for encountering or overtaking other aircraft. In solving this problem, the position of a given aircraft at the moment when darkness or another aircraft passes a determined point on the route (for example, the terminal airport) is determined. The speed of the approach (as the sum or difference of the speeds) and the location of the encounter are then determined.

Example. An aircraft is flying from point *A* to point *B* at 2015 Moscow time. The mean airspeed is 350 km/hr. The distance between the points is 1100 km. Determine the time of the encounter of the aircraft with darkness if darkness reaches point *B* at 2103 and point *A* at 2218.

Solution. (a) The rate of movement of darkness from point *B* to point *A* is

$$W_d = \frac{1100}{1 \text{ hr } 15 \text{ min}} = 880 \text{ km/hr}.$$

(b) At the moment of the arrival of darkness at point *B*, the aircraft will be at the following distance from point *A*:

$$S_{instr} = W_a \, (2103 - 2015) = 350 \cdot 0.8 = 280 \text{ km}$$

and the following distance from point *B*:

$$S_{rem} = 1100 - 280 = 820 \text{ km};$$

(c) The speed of the approach of the aircraft to darkness is

$$W_{app} = 350 + 880 = 1230 \text{ km/hr}$$

(d) The aircraft's encounter with darkness occurs at:

$$T_{enc} = 2103 + \frac{820 \text{ km}}{1230 \text{ km/hr}} = 2143$$

at a distance from point *A*

$$S_{enc} = 280 + 350 \text{ km/hr} \cdot 0040 = 514 \text{ km}$$

Pre-flight preparation is completed on board the aircraft, where the following events occur: a general survey of the equipment, switching on the equipment and displaying the aircraft course, adjusting and inspecting the radio-engineering equipment, reading the pressure on the altimeters, and other operations in accordance with the manual on flight operation and the instructions on aircraft navigation for a given type of aircraft.

CHAPTER EIGHT

GENERAL PROCEDURE FOR AIRCRAFT NAVIGATION

1. General Methods of Aircraft Navigation along Air Routes

The accuracy and reliability of aircraft navigation along air routes may be ensured only when the crew correctly uses each individual type of aircraft navagational equipment and skillfully combines the operation of the entire complex of this equipment. /505

By total utilization of navigational equipment, we mean the correct combination of operating the navigational means at the disposal of the crew in order to

(1) Solve the navigational problems whose elements, measured by various navigational means, are the parent elements;

(2) Compensate for the operation of individual types of navigational equipment by means of other types of equipment;

(3) Select the most suitable means, under the given conditions, and duplicate them by other means.

When several navigational elements must be measured simultaneously (for example, when the aircraft's bearing is determined on the basis of the responses of a course instrument and a radiocompass, and the direction-finding time is read on the basis of a clock on board) the procedure for taking readings must be such as to ensure the rapid recording of the most unstable elements. In addition, the moments of recording simultaneously changing indications must be as close together as possible.

In our example, the aircraft's course must first be recorded; then it is desirable to record the course angle of the radio station with the smallest lapse of time, since these elements may change quickly and simultaneously due to the fluctuations in the longitudinal axis of the aircraft.

As regards the direction-finding time, it may be recorded after the first two elements by taking into account the approximate correction for time expended on the reading and recording of the first elements. /506

If the radiocompass has an indicator combined with the course instrument, the bearing readings do not change with fluctuations in the aircraft's longitudinal axis. In such cases, only the accuracy of measuring the time intervals between the moments of recording the bearings rather than the sequence of recording the elements is important.

Additional values obtained by calculations or on the basis of tables (the convergence of meridians, deviations of the compass and radiocompass) are recorded later, since their values for the basic recorded elements remain unchanged.

In all cases when precalculated or discrete readings are recorded (e.g., a precalculated bearing, the moment when a selected landmark passes a circular distance mark on a radar screen, etc.), the moment of time of the reading is recorded first and the measured element second, since the pilot remembers it.

Recordings of measured elements are arranged in a sequence which is most convenient for adding in navigational calculations, usually on the basis of a form of aircraft log established for aircraft-type data or on the basis of a special calculating form.

If one of the types of navigational equipment is used to correct readings by a navigational device of another type (e.g., to correct the aircraft's coordinates) which are calculated by an automatic navigational device or by means of aircraft radar, attention must be focused on the accuracy of selecting the correction location where the geometry of the solution to the problem will be most advantageous.

In our example, it is advantageous to correct the X-coordinate of the aircraft when the landmark mark located on the line of the given path passes through the circular distance mark on the radar screen, and to correct the Z-coordinate on the basis of distance marks with path bearings of landmarks 90 or 270°.

Duplicating the navigational measurements by different navigational means is obligatory in order to avoid gross errors in determining the location of the aircraft or to resolve ambiguity in the instrument readings.

In individual cases when the accuracy of measuring navigational elements by different means is approximately the same, the duplication of measurements may be useful for raising their accuracy. In these cases the mean value of the results of two measurements by two different means is assumed to be the measured value.

In addition, the duplication of navigational measurements by different means is an emergency arrangement in case of breakdowns in the operation of individual elements of the equipment on board the aircraft.

Examples. 1. In determining the location of an aircraft visually or by means of aircraft radar, the crew may commit an error in identifying a landmark. Even if the location of an aircraft is determined approximately by means of an electronic navigational indicator or by astronomical means, the probability of a gross error in determining the aircraft's position is practically excluded. /507

2. In determining the bearing of an aircraft on the basis of a fan-type beacon or a hyperbolic position line by phase measurements, the readings are ambiguous. The ambiguity is resolved by an additional determination of the sector or phase path by less accurate means.

3. The accuracy in determining the aircraft's location by means of the goniometric rangefinding system and by means of aircraft radar is the same under normal conditions. Duplicating the measurements permits an increase in the accuracy of determining the aircraft's location as a mean based on the results of two measurements.

The above examples are characteristic only of the general principles of the total utilization of aircraft navigational equipment in executing a flight.

The great variety of flight conditions, navigational equipment on air routes, and aircraft equipment has determined the various methods of their total utilization.

Specific recommendations for the total utilization of navigational means over individual sections of air routes are usually given in descriptions and instructions on executing flights along air routes.

On flights not along air routes, the procedure for using navigational devices is determined during the flight preparation after studying the navigational situation along the flight route.

2. Stages in Executing the Flight

Executing flight with the use of any aircraft navigational device may be divided into stages with the following characteristic features: (a) take-off and climb; (b) flight along the route; (c) descent and approach to the airport; and (d) maneuvering and landing approach.

The stages of climb, descent and maneuvering are executed at varying altitudes and airspeeds, and flight along the route is executed under more stable conditions.

The most complex stage for the aircraft crew is usually the maneuvering stage near the airport, combined with the landing approach, especially under complex meteorological conditions.

Despite the general features of each flight stage, the aircraft navigational procedure in them is nevertheless very different if various aircraft navigational devices are employed. /508

The principal differences in the aircraft navigational methods occur when any navigational device (geotechnical, radio-engineering and astronomical) is used without automatic measurement of the airspeed components along the coordinate axes, without calculating the aircraft's path, and in other cases when there are devices on board for solving these problems. Therefore, in describing the general procedure of aircraft navigation at different flight stages, we shall discuss its features with the application of automatic navigational devices and without them.

Take-Off and Climb

During flight preparation, the section of the path from the takeoff airport to the first control landmark on the route is plotted on the map by a straight line and the distance and direction to the landmark from the center of the airport is indicated.

However, in practice the course of the takeoff almost never coincides with the indicated flight direction. In addition, the takeoff direction changes depending on the wind direction, and the initial point of the first turn after takeoff may change the distance, depending on the takeoff conditions (pressure, temperature, speed, wind direction and aircraft flight weight). Therefore, the aircraft's approach to the first control landmark may not be executed on the basis of those rules which are used on the flight chart.

Without using automatic devices, the crew at this stage of flight is obliged to perform continuous visual orientation or to calculate the path of the aircraft in accordance with the directions of the straight lines between turns and the trajectories of the turns made to gain altitude above the airport, refining the aircraft's position by radio-engineering means.

After the maneuvering is completed, the aircraft's course to the first landmark of the flight route is taken in accordance with the aircraft's position before acquiring this course. The course is then corrected, depending on the aircraft's flight along the indicated arrival trajectory.

Before arrival at the landmark, in the case of aircraft with gas turbine engines, the necessary value of the linear lead for the turn which must be made to acquire the given route is determined.

For further flight along the route with climb, the calculated course is followed first. This course, prior to arrival at the given flight echelon, is refined two or three times by measuring the aircraft's drift angle or on the basis of successive recordings of the LA. /509

The aircraft's approach to the first landmark is simplified considerably when automatic aircraft navigational devices are used. In this case, the automatic device is adjusted for calculating the path in a coordinate system for the section of the path from the center of the airport to the first landmark. After takeoff at any point, the aircraft's coordinates are given in this system whenever possible.

After maneuvering is completed, the course to the first landmark is followed in accordance with the flight angle of the arrival coordinate system and in accordance with the angle of departure for the final point of the stage.

$$\gamma = \psi - \alpha,$$

where γ is the aircraft's course in the selected frame of reference; ψ is the flight angle of the stage in the same frame of reference; and α is the angle of departure determined on the basis of the formula

$$\alpha = \frac{Z}{S-X},$$

where Z is the lateral coordinate at the end of maneuvering; X is the longitudinal coordinate at the end of maneuvering; and S is the distance from the center of the aircraft to the landmark.

After this, the aircraft's course is refined in accordance with the drift angle of the aircraft.

Before arrival at the landmark, the Z-coordinate must gradually decrease to zero and the X-coordinate must approach S.

The turn for moving along the route is executed, taking into account the linear lead. Further flight along the route is executed with the given flight path angle, taking into account the drift angle on the basis of the readings of the Doppler meter.

Executing a Flight Along a Route

The general aircraft navigational procedure along a route without the use of automatic devices is governed by the presence of navigational devices of aircraft navigation.

In the beginning of the first stage of horizontal flight, the calculated course is followed. It is later refined on the basis of the results of measuring the drift angle or on the basis of subsequent recordings of the LA.

If the landmarks along the route are marked by radio beacons, the arrival of the aircraft at these points is noted. In the absence of such radio devices, the moment of passing a landmark is determined visually on the basis of the flight time or by means of aircraft radar.

The airspeed is determined and checked on the basis of the moments of passing the landmarks. When necessary, the airspeed for /510 maintaining the flight time along the given path is changed.

On the basis of the measured values of the drift angle and the airspeed, the speed and direction of the wind are determined and the direction of movement for the flight in the next section of the route is calculated on this basis.

Turns along the flight route (in aircraft with gas turbine engines) are executed taking the linear lead into account (LLT).

At the control points of the route where it has been specified by plan, the fuel consumption is checked by comparing the actual fuel surplus to the calculated surplus.

With a pilot-navigator chart, checking of the fuel consumption may be executed at any point on the route where it is convenient or after equal intervals of flight.

If aircraft navigation is executed by means of automatic navigational devices, the direction of movement in all cases is determined by measuring the drift angle on the basis of the Doppler meter readings; the airspeed is selected in such a way as to maintain a given flight speed along the route.

When the landmarks are passed, the aircraft's coordinates are refined and the readings of the coordinate meters are corrected. At the same time, systematic errors in the entire complex of navigational equipment occur. They are eliminated by introducing pertinent corrections into the readings of the course device.

At the turning points on the route, the frame of reference of the aircraft's coordinates is changed by establishing a new value for the flight path angle on the angle setter of the map and recalculating the aircraft's coordinates in a new frame of reference.

Checking the fuel consumption in flight, using automatic navigational devices, is done on the basis of the same rules as are employed without such devices.

In horizontal flight along the route, just as under conditions of climb and descent, the crew is obliged to follow the meteorological situation continuously and to take measures to circumvent dangerous weather phenomena which are observed visually or by means of aircraft radar.

When passing landmarks or at other points of the route, the crew transmits data on the flight conditions to the ground personnel: the measured value of the wind parameters, the air temperature, cloudiness, meteorological visibility, bumpy air, ice deposits, etc.

These data are necessary for supervising the flights, planning the takeoffs of other aircraft, and refining the situation for /511
forecasting the weather along the air routes at later times.

In approaching the calculated points for the closest possible approach of the aircraft to alternate airports, the crew must obtain data on the state of the weather at the landing airport and must make the final decision on whether to continue the flight or return to the alternate airport.

At the end of the flight route, the crew must obtain from the dispatching personnel of the landing airport, data on the conditions of the approach to the airport and the pattern of the landing approach maneuver. They must then calculate the initial point of descent from the altitude of the given flight echelon, proceeding from the given vertical speed and mean airspeed at the descent stage. The crew must also ask permission of the dispatcher (flight supervisor) for the aircraft's descent.

Descent and Entrance to the Region of the Landing Airport by an Aircraft

The distance of the initial point of descent before arrival in the vicinity of the airport is usually determined on the basis of the difference in the altitude of the flight echelon along the route and the given altitude for approaching the landing airport.

Here the descent time

$$t_{des} = \frac{H_{ech} - H_{app}}{V_y}$$

where H_{ech} is the flight altitude along the route; H_{app} is the given altitude of the approach to the airport; and V_y is the given vertical rate of the aircraft's descent.

The mean airspeed for the aircraft's descent is determined with sufficient accuracy if the tail-wind component is known at the beginning and end of the aircraft's descent stage:

$$W_{av} = V_{av} + \frac{u_{x ech} + u_{x app}}{2},$$

where V_{av} is the mean airspeed in the descent stage of the aircraft and u_x is the tail-wind component at the beginning and end of the descent.

If the tail-wind component at the end of the descent is not known by the crew, the calculation is performed on the basis of half the value of the wind vector at flight altitude, taking into account the aircraft's flight direction under descent conditions.

When the approaches to the airport are not limited by the local terrain, it is advantageous to begin the aircraft's descent 1 - 1.5 min prior to passing the calculated point in order that the given approach altitude to the airport be achieved beforehand and horizontal flight be executed at this altitude for a short length of time. /512

Otherwise, the aircraft approaches the airport at an altitude above that given. This slightly complicated the maneuvering before landing.

In the descent stage (just as during climb) in aircraft which do not have automatic navigational devices, the drift angle of the aircraft must be determined more often and the course of movement must be refined. With automatic devices, aircraft navigation in the descent stage is practically the same as aircraft navigation in horizontal flight.

We must bear in mind that on jet and turboprop aircraft, dangerous meteorological phenomena (thunderstorms, ice deposits, and violent bumps) are more often encountered under climb and descent conditions than in given echelons of horizontal flight. This requires more attention by the aircraft crew to detecting and circumventing these meteorological phenomena.

Maneuvering in the Vicinity of the Airport and the Landing Approach

The landing approach is the most complex and important flight stage, especially at airports with a mountainous relief, under complex meteorological conditions. In this stage of flight, the aircraft approaches the ground; flight is executed with a changing altitude and airspeed, with an unsteady wind. Under complex meteorological conditions, interference in radio reception is most probable. This complicates the use of radiocompasses. In addition, the landing approach is the final stage of flight and the crew is most fatigued. All this requires careful preparation by the crew in executing the aircraft's landing approach maneuver.

Before approaching the airport, the crew must obtain the necessary data for the conditions of maneuvering and the landing approach. The crew must establish altimeter pressure scales on the basis of the pressure at the landing airport level and must refresh in their memory the approach scheme specified for the given airport in accordance with the landing directions. The crew must also perform preliminary calculations for the approach.

In executing the maneuver, the crew must control the motion parameters of the aircraft by all available means and must follow the commands of the ground personnel who are controlling the execution of the landing approach. /513

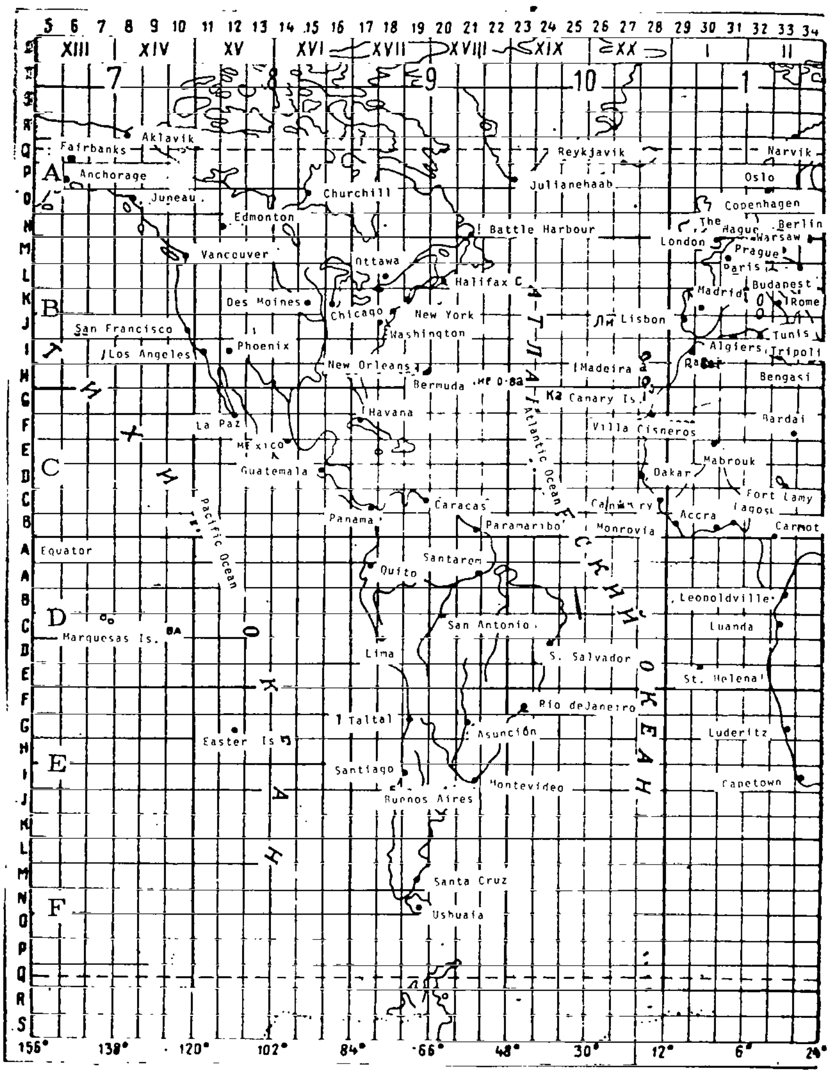
The aircraft's descent along the given trajectory on the landing path when the ground is not visible must be executed only up to the determined minimum altitude for the given airport. If the weather is lower than the established minimum and the aircraft does not transfer to visual flight at the established altitude, the crew must cease further descent and must act according to the instructions of the dispatcher (flight supervisor).

In transferring to visual flight, the crew must evaluate the available deviations from the given flight trajectory; if they do not exceed the permissible limits, the crew must visually direct the aircraft to the given trajectory and execute the landing. Otherwise, the crew must circle again and act according to the directions of the dispatcher (flight supervisor).

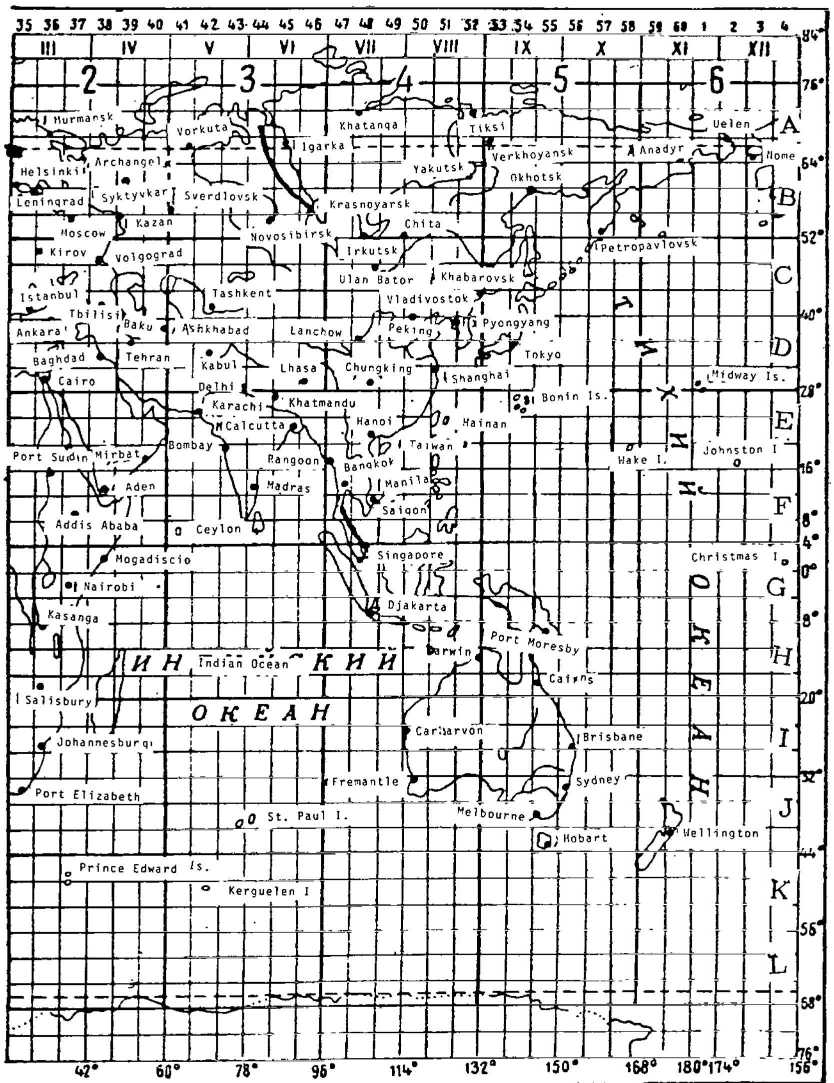
Supplement 1

Composite Chart of Topographical Maps

/514



Composite Chart of Topographical Maps (con't) /515



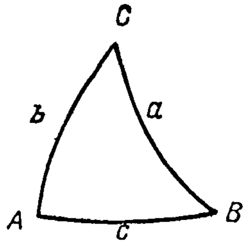
Supplement 2

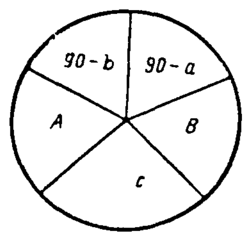
Spherical Trigonometry Formulas /516

In solving special navigational problems, especially problems of aviational astronomy, formulas of spherical trigonometry are used.

Spherical trigonometry formulas establish the interdependence between the sides and angles of a spherical triangle.

The sides of a spherical triangle a, b, c are the arcs of the central angles of a sphere α; the angles A, B, C are the surface spherical angles whose sides are the verticals from the points A, B and C. Therefore, both the sides and angles of the triangle are expressed in the same way (by angular degrees).





Formulas known as the law of cosines (the first group of formulas for spherical trigonometry) are the most widely used in navigation:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A;$$
$$\cos A = \cos B \cos C + \sin B \sin C \cos a.$$

These formulas may be written in three ways (for sides a, b, c and angles A, B, C).

The second formula of spherical trigonometry is the law of sines:

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}.$$

The third group of formulas is used much less often:

$$\sin a \cos b = \cos a \sin b \cos C + \sin c \cos B;$$
$$\sin a \operatorname{ctg} b = \operatorname{ctg} B \sin C + \cos a \cos C;$$
$$\sin A \cos B = \cos b \sin c - \cos c \sin B \cos A;$$
$$\sin A \operatorname{ctg} B = \operatorname{ctg} b \sin c - \cos c \cos A.$$

Each of these formulas may be written in six ways (in two ways for sines of the sides a, b, c and angles A, B, C).

If one of the surface angles (for example, C) is a straight line, the Napier rule is used to solve the triangle.

Five elements of the triangle (excluding the right angle C) are arranged in a circle, as shown in the Figure, thus designating the adjacent and nonadjacent elements of the triangle.

First Principle. The cosine of any element of a right triangle is equal to the product of the cotangents of the adjacent elements.

For example:

$$\cos c = \operatorname{ctg} A \operatorname{ctg} B.$$

Five such formulas may be written for elements A, $90-b$, $90-a$, B and c. /517

Second Principle. The cosine of each element is equal to the product of the sines of the nonadjacent elements.

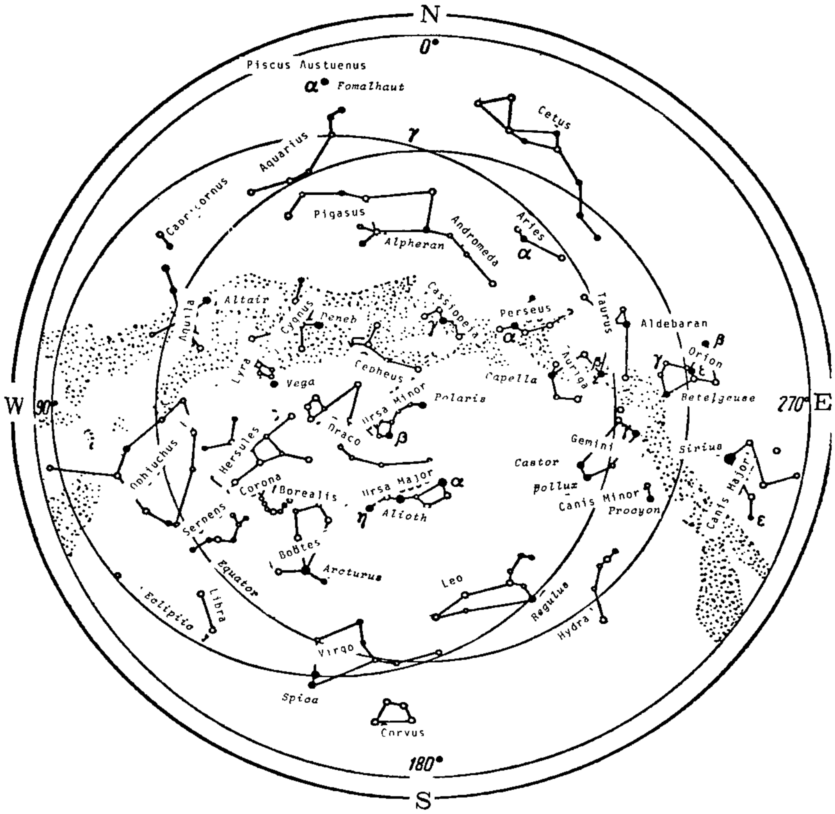
For example:

$$\cos A = \sin (90 - a) \sin B.$$

Five of these formulas may be written, one for each element.

Supplement 3

Map of the Heavens

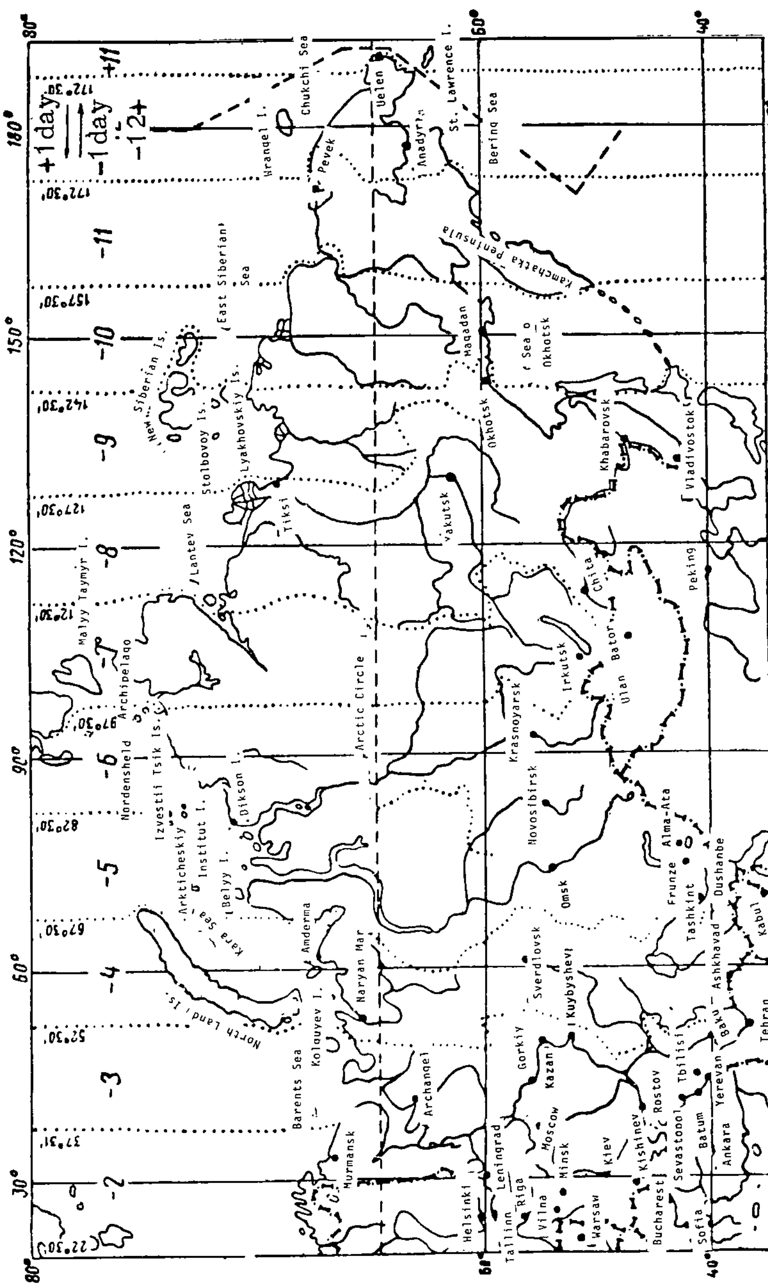


(Stars in italics)

Supplement 4

/518

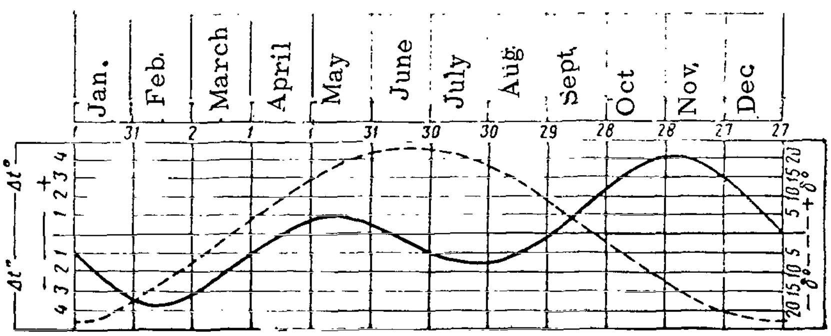
Map of Time Zones



Supplement 5

Table of Greenwich Hour Angles of the Sun and Chart of Their Corrections for the Flight Date /519

Moscow Time hr, min	Hour Angle, deg	Moscow Time hr, min	Hour Angle, deg	Moscow Time, hr, min	Hour Angle deg	Moscow Time, hr, min	Hour Angle deg
0.00	135	6.00	225	12.00	315	18.00	45
12	138	12	228	12	318	12	48
24	141	24	231	24	321	24	51
36	144	36	234	36	324	36	54
48	147	48	237	48	327	48	57
1.00	150	7.00	240	13.00	330	19.00	60
12	153	12	243	12	333	12	63
24	156	24	246	24	336	24	66
36	159	36	249	36	339	36	69
48	162	48	252	48	342	48	72
2.00	165	8.00	255	14.00	345	20.00	75
12	168	12	258	12	348	12	78
24	171	24	261	24	351	24	81
36	174	36	264	36	354	36	84
48	177	48	267	48	357	48	87
3.00	180	9.00	270	15.00	360	21.00	90
12	183	12	273	12	03	12	93
24	186	24	276	24	06	24	96
36	189	36	279	36	09	36	99
48	192	48	282	48	12	48	102
4.00	195	10.00	285	16.00	15	22.00	105
12	198	12	288	12	18	12	108
24	201	24	291	24	21	24	111
36	204	36	294	36	24	36	114
48	207	48	297	48	27	48	117
5,00	210	11.00	300	17.00	30	23.00	120
12	213	12	303	12	33	12	123
24	216	24	306	24	36	24	126
36	219	36	309	36	39	36	129
48	222	48	312	48	42	48	132



Supplement 6

Table of Values of the Function $\Phi(x - a)$ /52(

$x - a$	$\Phi(x - a)$	$x - a$	$\Phi(x - a)$	$x - a$	$\Phi(x - a)$	$x - a$	$\Phi(x - a)$
0,00	0,0000						
0,01	0,0040	0,66	0,2454	1,31	0,4049	1,96	0,4750
0,02	0,0080	0,67	0,2486	1,32	0,4066	1,97	0,4756
0,03	0,0120	0,68	0,2517	1,33	0,4082	1,98	0,4761
0,04	0,0160	0,69	0,2549	1,34	0,4099	1,99	0,4767
0,05	0,0199	0,70	0,2580	1,35	0,4115	2,00	0,4772
0,06	0,0239	0,71	0,2611	1,36	0,4131	2,02	0,4783
0,07	0,0279	0,72	0,2642	1,37	0,4147	2,04	0,4793
0,08	0,0319	0,73	0,2673	1,38	0,4162	2,06	0,4803
0,09	0,0359	0,74	0,2703	1,39	0,4177	2,08	0,4812
0,10	0,0398	0,75	0,2734	1,40	0,4192	2,10	0,4821
0,11	0,0438	0,76	0,2764	1,41	0,4207	2,12	0,4830
0,12	0,0478	0,77	0,2794	1,42	0,4222	2,14	0,4838
0,13	0,0517	0,78	0,2823	1,43	0,4236	2,16	0,4846
0,14	0,0557	0,79	0,2852	1,44	0,4251	2,18	0,4854
0,15	0,0596	0,80	0,2881	1,45	0,4265	2,20	0,4861
0,16	0,0636	0,81	0,2910	1,46	0,4279	2,22	0,4868
0,17	0,0675	0,82	0,2939	1,47	0,4292	2,24	0,4875
0,18	0,0714	0,83	0,2967	1,48	0,4306	2,26	0,4881
0,19	0,0753	0,84	0,2995	1,49	0,4319	2,28	0,4887
0,20	0,0793	0,85	0,3023	1,50	0,4332	2,30	0,4893
0,21	0,0832	0,86	0,3051	1,51	0,4345	2,32	0,4898
0,22	0,0871	0,87	0,3078	1,52	0,4357	2,34	0,4904
0,23	0,0910	0,88	0,3106	1,53	0,4370	2,36	0,4909
0,24	0,0948	0,89	0,3133	1,54	0,4382	2,38	0,4913
0,25	0,0987	0,90	0,3159	1,55	0,4394	2,40	0,4918
0,26	0,1026	0,91	0,3186	1,56	0,4406	2,42	0,4922
0,27	0,1064	0,92	0,3212	1,57	0,4418	2,44	0,4927
0,28	0,1103	0,93	0,3238	1,58	0,4429	2,46	0,4931
0,29	0,1141	0,94	0,3264	1,59	0,4441	2,48	0,4934
0,30	0,1179	0,95	0,3289	1,60	0,4452	2,50	0,4938
0,31	0,1217	0,96	0,3315	1,61	0,4463	2,52	0,4941
0,32	0,1255	0,97	0,3340	1,62	0,4474	2,54	0,4945
0,33	0,1293	0,98	0,3365	1,63	0,4484	2,56	0,4948
0,34	0,1331	0,99	0,3389	1,64	0,4495	2,58	0,4951
0,35	0,1368	1,00	0,3413	1,65	0,4505	2,60	0,4953
0,36	0,1406	1,01	0,3438	1,66	0,4515	2,62	0,4956
0,37	0,1443	1,02	0,3461	1,67	0,4525	2,64	0,4959
0,38	0,1480	1,03	0,3485	1,68	0,4535	2,66	0,4961
0,39	0,1517	1,04	0,3508	1,69	0,4545	2,68	0,4963
0,40	0,1554	1,05	0,3531	1,70	0,4554	2,70	0,4965

Table of Values of the Function Φ $(x-a)$, (con't)

/521

$x-a$	$\Phi(x-a)$	$x-a$	$\Phi(x-a)$	$x-a$	$\Phi(x-a)$	$x-a$	$\Phi(x-a)$
0,41	0,1591	1,06	0,3554	1,71	0,4564	2,72	0,4967
0,42	0,1628	1,07	0,3577	1,72	0,4573	2,74	0,4969
0,43	0,1664	1,08	0,3599	1,73	0,4582	2,76	0,4971
0,44	0,1700	1,09	0,3621	1,74	0,4591	2,78	0,4973
0,45	0,1736	1,10	0,3643	1,75	0,4599	2,80	0,4974
0,46	0,1772	1,11	0,3665	1,76	0,4608	2,82	0,4976
0,47	0,1808	1,12	0,3686	1,77	0,4616	2,84	0,4977
0,48	0,1844	1,13	0,3708	1,78	0,4625	2,86	0,4979
0,49	0,1879	1,14	0,3729	1,79	0,4633	2,88	0,4980
0,50	0,1915	1,15	0,3749	1,80	0,4641	2,90	0,4981
0,51	0,1950	1,16	0,3770	1,81	0,4649	2,92	0,4982
0,52	0,1985	1,17	0,3790	1,82	0,4656	2,94	0,4984
0,53	0,2019	1,18	0,3810	1,83	0,4664	2,96	0,4985
0,54	0,2054	1,19	0,3830	1,84	0,4671	2,98	0,4986
0,55	0,2088	1,20	0,3849	1,85	0,4678	3,00	0,49865
0,56	0,2123	1,21	0,3869	1,86	0,4686	3,20	0,49931
0,57	0,2157	1,22	0,3888	1,87	0,4693	3,40	0,49966
0,58	0,2190	1,23	0,3907	1,88	0,4699	3,60	0,499841
0,59	0,2224	1,24	0,3925	1,89	0,4706	3,80	0,499928
0,60	0,2257	1,25	0,3944	1,90	0,4713	4,00	0,499968
0,61	0,2291	1,26	0,3962	1,91	0,4719	4,50	0,499997
0,62	0,2324	1,27	0,3980	1,92	0,4726	5,00	0,49999997
0,63	0,2357	1,28	0,3997	1,93	0,4732		
0,64	0,2389	1,29	0,4015	1,94	0,4738		
0,65	0,2422	1,30	0,4032	1,95	0,4744		

Supplement 7

Units often Encountered in Aircraft Navigation and Their Values

Symbol of the unit	Geometric or Physical Meaning	Numerical Value and Dimension
π	Ratio of the circumference of a circle to its diameter	3.1416
Radian	Length of an arc of a circle, equal to the radius of the circle	57.3°
1°	Angular unit or length of arc	0.017453 rad
1′	" " " " " "	1/60° 0.000291 rad
1″	" " " " " "	1/60′, 0.000005 rad
1°	The mean value of the length of the arc of a great circle on the Earth's surface at 1°	111.1 km
1′	Length of the arc of a great circle (nautical mile)	1.8523 km /522
1″	Length of the arc of a great circle	30.9 m
t°C	Temperature on the Celsius scale	-
T°K	Absolute temperature (Kelvin scale)	t°C + 273°
R	Radius of the Earth's ellipsoid reduced to a sphere	6371 km
a	Semimajor axis of the Earth's ellipsoid	6378.245 km
b	Semiminor axis of the Earth's ellipsoid	6356.863 km
e	Eccentricity of the ellipse formed by the intersection of the Earth by a plane of the meridian	0.006689
e	Napier number (foundation of natural logarithms)	2.718282
M	Modulus of the transfer from natural logarithms to common logarithms	0.43429

Symbol of the unit	Geometric or Physical Meaning	Numerical Value and Dimension
$M_1 = \frac{1}{M}$	Modulus of the transfer from common logarithms to natural logarithms	2.30259
B	Characteristic gas constant for air	29.27 m/deg
κ	Specific heat ratio for air C_p/C_V	1.4
ρ	Mass density of the air near the ground under standard atmospheric conditions	0.125 kg·sec^2/m^4
γ	Weight density of the air near the ground under standard atmospheric conditions	1.226 kg/m^3
t_{gr}	Temperature gradient to an altitude of 11,000 m (standard atmospheric conditions)	6.5 deg/km
a	Speed of sound near the ground under standard atmospheric conditions	340.2 m/sec
g	Acceleration of gravity	9.81 m/sec^2
1 mm Hg	Unit of measurement of atmospheric pressure	1.36 g/cm

CONVERSION FROM ENGLISH AND AMERICAN UNITS TO METRIC UNITS

Statue mile	1.609 km
Foot	0.3048 m
Inch, Hg	254 mm Hg
Millibar	0.7501 mm Hg
Gallon (Imperial)	4.546 l
Gallon (American)	3.785 l

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